Instructions and Score Sheet (hand in with answers)

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Note: 445 students may answer 645 questions for extra credit, but max total grade will still be 100

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1. Remember that this is a graded homework assignment. It is the functional equivalent of a take-home exam.

2. You are to work **alone** or **in teams of two** and are **not to discuss the problems with anyone** other than the TAs or the instructor.

3. It is otherwise open book, notes, and web. But you should cite any references you consult.

4. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.

5. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.

6. Sign and hand in the score sheet as the first sheet of your assignment.

7. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment
The goal of this exercise is design of an image guided biopsy system. You have been given several components to work from. These include:

- **A biopsy probe.** This probe has a rectangular handle approximately 30x10x150 mm. A thin, stiff shaft approximately 200 mm long extends from the end of this handle, approximately 5 mm from one of the thin sides, and is the portion that may be inserted into the patient. The actual tissue-sampling portions are at the end of this shaft.

- **A 3D ultrasound system.** The probe for the ultrasound system has a cylindrical handle approximately 150 mm long and 30 mm in diameter. The probe is roughly “T” shaped, and the actual 3D US image sensor
is contained in a rectangular parallelepiped 30x30x90 mm in size. The ultrasound system has excellent image segmentation software, as well as US-CT registration software capable of locating an anatomic structure $S$ visible in CT. I.e., if the pose of the anatomic structure in CT is $F_{CS}$, the same structure can be located relative to the ultrasound image coordinates as $F_{IS}$.

- **An optical tracking device** mounted on a tripod. The optical tracker is capable of tracking small markers to an absolute accuracy of $\delta = 0.25 \text{ mm}$ relative to the optical tracker. However, the tripod on which the tracker is mounted is subject to small random displacements $\Delta F_T \approx [l + sk(\alpha_T), \bar{c}_T]$ relative to the floor.

- **A collection of 3 mm stick-on markers** capable of being tracked by the optical tracker. These are very adhesive and can be attached to any object in the room. Note that when markers are attached to something, only the *approximate* position (within about 3 mm) of the marker relative to the object can be controlled during the placement process.

- **A 300x300 mm metal plate.** This plate has a small dimple in the middle. Also, rigidly attached to the plate is a tube with approximate length 150 mm. The inner diameter of this tube closely matches the outer diameter of the shaft of the biopsy probe.
An ultrasound phantom with a corresponding CT scan in which some anatomic structures visible in CT and ultrasound have been identified. Segmented models of these structures are available for use by the ultrasound-to-CT registration software. The phantom has a roughly ovoid cross section, and is 200 mm thick and 400 mm wide.
Question 1

A. Suppose that a marker is located at \( \vec{m} \) relative to the tracker and is motionless relative to the floor. What is the apparent uncertainty introduced into the position of \( \vec{m} \) due to the small displacements \( \Delta F_T \) and the tracking error \( \delta \)?

B. Give a numerical estimate based on \( |\vec{m}| = 2000 \text{ mm}, \delta \leq 0.25 \text{ mm}, \quad |\vec{\alpha}_T| \leq 0.01, \quad \text{and} \quad |\vec{e}| \leq 1 \text{ mm}. \)

C. Suppose that you place markers at the corners of the metal plate. Describe a process for defining a coordinate system associated with these markers and describe how you would find the pose \( F_{TP} \) of this coordinate system relative to the tracker. Here you do not have to recite all the detailed steps of mathematical algorithms described in class, but you should identify them clearly (e.g., “Arun’s Method” or “Horn’s Method”) and say what the inputs and outputs are. You should include sufficient detail about the formulation, sensing, and workflow so that it is clear how the known algorithms are being invoked.

D. How accurately can you find the position relative to the plate of an additional marker located at a displacement \( \vec{m}_{Pk} \) relative to the plate, where \( |\vec{m}_{Pk}| \leq 400 \text{ mm} \), under the assumptions of Question 1B.
E. Suppose that there is some geometric distortion associated with the tracker. Is there any way that you can check for this distortion, using the equipment provided?

**Question 2**

One requirement for the biopsy system is to determine the position of the tip of the biopsy probe and also the direction of the shaft relative to the tracker. If the system is capable of determining the pose $F_{TB}(t)$ of a coordinate system associated with the biopsy probe at time $t$, then the problem reduces to finding the position $\mathbf{p}_{\text{tip}}$ of the biopsy tip and the direction $\mathbf{n}_{\text{shaft}}$ relative to the biopsy probe coordinate system.

A. Describe a workflow and algorithm outline for determining $\mathbf{p}_{\text{tip}}$ and $\mathbf{n}_{\text{shaft}}$. Note that this will probably involve using some of the stick-on markers. You should indicate where you plan to place them and also should include information about workflow, sensing, and formulation. Again, you do not have to recite internal details of any point-cloud-to-point-cloud registration algorithms. But make clear how they are being used. Your answer will probably include a few sketches, as well as step-by-step workflow and formulas. **Hint:** There are some pretty clear suggestions in the lecture notes and programming assignment about how to do this.
B. Assuming that you have determined $\mathbf{p}_{\text{tip}}$ and $\mathbf{n}_{\text{shaft}}$ accurately, how accurately will you be able to determine $\mathbf{F}_{TB}(t)\mathbf{p}_{\text{tip}}$ and $\mathbf{R}_{TB}(t)\mathbf{n}_{\text{shaft}}$ (i.e., what can you say about the angle between the computed value of $\mathbf{R}_{TB}(t)\mathbf{n}_{\text{shaft}}$ and the actual vector? Note that this might depend somewhat on where you put the markers onto the biopsy probe.

C. What can you say about the accuracy with which you can determine $\mathbf{p}_{\text{tip}}$ and $\mathbf{n}_{\text{shaft}}$?

**Question 3**

Our image-guided interventional system will also require that we be able to determine the relative transformation $\mathbf{F}_{TI}(t)$ between the tracker and the 3D ultrasound image. Again, this will probably require that you define a coordinate system using markers stuck to the ultrasound probe that will enable you to compute a transformation $\mathbf{F}_{TU}(t)$ between the tracker and the ultrasound probe and then do some sort of calibration process to determine a transformation $\mathbf{F}_{UI}$ between the probe body and the actual 3D image.

A. Describe how you will determine $\mathbf{F}_{TU}(t)$ and provide an estimate of how accurately you can determine $\Delta \mathbf{F}_{TU}(t) \approx [I + sk(\alpha_{TU}), \xi_{TU}]$. Your answer will doubtless include some markers, and you should explain where you
would place them on the probe and some analysis to justify your accuracy estimates. A sketch or two may be useful.

B. Describe a workflow and algorithm outline for determining $F_{UI}$ and give a formula for computing $F_{TI}(t)$, following the same general guidelines as those for Question 2A. **Hint:** you may want to review the lecture notes on calibration for some ideas about how to go about this.

C. Assume that your answer to Question 3A has enabled you to compute $F_{UI}$ to very high accuracy. Suppose also that you are able to locate a biopsy target on an actual patient at a location $\mathbf{c}(t)$ relative to a 3D US image when your system reports a value of $F_{TU}(t)$. Describe a workflow and human-machine interfaces that will enable you to perform the biopsy (i.e., to insert the biopsy device into the patient along a straight line to the target and place the tip onto the target.

D. Estimate how accurately you can place the biopsy tool onto the target.