Streaming Property Testing of Visibly Pushdown Languages

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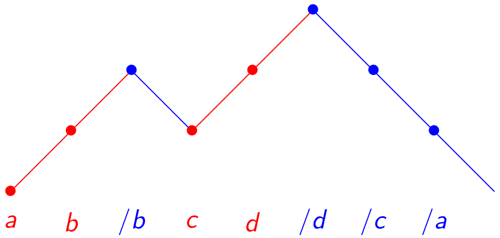
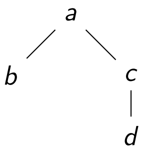
Visibly Push-Down Languages

Definition

A VPL is a language of $\Sigma = \Sigma_+ \cup \Sigma_- \cup \Sigma_=$ that is recognized by a stack automaton that pushes when it reads a symbol in Σ_+ and pops when it reads a symbol of Σ_-.

In particular, a regular tree language with the tree read in DFS order (such as an XML document) is a VPL.





Motivation and context

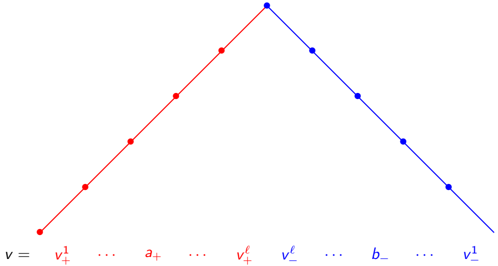
- Checking the validity of large documents needs to be done efficiently.
- High stack $\implies$ cannot be done with small memory in streaming.
- An efficient property tester can pre-reject some documents before a more costly check.

VPLs are hard to recongize in streaming and hard to test for in the query model:

- Recognizing some VPLs in streaming requires memory $\Omega(n)$. (Disjointness)
- A query-model property tester for the parenthesis language requires $\Omega(n^{1/11})$ queries. [Parnas, Ron, Rubinfeld '03]

Non-alternating sequences

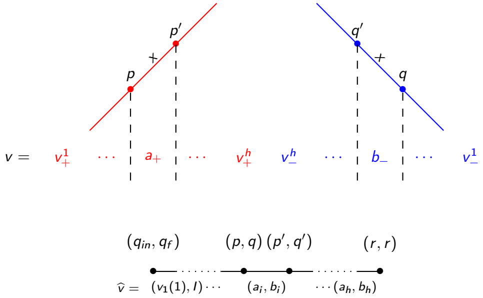
Consider $v = v_+ v_-$. We still want to know if $v \in L$.



- Known to be hard to decide exactly (encoding of Set Disjointness).
- Any solution may give insight to general problem.

Non-alternating sequences as elements of regular languages

"Slices" of v can be interpreted as a word $\widehat{v}$, with $\widehat{v}$ in some regular language if and only if $v \in L$.



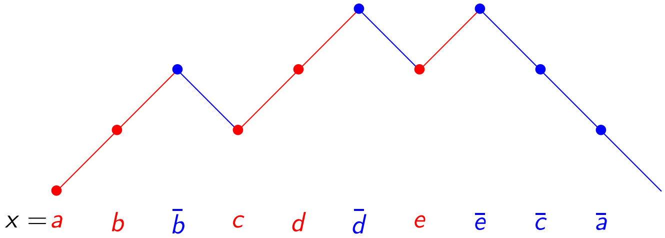
Recognizing non-alternating sequences with a tester for regular languages

- There is an algorithm for testing regular languages in $\mathrm{O}(1/\varepsilon^2)$ **non-adaptive** queries [Alon, Krievelich, Newman, Szegedy '00].
- To get sampling of $\widehat{v}$, remember sampled letters in v_+ (memory $\mathrm{O}(1/\varepsilon. \log n)$ for the heights) then read letters of matching height in v_-.
- Can do more than just accept/reject : can test for all pairs of states (p, q) if there is a run of $\mathcal{A}$ on v from p to q.

We now have a black-box streaming tester for non-alternating sequences that outputs some $R \subset Q \times Q$ indicating the possible beginning and end states for v. From this we build an algorithm for the general problem.

From non-alternating sequences to the general problem: general idea

- Input $x \in \Sigma^*$.
- Find v a "peak" in x and use the non-alternating sequence tester on it.
- Repeat this process



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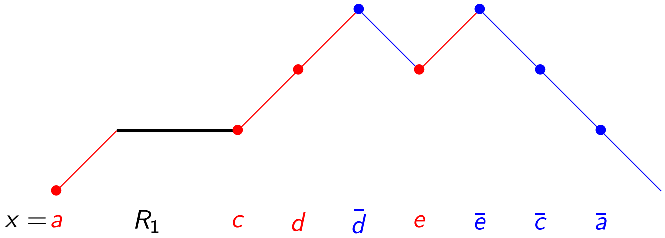

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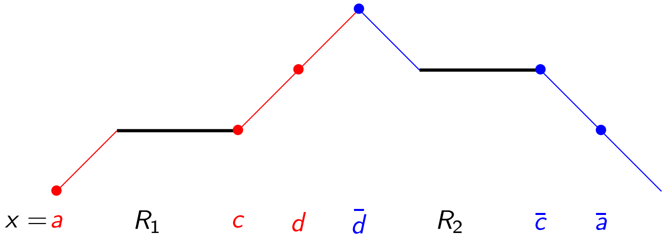
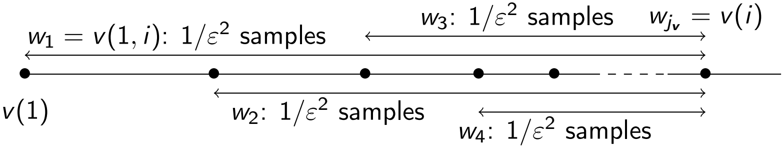


From non-alternating sequences to the general problem: sampling

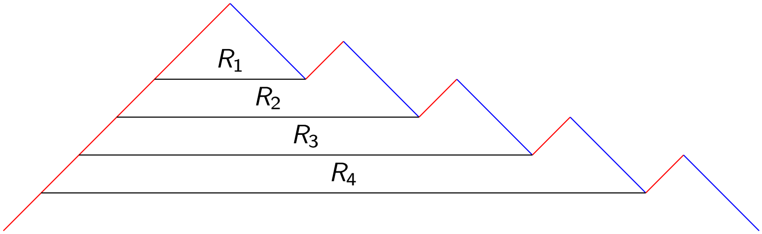
- To compute R from a peak v we need $1/\varepsilon^2$ samples inside the peak.
- We do not know in advance how large the peaks will be.
- Perform $(1+\varepsilon)$-suffix sampling: reservoir sampling on several suffixes $w_1, \ldots, w_{j_v}$, each $(1+\varepsilon)$ times large than the last.



Total amount of samples : $\log(|v|)/(\varepsilon^2 \log(1+\varepsilon)) \approx \log(|v|)/\varepsilon^3$.

From non-alternating sequences to the general problem: handling R's

- Each R corresponds to some v' potentially $\varepsilon|v|$-far from peak v.
- If too many R's within R's, risk of accumulation of error.



- Solution: not compute R immediately, wait to see if the next peak is much smaller
- This has a cost : $\log n$ peaks waiting in the stack
- $\log n$ potential nested R's mean we have to use $\varepsilon' = \varepsilon / \log n$ for the tester for peaks.

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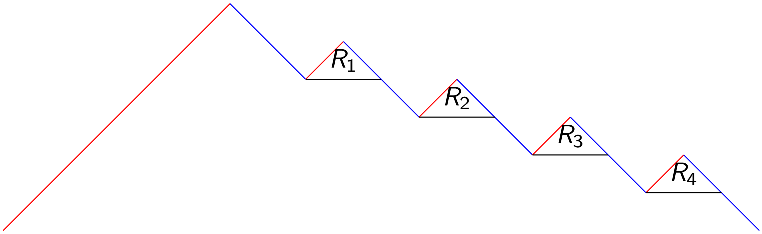

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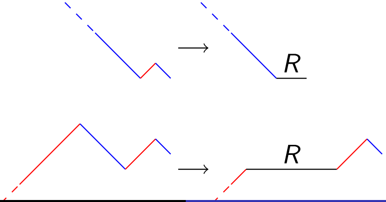
Algorithm for the general case

- Use $\varepsilon' = \varepsilon / \log n$ because of error accumulation.
- Maintain $\log^3 n/\varepsilon^2$ independent and well-distributed sampling of factors of size $\log n/\varepsilon$.
- Because computing R's messes with the sampling, we in fact need memory $\mathrm{O}(\log^6 n/\varepsilon^4)$.
- Maintain a stack of past peaks not transformed into a R yet.
- If a peak is finished (i.e. returned to starting height), compute the R, get previous peak out of the stack
- If current peak has at least half the weight of previous peak (in the stack), remove that peak from the stack and compute the R.
- Total memory cost: $\mathrm{O}(\log^7 n/\varepsilon^4)$.



Future directions

There may still be some hope of reducing the memory cost :

- Maybe each element of the stack does not need to preserve all the sampling as it grows older. This would remove a $\log n$ factor.
- One of the $\log n$ factors is due to the assumption that all R's are correct (up to a relative error of ε) with high probability. Maybe we can afford a few completely wrong R's.
- The high stack, small peaks, and nested R's are what makes our algorithm costly. But they mostly occur when the height is low, and we have an exact algorithm for checking VPLs with memory cost $\text{height}(x)$ (run the automaton). Can we find a compromise?

Thank you for your attention