Homework Assignment 4 – 600.445/645 Fall 2015

Instructions and Score Sheet (hand in with answers)

Name

Email

Other contact information (optional)

Signature (required)  I have followed the rules in completing this assignment

Name

Email

Other contact information (optional)

Signature (required)  I have followed the rules in completing this assignment

Please indicate whether you are taking 600.445 or 600.645 (Circle one)

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1. Remember that this is a **graded** homework assignment. It is effectively an exam.

2. You are to work alone or in a team of two people and are not to discuss the problems with anyone other than the TAs or the instructor.

3. Put your names and email address on each sheet and number the sheets.

4. You are encouraged to make free use of any published materials, the web, etc. in developing your answer but a) you must give full and proper citations to any references consulted and b) you may not consult, discuss, or otherwise communicate about this assignment with any human being except your lab partner, the course instructor, or the TAs. The one exception is that you should not refer to previous years’ homework.

5. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.

6. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.

7. Sign and hand in the score sheet as the first sheet of your assignment.

8. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment.

9. Attach the grade sheet as the first sheet and attach all sheets together.

10. You must include a self-addressed, seal-able 8 ½ x 11 inch envelope if you expect to the homework to be returned (per JHU’s interpretation of FERPA).
Scenario

Consider the robotically-assisted osteotomy system shown in Fig. 1. This system has a workstation (not shown), an optical tracking system, and a robot equipped with two force/torque (F/T) sensors. One of these sensors is attached to a handle, which, in turn is attached to the tooling attachment plate of the robot. When the human user exerts forces or torques on this handle, the F/T sensor senses these values and the workstation computes a corresponding F/T vector $\eta_h = [f_h, \tau_h]$ resolved in the coordinate system of the robot’s tooling attachment plate.

The robot has another F/T sensor that is also attached to the tooling plate and supports a tool holder, to which a bone forceps is attached. When forces or torques are applied to the tool, this sensor measures them and the workstation computes an F/T vector $\eta_g = [f_g, \tau_g]$, also resolved in the coordinate system of the robot’s tooling attachment plate.

We will adopt the notation $\xi = [\alpha, \epsilon]^T$ to indicate a set of small orientation and position variables. We will use $\Delta F(\xi) \approx [I + sk(\alpha), \epsilon]$ to indicate the corresponding pose change.
The workstation is able to read the joint values \( \ddot{\mathbf{q}}(t) \) of the robot and has a function \( \mathbf{F}_{\text{kine}}(\ddot{\mathbf{q}}) \) that computes the pose of the tooling plate relative to the base coordinate system of the robot. The workstation also has a function

\[
\mathbf{J}_{\text{kine}}(\ddot{\mathbf{q}}) = \begin{bmatrix}
\mathbf{J}_\alpha(\ddot{\mathbf{q}}) \\
\mathbf{J}_\varepsilon(\ddot{\mathbf{q}})
\end{bmatrix}
\]

such that for small changes \( \Delta \ddot{\mathbf{q}} \), the corresponding pose of the robot’s tooling plate pose is given by \( \mathbf{F}_{\text{kine}}(\ddot{\mathbf{q}} + \Delta \ddot{\mathbf{q}}) = \Delta \mathbf{F}_{\text{kine}} \mathbf{F}_{\text{kine}}(\ddot{\mathbf{q}}) \), where \( \Delta \mathbf{F}_{\text{kine}} \approx [\mathbf{I} + sk(\dot{\mathbf{\xi}}_{\text{kine}}), \dot{\mathbf{\xi}}_{\text{kine}}^T] \) and

\[
\ddot{\mathbf{\xi}}_{\text{kine}} = \begin{bmatrix}
\dot{\alpha}_{\text{kine}} \\
\dot{\varepsilon}_{\text{kine}}
\end{bmatrix} = \mathbf{J}_{\text{kine}}(\ddot{\mathbf{q}}) \Delta \ddot{\mathbf{q}} = \begin{bmatrix}
\mathbf{J}_\alpha(\ddot{\mathbf{q}}) \\
\mathbf{J}_\varepsilon(\ddot{\mathbf{q}})
\end{bmatrix} \Delta \ddot{\mathbf{q}}
\]

There is also an optical tracking system interfaced to the robot. A calibration step has been performed, so that the transformation between optical tracker and robot coordinates is given by \( \mathbf{F}_0 = [\mathbf{R}_0, \mathbf{p}_0] \). There are two “tracker bodies” attached to the patient. Tracker Body 1 is attached to the main part of the skull and Tracker Body 2 is attached to a bone fragment held by the forceps tool. The tracking system returns the poses of these two bodies as \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \), respectively.

The robot’s mid-level motion control runs at a regular clock period of \( \Delta t \). Thus, an incremental joint motion of \( \Delta \ddot{\mathbf{q}} \) corresponds to joint velocities of

\[
\dot{\mathbf{q}} = \frac{\Delta \ddot{\mathbf{q}}}{\Delta t}
\]

**Note:** We sometimes describe haptic interfaces in which the human pushes on the robot and the robot moves accordingly as “admittance-type” interfaces. Similarly, we refer to interfaces where the robot pushes back on the human in response to motion by the human as “impedance-type” interfaces. The “steady hand” robot virtual fixtures described in class were of the admittance type.
Fig. 2: Steady Hand Cutting System

Fig. 2 shows a variation of the system in Fig. 1, in which the bone forceps have been replaced by a cutting tool. The tip of the cutting tool is at displacement $\vec{p}_{\text{tool}}$ relative to the endplate of the robot.

$$\eta_g = [f_g, \tau_g]$$

$$\eta_h = [f_h, \tau_h]$$
Questions

1. Give an expression for computing $F_3 = \text{the pose of Tracker Body 2, relative to the robot’s tooling plate}$ (not relative to the optical tracking system).

2. Given a small change in the joint angles $\Delta \mathbf{q}$, we earlier had an expression
   \[ F_{\text{kins}} (\mathbf{q} + \Delta \mathbf{q}) \approx \Delta F_{\text{kins}}(\mathbf{q}) F_{\text{kins}} (\mathbf{q}) \] where $\xi_{\text{kins}} = J_{\text{kins}}(\mathbf{q}) \Delta \mathbf{q}$. Give an expression for
   \[ \xi_{\text{plate}} = [\alpha_{\text{plate}} \cdot \varepsilon_{\text{plate}}] \] for the corresponding small pose change expressed relative to the tooling plate coordinate system. I.e., we want an expression for $\xi_{\text{plate}} = [\alpha_{\text{plate}} \cdot \varepsilon_{\text{plate}}]$, where
   \[ F_{\text{kins}} (\mathbf{q} + \Delta \mathbf{q}) \approx F_{\text{kins}} (\mathbf{q}) \Delta F_{\text{plate}}(\xi_{\text{plate}}) \] in terms of $J_{\text{kins}}(\mathbf{q})$, $F_{\text{kins}} (\mathbf{q})$, and $\Delta \mathbf{q}$.

3. Given a small change in the joint angles $\Delta \mathbf{q}$, what is the corresponding change
   \[ \Delta F_{12} \approx [I + sk(\alpha_{12}) \cdot \varepsilon_{12}] \] in the pose of the Tracker Body 2 relative to Tracker Body 1. Here you may assume that you have computed the answer to question 1 correctly, so you can just use $F_3 = [R_3, \mathbf{p}_3]$ as a constant value.

4. Suppose that we have a gain matrix $K_h$ such that handle F/T values $\mathbf{h}_{\text{kin}}$ should produce an incremental motion $\Delta F_h(\xi_{\text{h}} = K_h \mathbf{h}_{\text{kin}})$ expressed in the coordinate system of the tooling plate. Using the “admittance style” virtual fixtures discussed in class, express an optimization problem that will produce a set of incremental joint values $\Delta \mathbf{q}$ that will produce this desired incremental motion. You can assume that your answer to Question 2 has enabled you to correctly compute a $J_{\text{plate}}(\mathbf{q})$ such that
   \[ \xi_{\text{plate}} = J_{\text{plate}}(\mathbf{q}) \Delta \mathbf{q} \] for handle $h$.

5. Suppose now that the workstation has determined some desired pose for the bone fragment such that Rigid Body 2 would have pose $F_{\text{des}} = F F_{12}$. Set up an optimization problem to implement admittance control guiding with a guidance virtual fixture to assist the surgeon achieve the desired alignment. The effect should be some “spring-like” behavior pulling the surgeon’s hand to achieve the desired alignment.

6. Now suppose that we have a point $\mathbf{a}$ defined on the skull base relative to $F_1$ and another point $\mathbf{b}$ on the moving fragment defined relative to $F_2$. Modify your admittance virtual fixture to help the surgeon align the bone fragment so that point $\mathbf{b}$ is at position $\mathbf{a}$. Your virtual fixture should allow both position and orientation changes of the moving bone fragment.

7. How would you modify the optimization problem of Question 4 so that no point within a distance $\rho$ of the tooling plate moves at a linear velocity greater than $v_{\text{max}}$ relative to the robot base. Here, I am looking for a quadratic constraint.

8. Suppose that your optimizer is only able to handle linear constraints, how would you deal with this?

9. Consider the system in Fig. 2. Define an optimization problem implementing an admittance virtual fixture that permits only translation of the cutting tool (i.e., keeps the orientation constant) and scales the forces required to move the robot so that the force exerted by the
tool on the patient is 0.01 times the force exerted by the surgeon on the control handle. **Hint:** Recall that a linear force $\mathbf{f}$ exerted at a point $\mathbf{d}$ on a rigid body produces a torque $\mathbf{\tau} = \mathbf{f} \times \mathbf{d}$ at the origin of that body.

10. For the system in Fig. 2, suppose that we have a triangular mesh model of the skull that has been registered relative to Rigid Body 1. Suppose that we know the positions of three vertices $[\mathbf{a}, \mathbf{b}, \mathbf{c}]$ of a triangle in this mesh relative to Rigid Body 1. The outward facing normal of this triangle is $\mathbf{n} = (\mathbf{a} \times \mathbf{b}) / \|\mathbf{a} \times \mathbf{b}\|$. Further, suppose that this triangle is the one “closest” to the cutter tip. Define an optimization problem implementing admittance-controlled guiding with a forbidden region virtual fixture that prevents the cutter tip from penetrating more than a distance $\delta$ below the surface defined by the triangle while still permitting admittance control of the motions via the handle force sensor and also keeping the orientation of the cutter constant.

11. How would you modify your answer to Question 10 so that the robot’s resistance to motion in the $-\mathbf{n}$ direction increases as the cutter approaches the target depth $\delta$ below the triangle surface? The resistance to motion in this direction should be no greater than motion in free space when the cutter is “above” the triangle, but should increase linearly as the cutter approaches the target depth.