Homework Assignment 4 – 600.445 Fall 2012

Instructions and Score Sheet (hand in with answers)

<table>
<thead>
<tr>
<th>Name</th>
<th>Email</th>
<th>Other contact information (optional)</th>
<th>Signature (required)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>I have followed the rules in completing this assignment</td>
</tr>
</tbody>
</table>

Name
Email
Other contact information (optional)
Signature (required) I have followed the rules in completing this assignment

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. Remember that this is a graded homework assignment. It is effectively an exam.

2. You are to work alone or in a team of two people and are not to discuss the problems with anyone other than the TAs or the instructor.

3. Put your names and email address on each sheet and number the sheets.

4. You are encouraged to make free use of any published materials, the web, etc. in developing your answer but a) you must give full and proper citations to any references consulted and b) you may not consult, discuss, or otherwise communicate about this assignment with any human being except your lab partner, the course instructor, or the TAs. The one exception is that you should not refer to previous years' homework.

5. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.

6. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.

7. Sign and hand in the score sheet as the first sheet of your assignment

8. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment.

9. Attach the grade sheet as the first sheet and attach all sheets together.

10. You must include a self-addressed, seal-able 8 ½ x 11 inch envelope if you expect to the homework to be returned (per JHU’s interpretation of FERPA).
Scenario

Figure 1: Pre-drilled pelvis fixation plate holes

Consider the situation shown in Figure 1 (left). The patient has a severely broken bone (here, a pelvis). The goal is to reduce the fracture (i.e., realign the bone fragments) and bolt them together using screws and plates as shown in Figure 1 (right). The normal procedure goes roughly like this:

1. Reduce the fracture.
2. Align the plates with the bone fragments.
3. Drill pilot holes using the plates as a drill template.
4. Insert bone screws through the plates to secure the bones together.

However, we will here consider an alternative procedure, based on CT-based planning. The preoperative sequence is roughly as follows:

1. Obtain a CT scan of the patient, and segment the scan to identify the bone fragments. Let $F_1$ and $F_2$ be the coordinate systems associated with each bone fragment.
2. In the planning system, determine how bone fragment 1 needs to be repositioned relative to bone fragment 2 so that the fracture is reduced. I.e., determine a coordinate transformation $F_{red}$ such that $F_1 = F_{red} F_2$ when the fracture is reduced.
3. Determine the design of plates to hold the bone fragments in this relative pose. Determine the locations $\mathbf{h}_{1k}$ and $\mathbf{h}_{2k}$ relative to $F_1$ and $F_2$ of holes to be pre-drilled into the bones to hold the plates (and, hence, the bone fragments) in the right pose.
4. Prefabricate the plates.

In the operating room, the bone fragments are to be registered to the preoperative plan, the holes are to be drilled in the desired places, and the plates are to be attached to reduce the fracture. For the purpose of this exercise, we may assume that the procedure will be successful if the holes are drilled within approximately 2 mm of the planned locations, though higher accuracy may in some cases be desirable.
Your problem is to design a system to perform the intraoperative portion of this procedure. Very broadly, the steps are:

1. Set up the needed equipment and perform any needed calibrations
2. Set up the patient (e.g., attach reference markers to patient [if needed], etc.)
3. Perform the registration of the intraoperative system to the preoperative plan
4. Perform the drilling of the holes (or assist the surgeon in drilling them)
5. Attach the plates & screws

Each of these steps may be performed in one of several ways, depending on the equipment and algorithmic approach chosen.
Available apparatus and software

We assume that the following are available for designing the system:

- A computer workstation.
- A surgical drill and other surgical tools
- An optical navigational tracking system interfaced to the workstation.
- A mobile x-ray c-arm interfaced to the workstation. The detector on the x-ray system is a flat panel with no distortion. The x-ray source is located at position \( \mathbf{s} = [0,0,1000] \) relative to the coordinate system \( \mathbf{F}_D \) of the detector.
- A large number (as many as you want) of markers that can be detected by the optical tracking system and by the x-ray system. The optical tracker can determine the position of any marker to an absolute accuracy of 0.3 mm over a work volume of 1.5 meters cubed. The workstation has image processing software for determining the position of the projected image of any marker. If a marker has position \( \mathbf{v} = [x,y,z] \) relative to \( \mathbf{F}_D \) then the projected image will be at

Figure 2: Available Equipment
\[ \bar{u} = \text{spotfind}(\text{proj}(\hat{s}, \hat{v})) = \frac{1000}{1000 - z} [x, y, 0] \] in the coordinate system of the detector.

Further, the value of \( u \) will be accurate to 0.2 mm. I.e., \( \| \Delta \bar{u} \| \leq 0.2 \text{ mm} \).

- The workstation also has available software for solving a single image 2D-3D registration problem. In particular, if \( \text{Image}_{\text{xray}} \) is an x-ray image of the patient, we have a function \( \text{F}_{\text{CD}} = \text{Register}(\text{Image}_{\text{xray}}, \text{Image}_{\text{CT}}) \) such that if a point \( \hat{c} \) is defined in CT coordinates, then the corresponding 3D point relative to the detector is \( \bar{d} = \text{F}_{\text{CD}} \cdot \hat{c} \). The uncertainty in this registration is \( \Delta \text{F}_{\text{CD}} \approx [I + sk(\alpha_{\text{CD}}, \varepsilon_{\text{CD}})] \). In this case, the uncertainty is given in detector coordinates, so that \( \text{F}^*_{\text{CD}} = \Delta \text{F}_{\text{CD}} \cdot \text{F}_{\text{CD}} \) and thus \( \Delta \bar{d} \approx \alpha_{\text{CD}} \times \bar{d} + \varepsilon_{\text{CD}} \). For this algorithm, we know

\[
\begin{align*}
\left| \alpha_{\text{CD}} \right| & \leq [0.0001, 0.0001, 0.00002] \text{ radians} \\
\left| \varepsilon_{\text{CD}} \right| & \leq [0.5, 0.5, 7.5] \text{ mm}
\end{align*}
\]

**Note:** The \( \alpha_{\text{CD}} \) and \( \varepsilon_{\text{CD}} \) values are not very realistic, but were chosen to make the problem easier.

- The markers may be attached to any surface (including the patient), but may be attached to the patient only during the actual procedure (i.e., after the CT scan is made). They may be permanently attached to any operating room equipment or fabricated equipment. The markers may be assumed to remain in fixed positions after being attached to equipment or other objects, but the *exact* positions cannot be controlled by the design of the things they are attached to. They can, of course, be measured using the optical tracker.

- There is a workshop able to fabricate any brackets, calibration fixtures, or other similar simple apparatus. The materials may include aluminum, steel, titanium, and various radiolucent, autoclavable plastics.
**The actual assignment**

As mentioned above, you are to design a system to enable the surgeon carry out the intraoperative procedure (up to drilling the holes for the plates), together with the workflow and procedural steps, using the apparatus provided, together with such simple fixtures as you may design (e.g., an optical tracking “rigid body” with multiple markers to attach to the drill). There is no single correct solution to this problem using the equipment provided, but it is important that you explain clearly what your solution is and justify your approach.

Again, the goal is to get the plate holes drilled to within 2 mm of the desired positions on the bone fragments.

1. Describe the system setup for this procedure, including sketches and verbal descriptions for
   - Any brackets or other fabricated apparatus, with rough dimensions showing where any markers are to be attached.
   - Description of where you will attach any other markers (on patient, on x-ray system, OR table, or wherever)
   - Description of any calibration procedures that must be done, and when they should be done.
   - Description of the approximate positioning of apparatus relative to the patient.

Here, we are not seeking beauty or actual mechanical design drawings. Rather, the goal is simply to provide enough information to enable one of ordinary skill in the art to implement your design. Also, the focus is on basic design, so things like material choice is beside the point, though you may want to say if something needs to be transparent to x-rays.

2. Describe the step-by-step procedural flow, giving a clear statement of what is to be done at each step, what information is to be sensed, what is to be computed, and how this information is to be used. Here, you should also clearly define any variables that you may need to use in Parts 3 and 4 (below).

3. For any steps in Part 2 that involve computations, give sufficient formulas so that it is clear how the computations are to be done. Here, you can use the names of known methods (e.g., Arun’s or Horn’s method for point cloud to point cloud registration) with suitable citations to papers or lecture notes. But the formulations should make very clear what the inputs and outputs are in terms of your defined variables.

4. Conduct an analysis to demonstrate that your proposed method will produce the desired accuracy. **Hint:** performing this analysis may be a factor in determining some of the key dimensions in Part 1.

5. Suppose that a generous billionaire has now given you the funds to implement your system. Outline the steps that you would take to get this system into clinical use. Give rough estimates for the amount of effort needed for each step and roughly how long each step will require.

**Hint:** there may be some intraoperative motion of the bone fragments relative to things in the room. We discussed ways to deal with this in class.