# Theory of Network Communication 

Fall 2004

## Assignment 2

Problem 1 (3 points):
Show that the $b$-ary DeBruijn graph of dimension $d$ has a degree of $2 b$ and a diameter of $d(2$ points). Express $d$ in terms of $n$ (the number of nodes) and $b$ in order to show that the DeBruijn graph can be used to prove Theorem 3.8 (1 point).

Problem 2 (2 points):
Compute the expansion of an $n \times n$-torus when $n$ is even. It is sufficient here to guess the worst-case set $U$ (1 point) and to compute the value $c(U, \bar{U}) / \min \{c(U), c(\bar{U})\}$ (1 point).

Problem 3 (2 points):
Consider the concurrent multicommodity flow problem given in Figure 1. Try to find an optimal feasible solution for it (i.e. a solution in which the flows do not exceed the edge capacities), and compute from this the concurrent max-flow $f$.


Figure 1: A concurrent multicommodity flow problem with two source-destination pairs with $d_{1}=d_{2}=1$.

Problem 4 (3 points):
Consider the pentagon, i.e. a cycle with 5 nodes, and suppose that the capacity of all edges is equal to 1 . Compute the flow number of the pentagon. Start here by looking at the system of shortest paths connecting any source-destination pair and use this system to bound the dilation and congestion of a best possible solution for the special BMFP $\mathcal{B}$. Conclude from this on the flow number of the pentagon.

