Distributed Submodular Maximization in Massive Datasets

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Joint work with

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Combinatorial Optimization

- Given
  - A set of objects V
  - A function f on subsets of V
  - A collection of feasible subsets I
- Find
  - A feasible subset of I that maximizes f
- Goal
  - Abstract/general f and I
  - Capture many interesting problems
  - Allow for efficient algorithms

Submodularity

We say that a function $f : 2^V \to \mathbb{R}_+$ is submodular if:

$$f(A) + f(B) \geq f(A \cup B) + f(A \cap B)$$

We say that f is monotone if:

$$f(A) \leq f(B), \quad \forall A \subseteq B$$

Alternatively, f is submodular if:

$$f(A \cup \{x\}) - f(A) \geq f(B \cup \{x\}) - f(B)$$

for all $A \subseteq B$ and $x \notin B$

Submodularity captures diminishing returns.

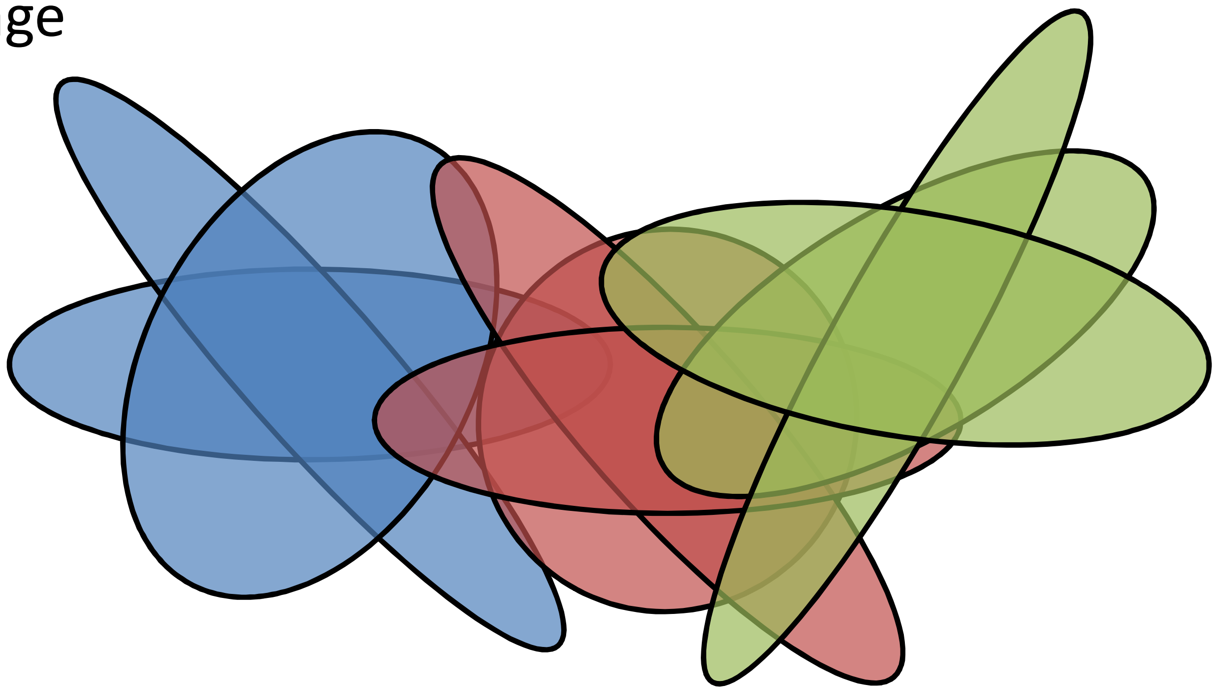
Submodularity

Examples of submodular functions:

- The number of elements covered by a collection of sets
- Entropy of a set of random variables
- The capacity of a cut in a directed or undirected graph
- Rank of a set of columns of a matrix
- Matroid rank functions
- Log determinant of a submatrix of a psd matrix

Example: Multimode Sensor Coverage

- We have distinct locations where we can place sensors
- Each sensor can operate in different modes, each with a distinct coverage profile
- Find sensor locations, each with a single mode to maximize coverage



Example: Identifying Representatives In Massive Data

Example: Identifying Representative Images

- We are given a huge set X of images.
- Each image is stored multidimensional vector.
- We have a function d giving the difference between two images.
- We want to pick a set S of at most k images to minimize the loss function:

$$L(S) = \frac{1}{|X|} \sum_{e \in X} \min_{r \in S} d(e, r)$$

- Suppose we choose a distinguished vector e_0 (e.g. 0 vector), and set:

$$f(S) = L(\{e_0\}) - L(S \cup \{e_0\})$$

- The function f is submodular. Our problem is then equivalent to maximizing f under a single cardinality constraint.

Need for Parallelization

- Datasets grow very large
  - TinyImages has 80M images
  - Kosarak has 990K sets
- Need multiple machines to fit the dataset
- Use parallel frameworks such as MapReduce

Problem Definition

- Given set V and submodular function f
- Hereditary constraint I (cardinality at most k, matroid constraint of rank k, …)
- Find a subset that satisfies I and maximizes f
- Parameters
  - n = |V|
  - k = max size of feasible solutions
  - m = number of machines

Greedy Algorithm

Initialize S = {}

While there is some element x that can be added to S:

Add to S the element x that maximizes the marginal gain $f(S \cup \{x\}) - f(S)$

Return S

Greedy Algorithm

- Approximation Guarantee
  - 1 - 1/e for a cardinality constraint
  - 1/2 for a matroid constraint
- Inherently sequential
- Not suitable for large datasets

Mirzasoleiman, Karbasi, Sarkar, Krause '13

Distributed Greedy

Performance of Distributed Greedy

- Only requires 2 rounds of communication
- Approximation ratio is:

$$O\left(\frac{1}{\sqrt{\min(m,k)}}\right)$$

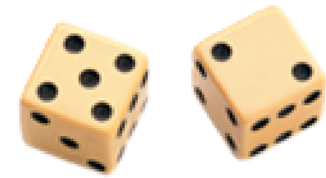
(where m is number of machines)

- Can construct bad examples
- Lower bounds for the distributed setting

(Indyk et al. '14)

Power of Randomness

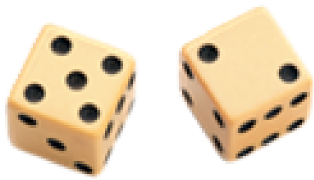
Power of Randomness



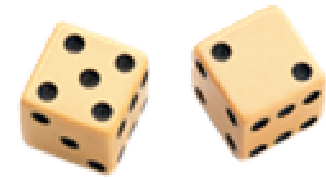
- Randomized distributed Greedy
  - Distribute the elements of V randomly in round 1
  - Select the best solution found in rounds 1 & 2
- Theorem: If Greedy achieves a C approximation, randomized distributed Greedy achieves a C/2 approximation in expectation.
- Related results: [Mirrokni, Zadimoghaddam ’15]

Intuition

- If elements in OPT are selected in round 1 with high probability
  - Most of OPT is present in round 2 so solution in round 2 is good
- If elements in OPT are selected in round 1 with low probability
  - OPT is not very different from typical solution so solution in round 1 is good



Power of Randomness



- Randomized distributed Greedy
  - Distribute the elements of V randomly in round 1
  - Select the best solution found in rounds 1 & 2
- Provable guarantees
  - Constant factor approx for several constraints
- Generality
  - Same approach to parallelize a class of algorithms
  - Only need a natural consistency property
  - Extends to non-monotone functions

Optimal Algorithms?

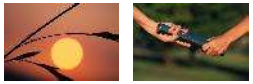
- Near-optimal algorithms?
- Framework to parallelize algorithms with almost no loss?

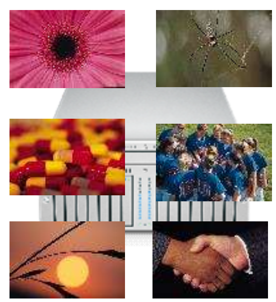
YES, using a few more rounds



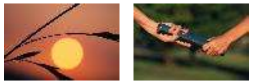


Core Set

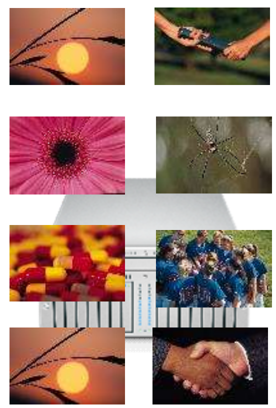




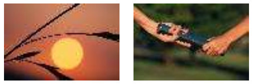
Core Set



Send Core Set
to every machine



Core Set

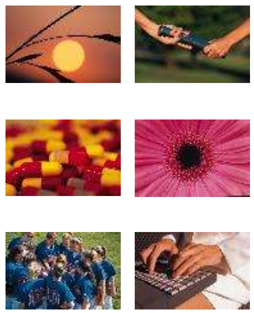


Core Set

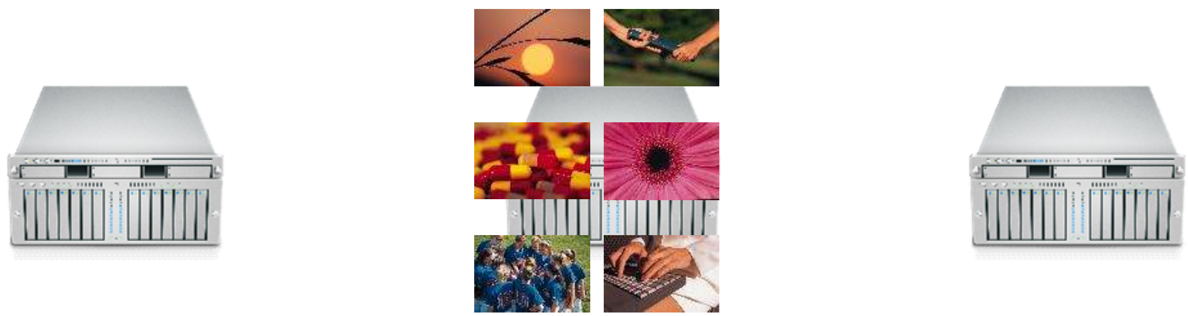
Core Set

Grow Core Set
over $1/\varepsilon$ rounds

Core Set

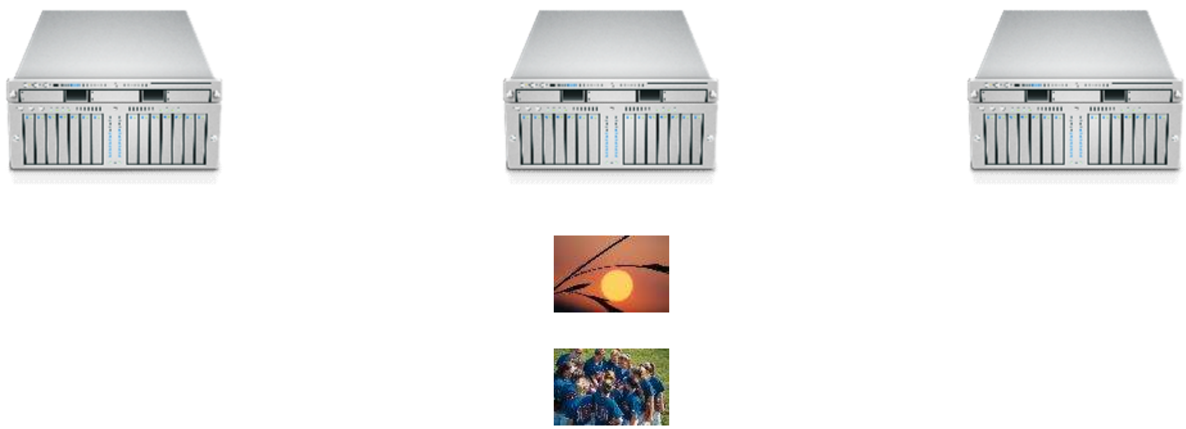


Grow Core Set
over $1/\varepsilon$ rounds



Core Set

Grow Core Set
over $1/\boldsymbol{\varepsilon}$ rounds



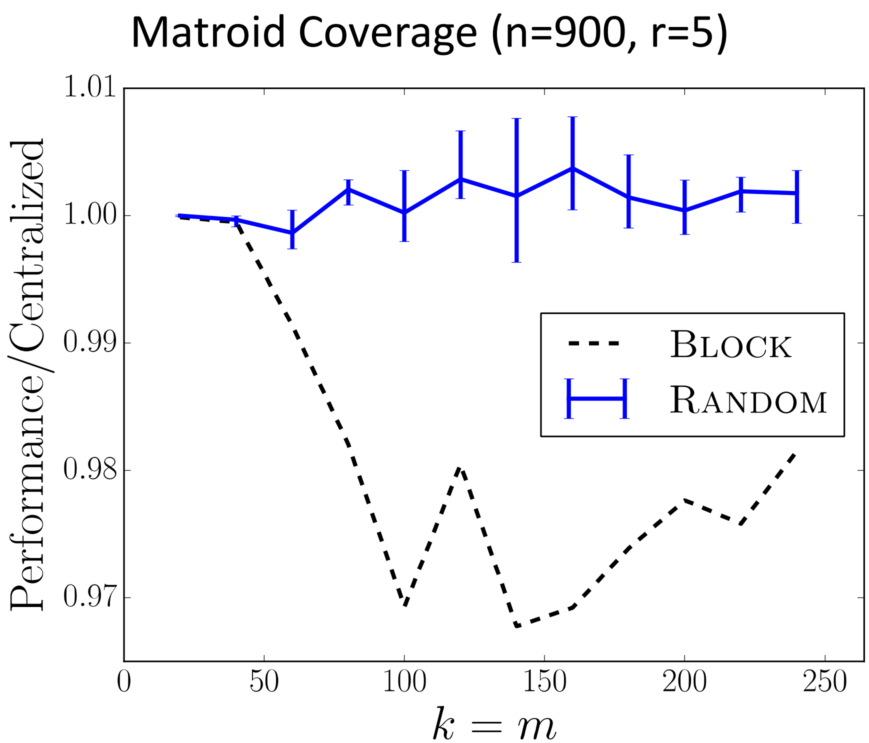
Core Set

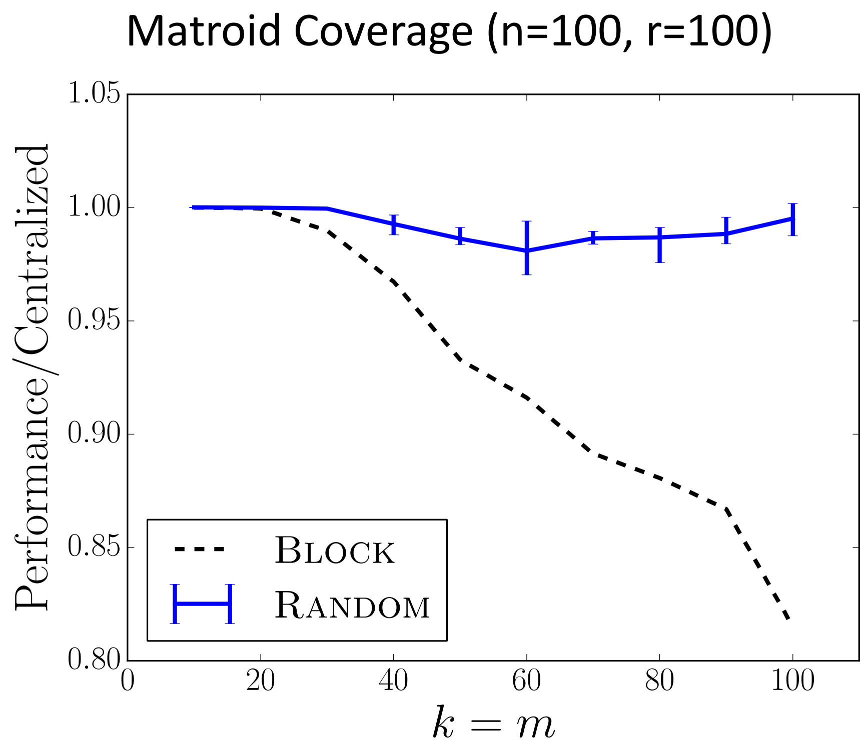
Grow Core Set
over $1/\varepsilon$ rounds

Leads to only an ε loss
in the approximation

Intuition
Each round adds an ε fraction
of OPT to the Core Set

Matroid Coverage Experiments





It's better to distribute ellipses from each location across several machines!

Thank You!

Questions?