Theory of Network Communication Fall 2004 Assignment 2

Problem 1 (3 points):

Show that the *b*-ary DeBruijn graph of dimension d has a degree of 2b and a diameter of d (2 points). Express d in terms of n (the number of nodes) and b in order to show that the DeBruijn graph can be used to prove Theorem 3.8 (1 point).

Problem 2 (2 points):

Compute the expansion of an $n \times n$ -torus when n is even. It is sufficient here to guess the worst-case set U (1 point) and to compute the value $c(U, \bar{U}) / \min\{c(U), c(\bar{U})\}$ (1 point).

Problem 3 (2 points):

Consider the concurrent multicommodity flow problem given in Figure 1. Try to find an optimal feasible solution for it (i.e. a solution in which the flows do not exceed the edge capacities), and compute from this the concurrent max-flow f.

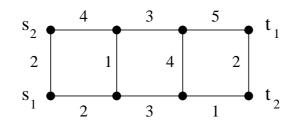


Figure 1: A concurrent multicommodity flow problem with two source-destination pairs with $d_1 = d_2 = 1$.

Problem 4 (3 points):

Consider the pentagon, i.e. a cycle with 5 nodes, and suppose that the capacity of all edges is equal to 1. Compute the flow number of the pentagon. Start here by looking at the system of shortest paths connecting any source-destination pair and use this system to bound the dilation and congestion of a best possible solution for the special BMFP \mathcal{B} . Conclude from this on the flow number of the pentagon.