Algorithms for Querying Noisy Distributed/Streaming Datasets

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Sublinear Algo Workshop @ JHU

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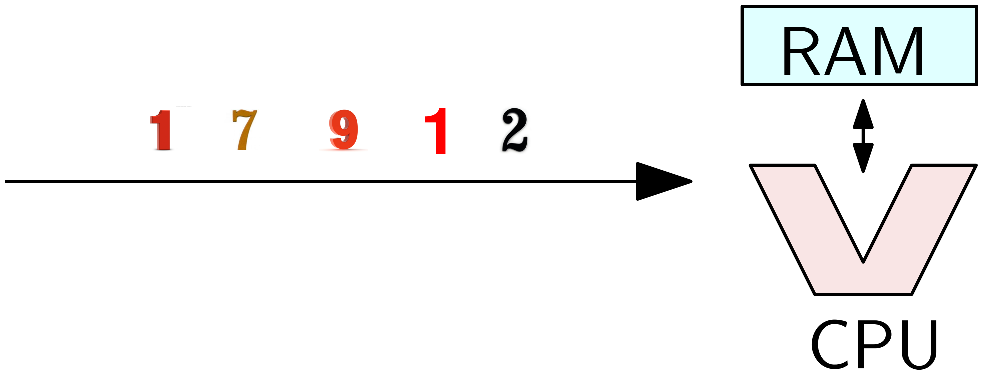
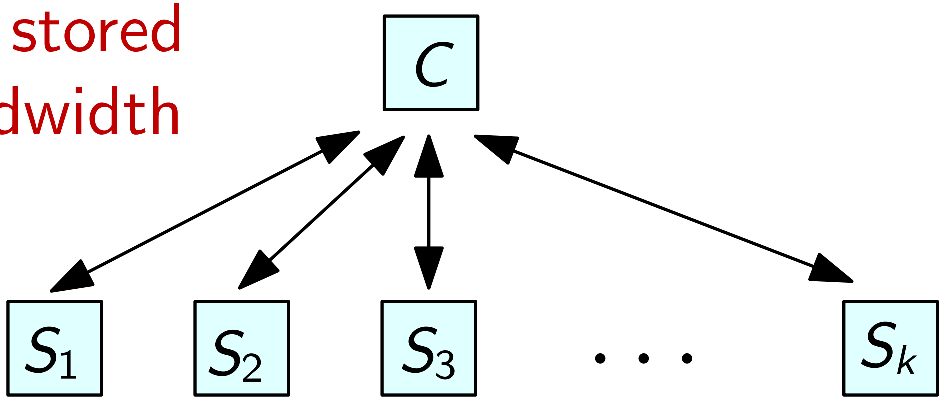
The “big data” models

- The **streaming** model (Alon, Matias and Szegedy 1996)
  – *high-speed* online data
  – *limited* storage



- The **k-site** model
  – data is *distributedly* stored
  – *limited* network bandwidth



k-site model

k **sites and** 1 **coordinator**.

– each site has a 2-way communication channel with the coordinator.

– each site S_i has a piece of data x_i. The coordinator has $\emptyset$.

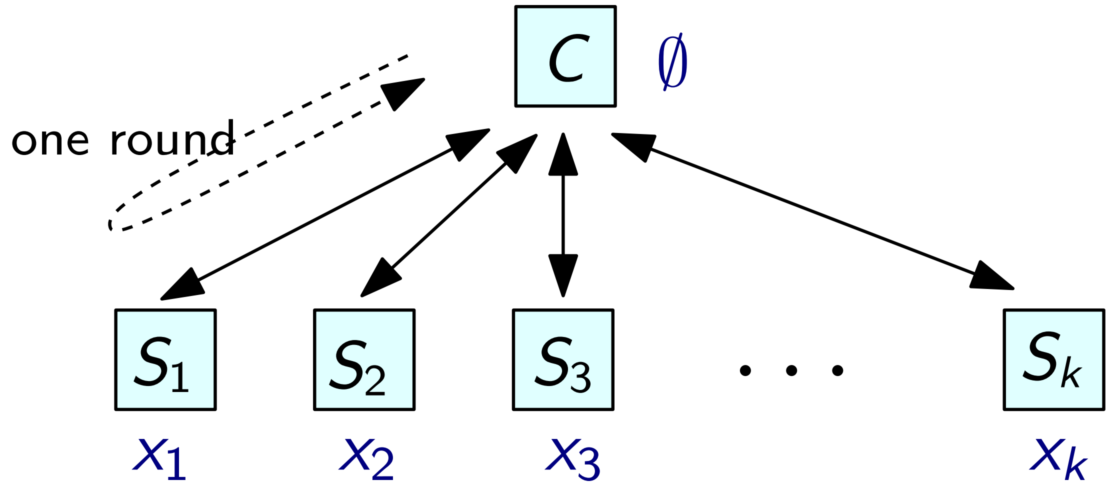
Task: compute $f(x_1, \ldots, x_k)$ together via communication.

– The coordinator reports the answer.

– computation is divided into rounds.

Goal: minimize both

- total #bits of comm. (o(Input); best polylog(Input))
- and #rounds ($O(1)$ or polylog(Input)).



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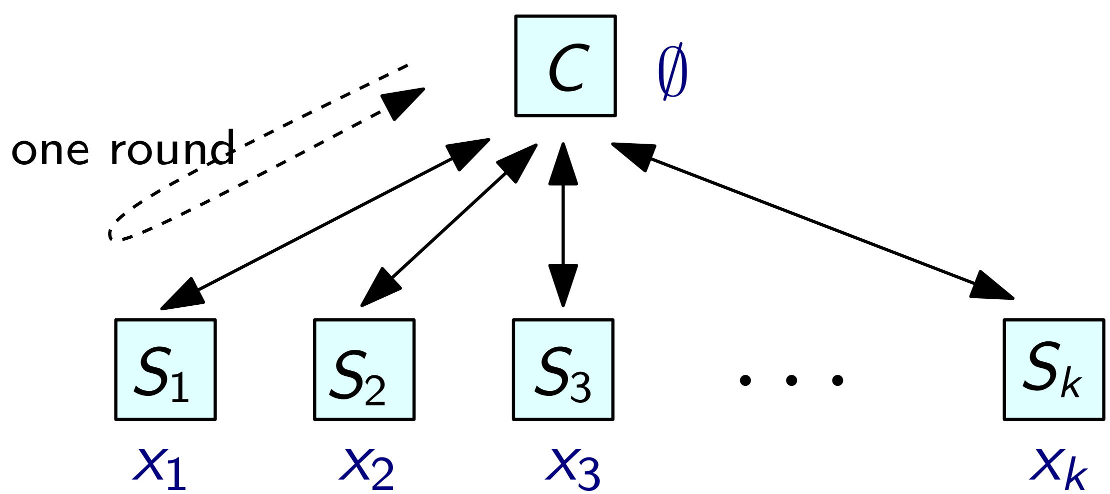
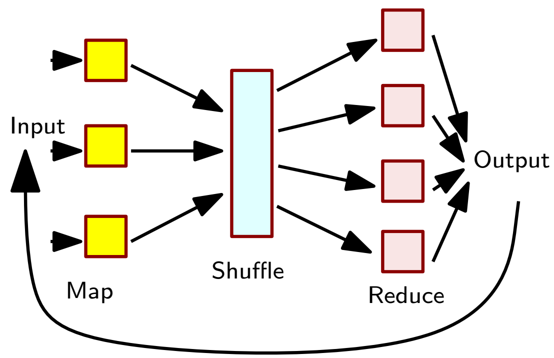


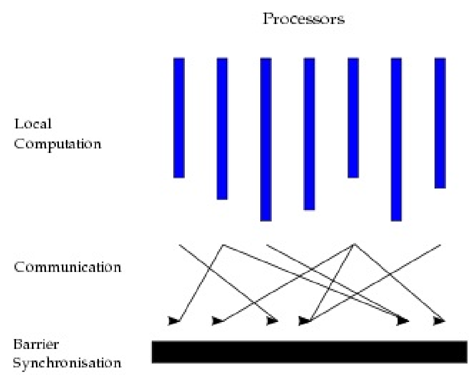
– no constraint on #bits can be sent or received by each site at each round. (usually balanced)
– do not count local computation (usually linear)

k-site model (cont.)

Communication $\rightarrow$ time, energy, bandwidth, ...

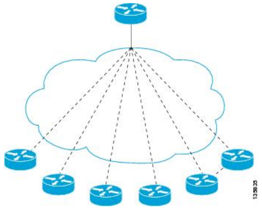


The MapReduce model.



The BSP model.

Also network monitoring, sensor networks, etc.



Abstraction

k-site model (cont.)

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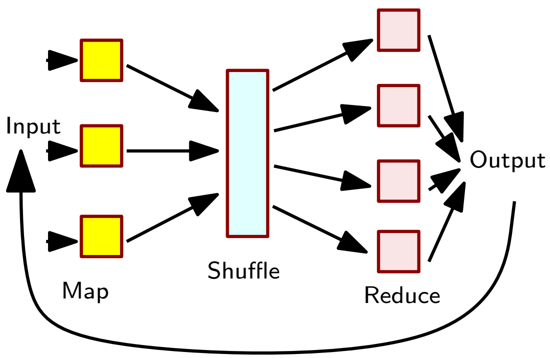

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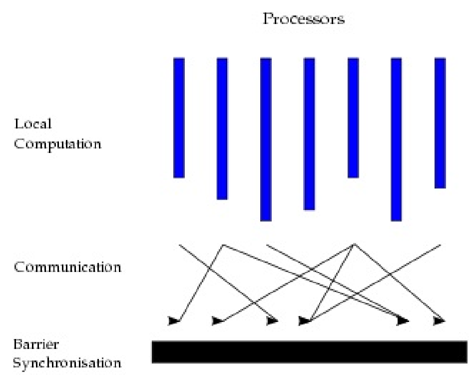

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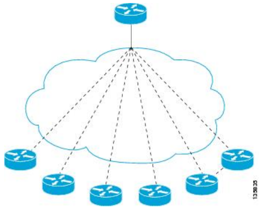

Abstraction

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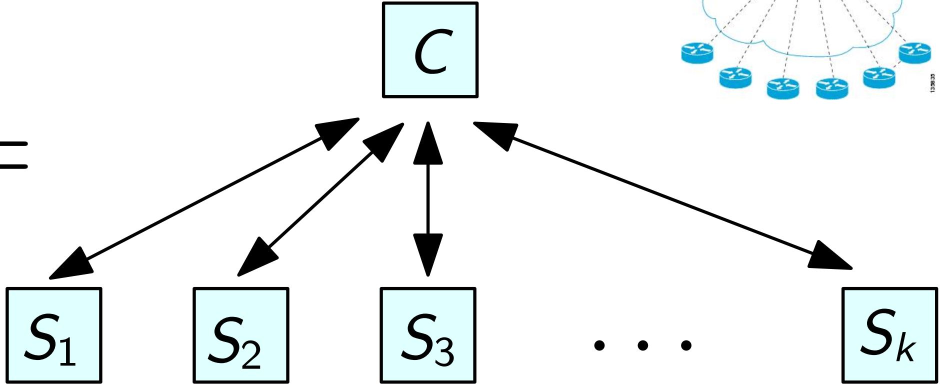
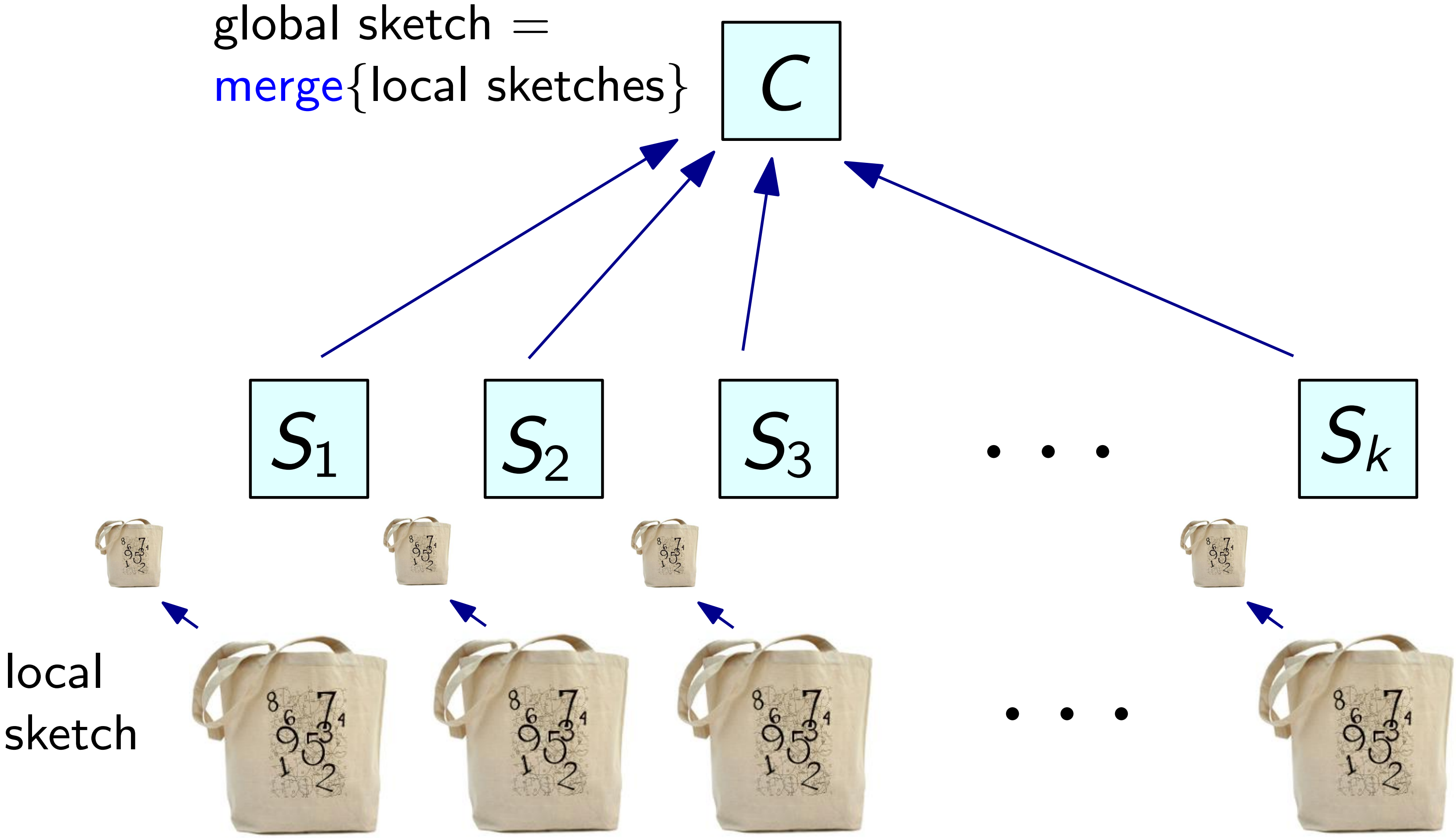


We will start with the k-site model, and will mention the streaming model at the end

Sketching

Q: How many distinct elements (F_0) in the **union** of the k bags?



Linear sketching

- **Random linear mapping** $M : R^n \to R^k$ where $k \ll n$.

$$\begin{bmatrix} \quad M \quad \end{bmatrix} \begin{bmatrix} x \\ \\ \\ \end{bmatrix} = \begin{bmatrix} Mx \end{bmatrix} \longrightarrow g(Mx) \approx f(x)$$

linear mapping

sketching vector

The data. e.g.,
a frequency vector

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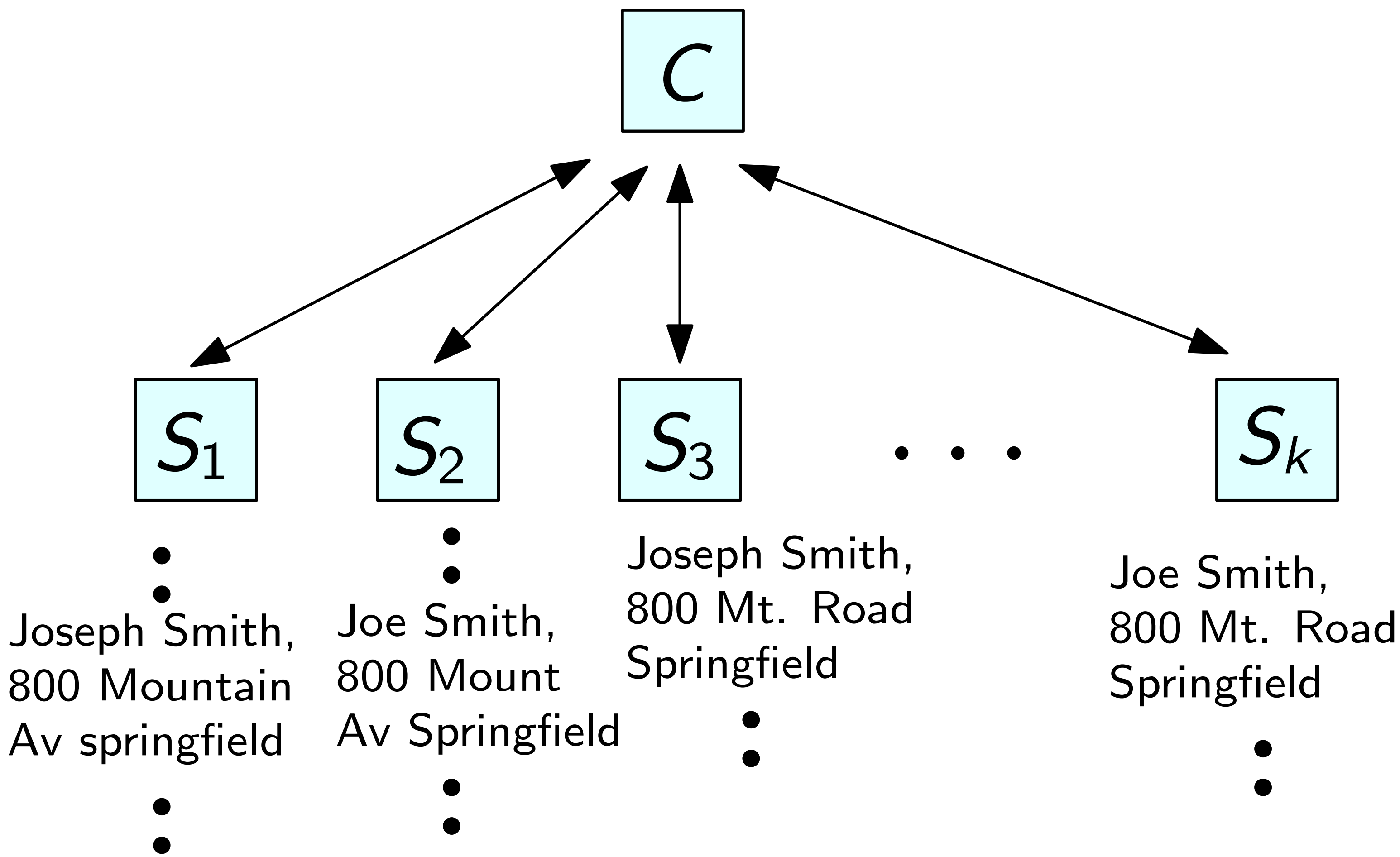
- **Perfect** for distributed and streaming computation

- **Simple and useful**: used in many statistical/graph/algebraic problems in streaming, compressive sensing, ...

But what if the data is noisy?

Real world distributed datasets are often noisy!



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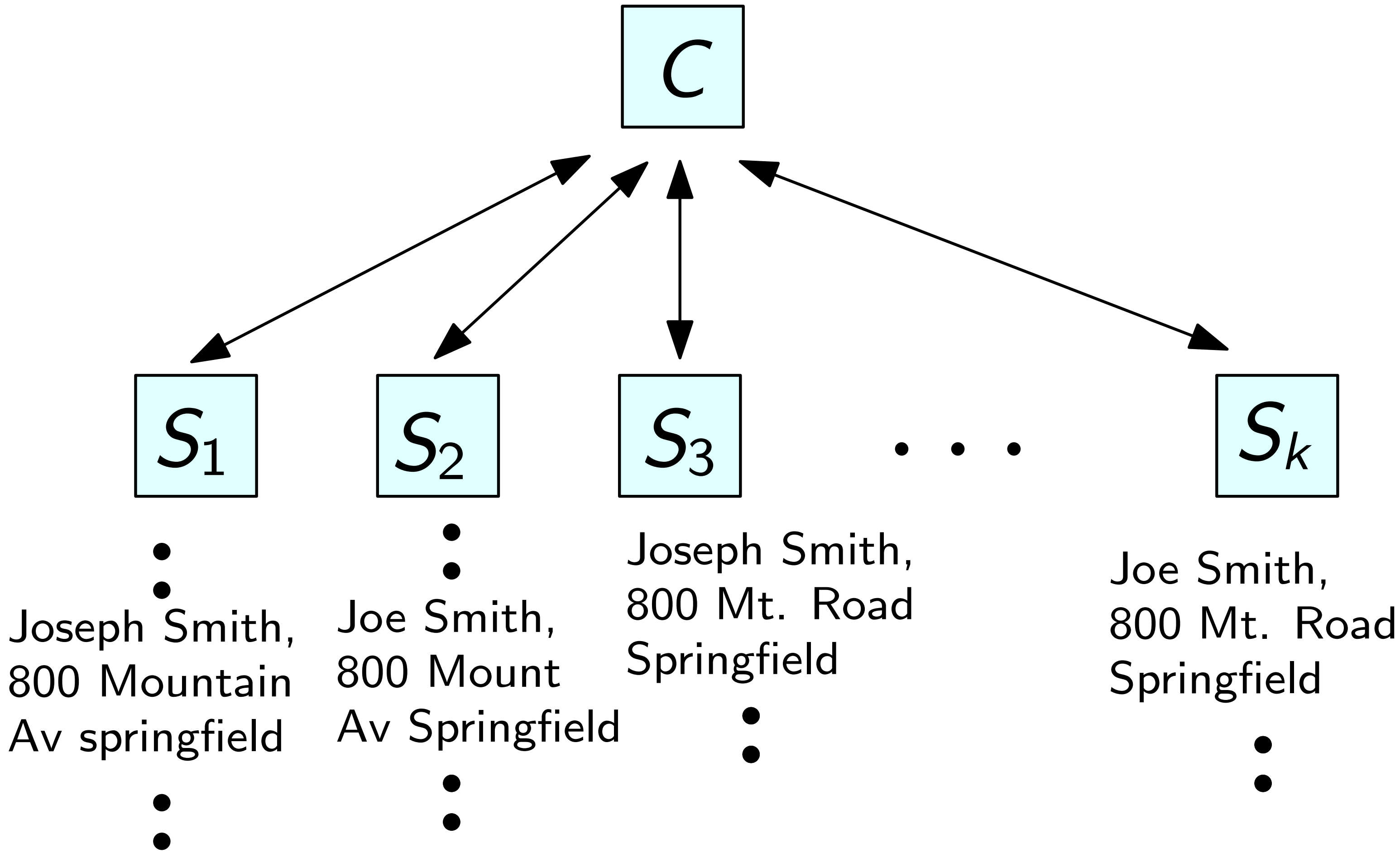

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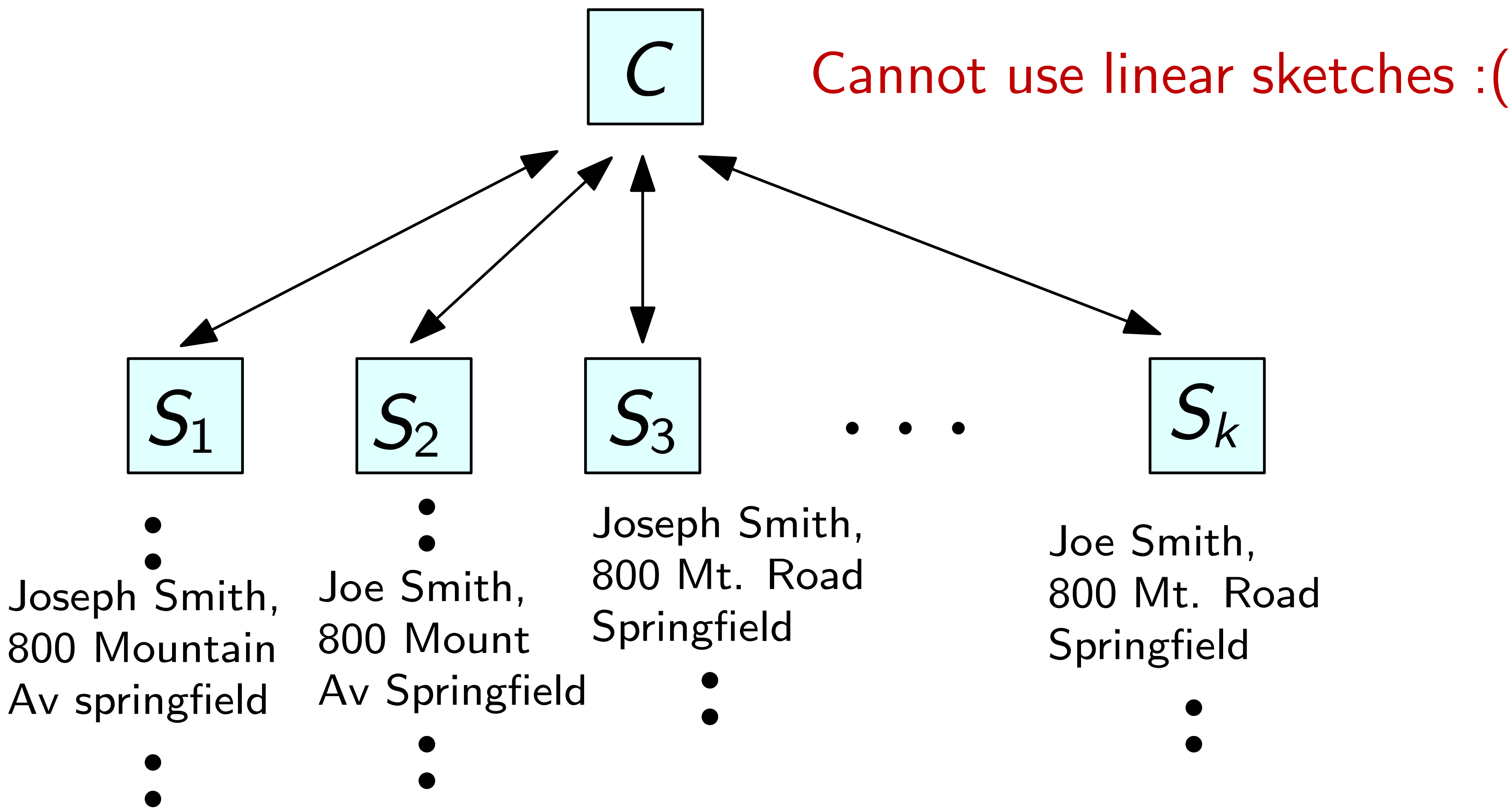

Noisy data is universal

- Music, Images, ...
  After compressions, resize,
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- 

  “sublinear algorithm workshop 2016”
  “JHU sublinear algorithm”
  “sublinear John Hopkins”

Queries of the same meaning sent to Google

Related to Entity Resolution

- Related to Entity Resolution: Identify and link/group different manifestations of the same real world object.

  Very important in data cleaning / integration. Have been studied for 40 years in DB, also in AI, NT.

  E.g. [Gill& Goldacre'03, Koudas et al.'06, Elmagarmid et al.'07, Herzog et al.'07, Dong& Naumann'09, Willinger et al.'09, Christen'12] for introductions, and [Getoor and Machanavajjhala'12] for a toturial.

  Centralized, detect items representing the same entity, merge/output all distinct entities.

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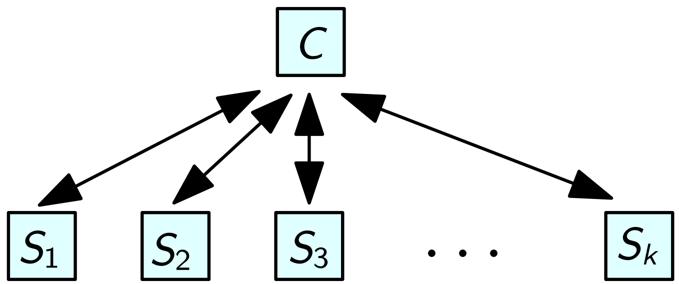
- In the big data models, we want communication/space-efficient algorithms (o(input size)); cannot afford a comprehensive de-duplication.

Our problems and goal

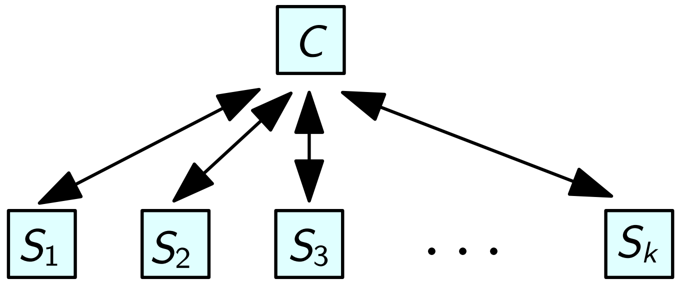


Problem: how to perform in the k-site model robust statistical estimation comm. efficiently?

Assume all parties are provided with an oracle (e.g., a distance function and a threshold) determining whether two items u, v rep. the same entity (denoted by $u \sim v$) or not

We will design a framework so that users can plug-in any "distance function" at run time.

Our problems and goal



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We will design a framework so that users can plug-in any "distance function" at run time.

Goal: minimize communication & #rounds

Remarks

Remark 1. We do not specify the distance function in our algorithms, for two reasons:

(1) Allows our algorithms to work with *any* distance functions.

(2) Sometimes it is very hard to assume that similarities between items can be expressed by a well-known distance function: "AT&T Corporation" is closer to "IBM Corporation" than "AT&T Corp" under the edit distance!

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Remark 2. We assume transitivity: if $u \sim v$, $v \sim w$ then $u \sim w$. In other words, the noise is "well-shaped".

One may come up with the following problematic situation: we have $a \sim b$, $b \sim c$, $\ldots$, $y \sim z$, however, $a \not\sim z$.

For many specific metic spaces, our algorithms still work if the number of "outliers" is small.

Remarks (cont.)

Remark 3. Clustering will help?

Answer: NO. #clusters can be linear.

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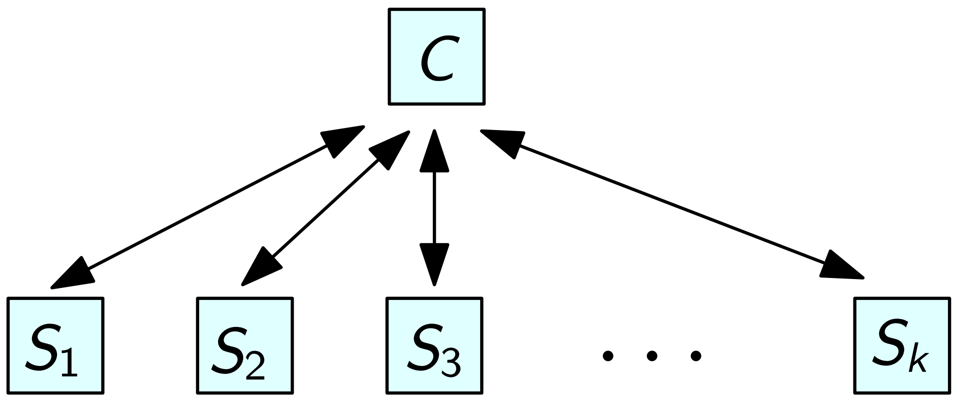
Answer: NO. #clusters can be linear.

Remark 4. Does there exist a magic hash function that
(1) map (only) items in same group into same bucket
and
(2) can be described succinctly?

Answer: NO

For specific metrics, tools such as LSHs may help

A few notations



- We have k sites (machines), each holding a multiset of items S_i.
- Let multiset $S = \bigcup_{i \in [k]} S_i$, let $m = |S|$.
- Under the transitivity assumption, S can be partitioned into a set of groups $\mathcal{G} = \{G_1, \ldots, G_n\}$. Each group G_i represents a distinct universe element.
- $\tilde{O}(\cdot)$ hides $\text{poly}\log(m/\epsilon)$ factors.

Our results

	noisy data		noise-free data
	(comm.) items	rounds	bits
F_0	$\tilde{O}(\min\{k/\epsilon^3, k^2/\epsilon^2\})$	$\tilde{O}(1)$	$\Omega(k/\epsilon^2)$ [WZ12,WZ14]
L_0-sampling	$\tilde{O}(k)$	$\tilde{O}(1)$	$\Omega(k)$
F_p $(p \geq 1)$	$\tilde{O}((k^{p-1} + k^3)/\epsilon^3)$	$O(1)$	$\Omega(k^{p-1}/\epsilon^2)$ [WZ12]
(ϕ, ϵ)-HH	$\tilde{O}(\min\{k/\epsilon, 1/\epsilon^2\})$	$O(1)$	$\Omega(\min\{\frac{\sqrt{k}}{\epsilon}, \frac{1}{\epsilon^2}\})$ [HYZ12,WZ12]
Entropy	$\tilde{O}(k/\epsilon^2)$	$O(1)$	$\Omega(k/\epsilon^2)$ [WZ12]

1. *p-th frequency moment* $F_p(S) = \sum_{i \in [n]} |G_i|^p$.

   We consider F_0 and F_p $(p \geq 1)$, and allow a $(1+\epsilon)$-approximation.

2. *L_0-sampling* on S: return a group G_i (or an arbitrary item in G_i) uniformly at random from $\mathcal{G}$.

3. *(ϕ, ϵ)-heavy-hitter* of S $(0 < \epsilon \leq \phi \leq 1)$ (definition omitted)

4. *Empirical entropy*: $\text{Entropy}(S) = \sum_{i \in [n]} \frac{|G_i|}{m} \log \frac{m}{|G_i|}$.

   We allow a $(1+\epsilon)$-approximation.

Take-home message:

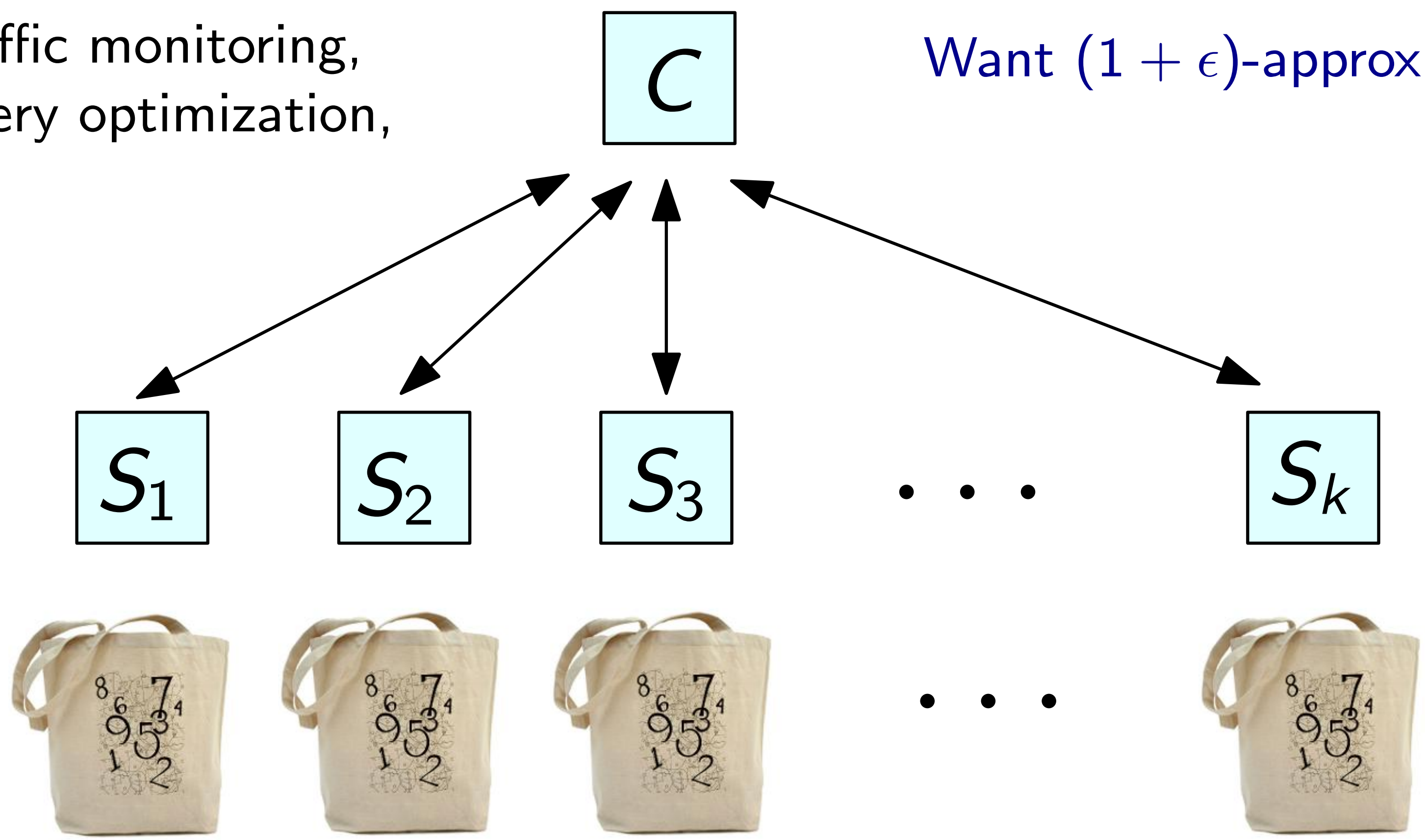
In the distributed setting, we can handle well-shaped noise in several statistical estimations almost for free in terms of communication

Rest of the talk: Algorithms for F_0

Q: How many distinct elements/groups in the **union** of the k bags?

Important in:
traffic monitoring,
query optimization,
...

Want $(1+\epsilon)$-approximation



1. Simple-Sampling

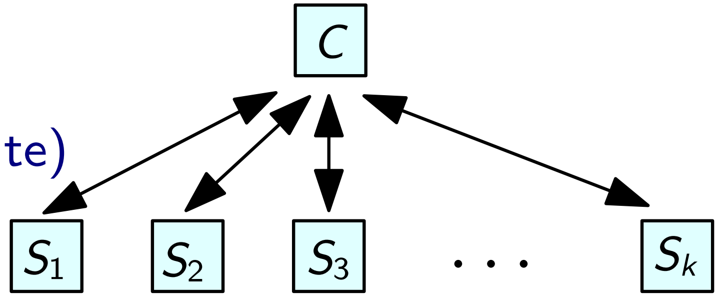
Simple.
$\tilde{O}(k^2/\epsilon^2)$ comm. 2 rounds.

2. Advanced-Sampling

A bit more complicated.
$\tilde{O}(k/\epsilon^3)$ comm. $\tilde{O}(1)$ rounds

Better than $\tilde{O}(k^2/\epsilon^2)$ bits in the sense that
(1) we want to scale on k
(2) used in the algo for ℓ_0-sampling with $\epsilon = \Theta(1)$

Simple-Sampling



Algorithm Simple-Sampling

(assuming local de-duplication is done at each site)

1. Let $m = |S| = \sum_{i \in [k]} |S_i|$.
2. For $j = 1, \ldots, \eta = \Theta(k/\epsilon^2)$
   (a) jointly sample a random item $u_j \in S$;
   Let G_{u_j} be the group containing u_j.
   (b) jointly compute $\left|G_{u_j}\right|$, and set $X_j = 1/\left|G_{u_j}\right|$.
3. Output $\frac{m}{\eta}\sum_{j \in [k]} X_j$.

Theorem

Simple-Sampling gives a $(1+\epsilon)$-approximation of F_0 with probability $2/3$ using $\tilde{O}(k^2/\epsilon^2)$ bits and 2 rounds.

Advanced-Sampling

Main idea: reduce the variance of X_j in Simple-Sampling

– If we can partition all groups in $\mathcal{G}$ into classes $\mathcal{G}_0, \ldots, \mathcal{G}_{\log k}$ such that $\mathcal{G}_\ell = \{G \in \mathcal{G} \mid |G| \in (2^{\ell-1}, 2^\ell]\}$, and run Algo Simple-Sampling on each class individually, we can shave a factor of k in the number of samples X_j needed ($\eta : k/\epsilon^2 \to 1/\epsilon^2$).

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Our techniques:

local hierarchical partition

$+$ distributed rejection sampling

Advanced-Sampling (cont.)

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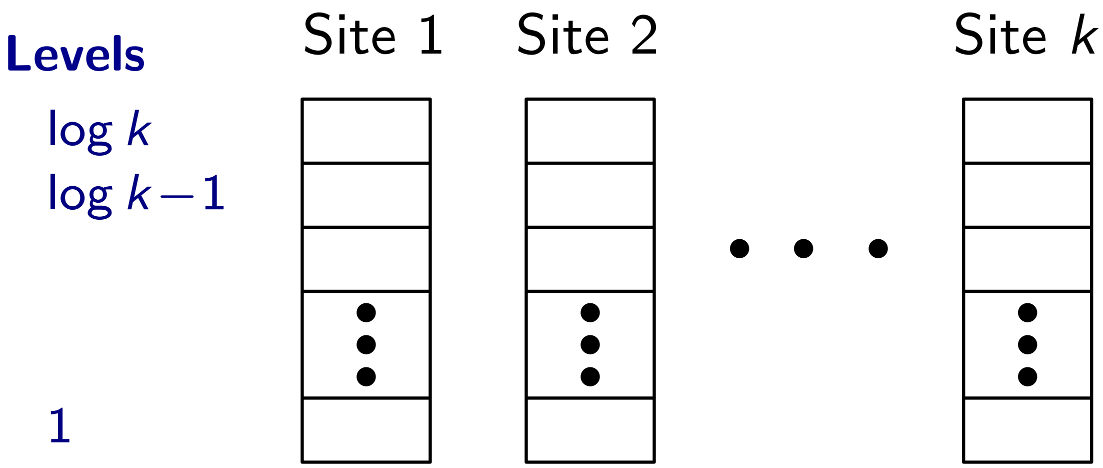
Local hierarchical partition: at site i about $|S_i|/2^\ell$ at level ℓ.



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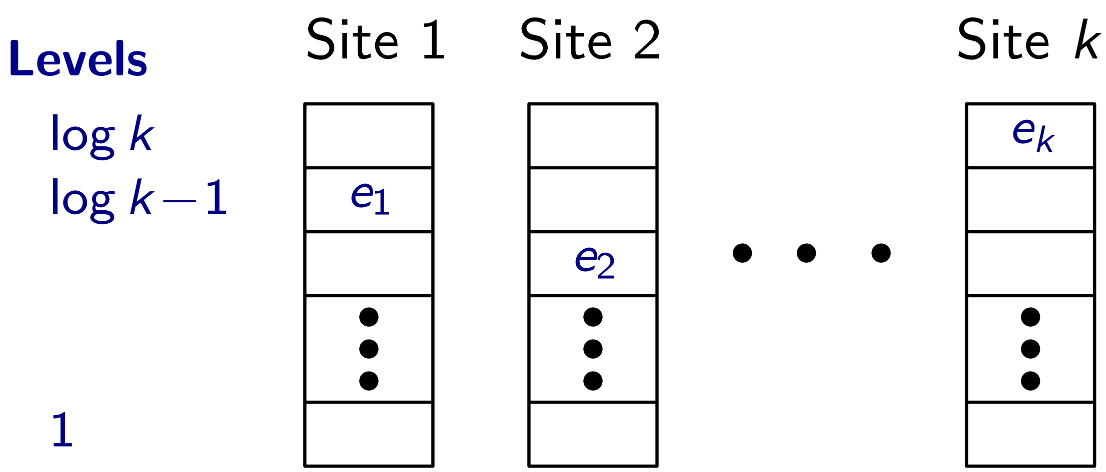

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Have inconsistency, $u \sim v$ but u, v are sampled at different levels at different sites.

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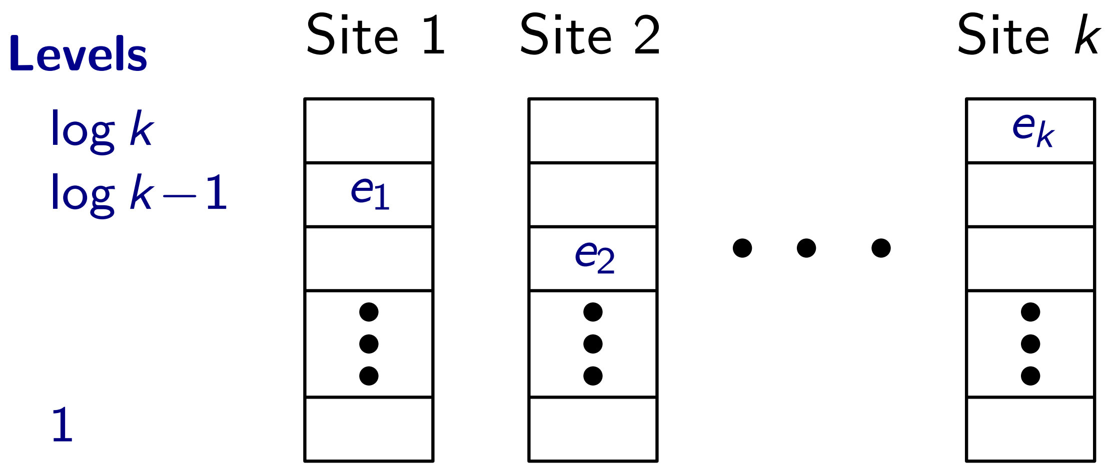

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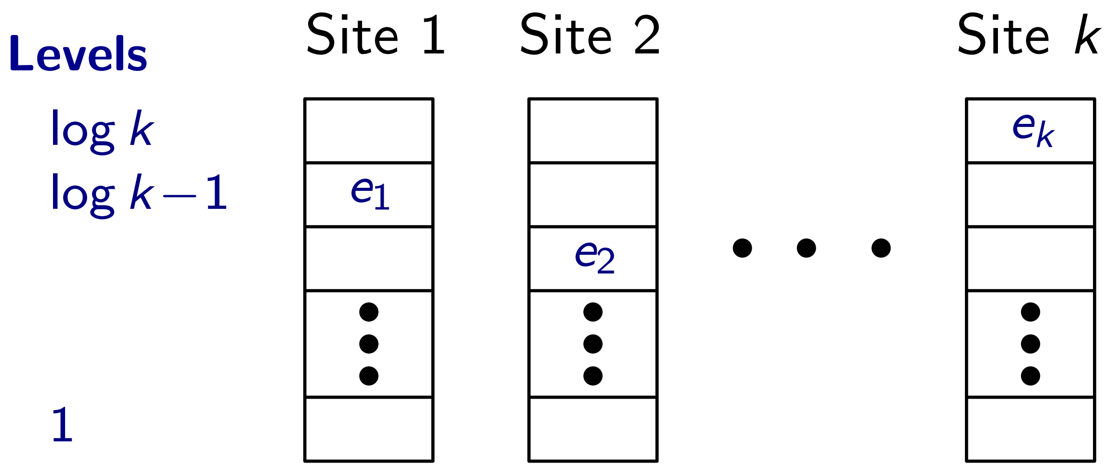

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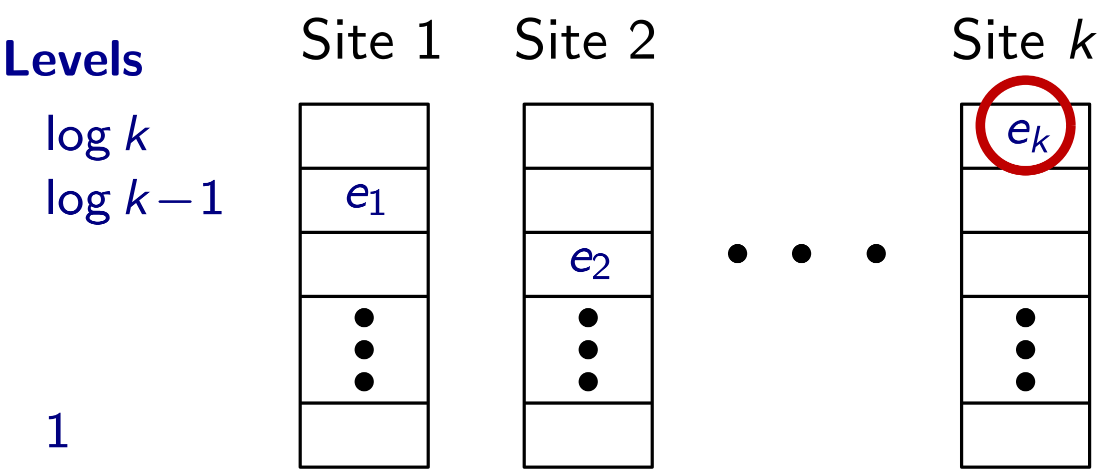

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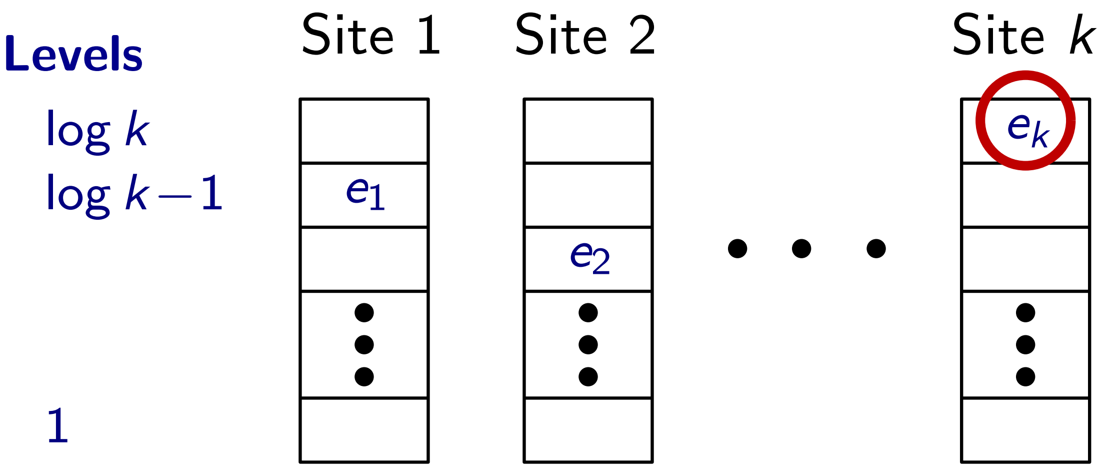

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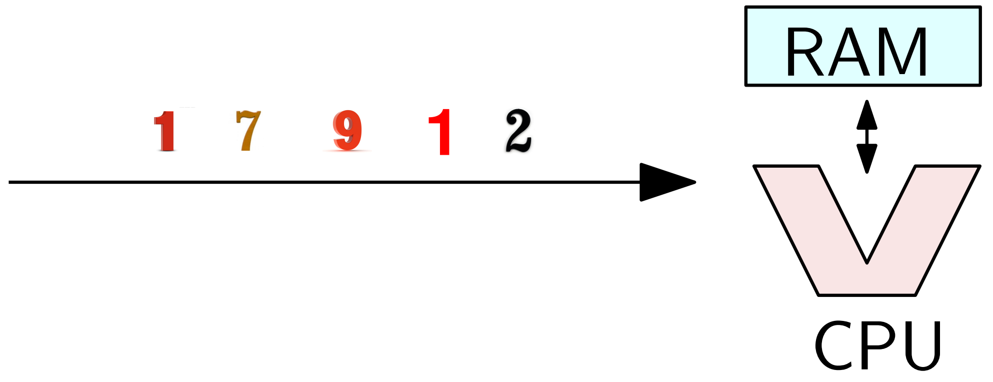
Repeat until we get $\tilde{O}(1/\epsilon^2)$ X_j's for **each** level of groups, and then run the estimation of *Simple-Sampling* for each level.

Other problems

1. L_0-**sampling**: $\tilde{O}(k)$ communication and $\tilde{O}(1)$ rounds.
   – Use the algorithm for F_0 as a subroutine
2. p-**th frequency moment**: $\tilde{O}((k^{p-1} + k^3)/\epsilon^3)$ comm. and $\tilde{O}(1)$ rounds.
   – Adapt an algo by Kannan, Vempala and Woodruff. (COLT 2014)
3. (ϕ, ϵ)-**heavy-hitter**: $\tilde{O}(\min\{k/\epsilon, 1/\epsilon^2\})$ comm. and $O(1)$ rounds.
   – Easy
4. **Empirical entropy**: $\tilde{O}(k/\epsilon^2)$ comm. and $O(1)$ rounds.
   – Adapt an algo by Chakrabarti, Cormode and McGregor (SODA 2007) in streaming

Now a bit on the streaming model



The streaming model

Q: Can we adapt the algorithms for the k-site model to the streaming model?

– the *simple-sampling* needs to revisit the data (2 rounds)

– the *advanced-sampling* needs more rounds

Not sure if we can do it for general metric spaces.

Can do for some specific metric spaces. For example, for $O(1)$-Euclidean space and well-shaped datasets, there exists a streaming algo using space $\tilde{O}(1/\epsilon^2)$ (Chen, Z., 2016).

Experiments (streaming model)

- **Problem**: compute the number of robust distinct elements (F_0) in the streaming model

  Given a threshold α, partition items in the input set S to a *minimum* set of groups $\mathcal{G} = \{G_1, \ldots, G_n\}$ so that $\forall p, q \in G_i, d(p, q) \leq \alpha$.

- **Data**: $4,000,000$ images from `ImageNet`, converted into points in the Euclidean space

- **Computing environment**: a desktop PC with 8GB of RAM and a 4-core 3.40GHz Intel i7 CPU

Experiments (known α)

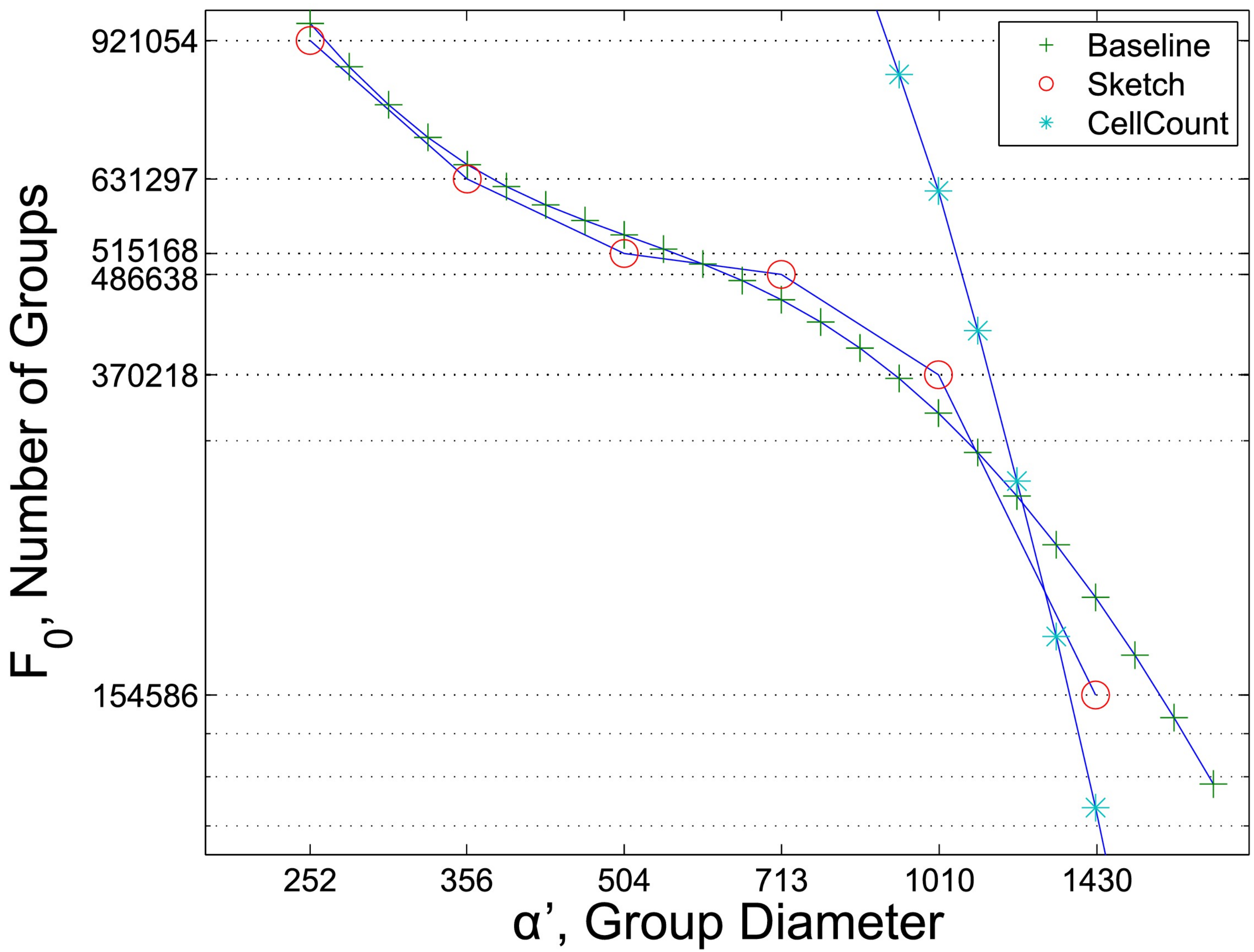
No. pts	9,000	18,000	36,000	72,000
`I500k100x5d`	22.8%	10.6%	8.3%	6.6%
`I500k10x5d`	15.8%	9.2%	6.7%	5.7%
`I500k2x5d`	5.2%	3.0%	2.8%	2.2%
`I4m2x5d`	6.0%	3.5%	3.3%	2.4%

Table 6: Vary duplication ratio; average error over 20 runs; median output of 6 sketches; known α.

No. pts	9,000	18,000	36,000	72,000	144,000
`I4m2x5d`	6.0%	3.5%	3.3%	2.4%	1.7%
`I4m2x10d`	5.8%	4.2%	3.4%	2.6%	1.5%
`I4m2x20d`	6.4%	4.4%	3.6%	2.0%	1.3%

Table 7: Vary dimensionality; average error over 20 runs; median output of 6 sketches; known α.

Experiments (unknown α)



`Baseline`
(greedy algo.)
$\Theta(n)$ space

`Sketch`
(our algo.)
$\tilde{O}(1/\epsilon^2)$ space

`CellCount`:
(streaming algo. for comparison)
$\tilde{O}(1/\epsilon^2)$ space

Dataset: I500k100x5d

Open problems

k-site model

- A number of bounds can possibly be improved. For example:
  - Can we get the optimal upper bound $\tilde{O}(k/\epsilon^2)$ for F_0?
  - Can we remove the k^3 factor in the communication cost for F_p?

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- Can we obtain efficient algorithms for L_p-sampling?

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Streaming model

- Algorithms for general metrics?
  (Now we can only do for some specific metrics use LSHs)

Thank you!
Questions?

– Communication-Efficient Computation on Distributed Noisy Datasets
Zhang, SPAA 2015

– Streaming Algorithms for Robust Distinct Elements
Chen and Zhang, SIGMOD 2016