Mixed Stochastic and Event Flows

Brownian Motion Modeling for Simulation Dynamics

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Outline

Motivation

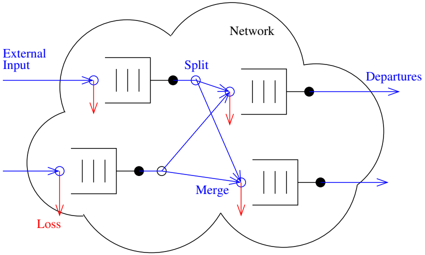
Scalable Network Simulation Models

- Future network design and modeling requires large scale, high fidelity simulations capability.
- Training requires real-time speedup of network simulations.
- Parallelization of network simulations not always useful due to lack of topological communities of interest.
- Hybrid analytic/event simulations appear to be an attractive alternative.

Hybrid Simulation Types

- Network models:
  - Partitioned models, e.g., packet edge and analytic core
  - Mixed node models, i.e., packet and analytic traffic mixed at each network queue.
- Analytic models:
  - Deterministic models, e.g., dynamics described by deterministic differential equations.
  - Stochastic models, e.g., Brownian motion models of queue dynamics.

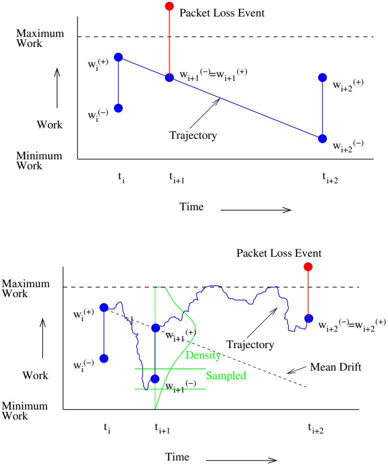
Challenges to Stochastic, Mixed Node Models



Simulation of network of queues.

- Mixing the two fundamentally different traffic types at a single, finite queue.
- Time dependent models of finite-sized queue dynamics.
- Splitting and merging mixed-traffic flows within network simulation.

Approach to Hybrid Stochastic Simulations



Contrasting deterministic versus stochastic fluid mixing at queue.

Brownian Motion Models

- Cumulative Distribution Function for a *GI/G/1/K* queueing system, $F(w, t|w_0, t = 0)$ satisfies the Fokker-Planck Equation [Kobayashi, 1974a], [Kobayashi, 1974b] and [Heyman, 1975]

$$\frac{\partial}{\partial t} F = -m \frac{\partial F}{\partial w} + \frac{1}{2}\sigma^2 \frac{\partial^2 F}{\partial w^2} \tag{1}$$

where $m = \lambda_s - \mu$, and $\sigma^2 = \lambda_s \times C_v^2 + \mu \times C_{v,\mu}^2$.

- The Fokker-Planck equation has an analytic solution

$$\begin{aligned} F(w, t|w_0, t = 0) &= \alpha \times \Phi\left(\frac{w - w_0 - mt}{\sigma\sqrt{t}}\right) \\ &+ \beta \times e^{2mw/\sigma^2} \Phi\left(\frac{-w - w_0 - mt}{\sigma\sqrt{t}}\right) \end{aligned} \tag{2}$$

$$\Phi\left(\frac{\pm w - w_0 - mt}{\sigma\sqrt{t}}\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\pm w - w_0 - mt}{\sigma\sqrt{t}}} e^{-x^2/2} dx \tag{3}$$

Boundary Conditions

- Initial Conditions:

  $F(w, t = 0) = H(w - w_0)$; $H(x)$ is the Heavy-side Function.

- Upper and lower limits on work in system:

  Common BCs -

$$\lim_{w \to 0} F(w, t | w_0, t = 0) = 0 \quad (4)$$

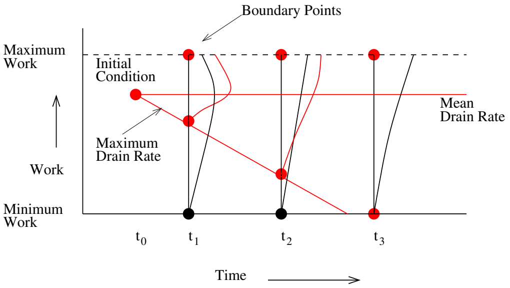
$$\lim_{w \to w_{max}} F(w, t | w_0, t = 0) = 1 \quad (5)$$

  Our alternative BCs -

$$\lim_{w \to max[0, w_0 - \mu t]} F(w, t | w_0, t = 0) = 0 \quad (6)$$

$$\lim_{w \to w_{max}} F(w, t | w_0, t = 0) = 1 \quad (7)$$

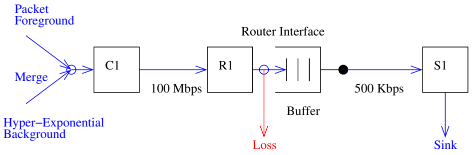
Rationale for Our Alternative BCs



Alternative BCs and their impact of density functions versus time.

- Buffer size sets maximum and minimum limits to work in queue.
- Maximum drain rate (assuming no arrivals) sets short term time-dependent limit on the minimum work in queue.

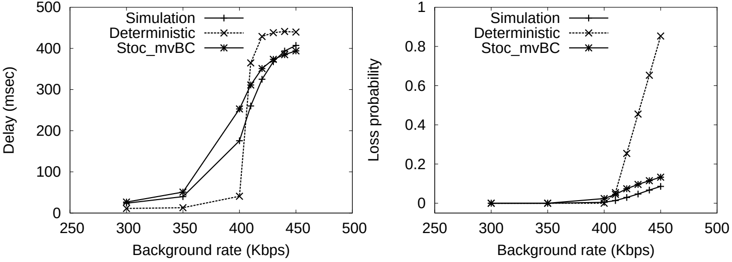
Simple Simulation Model



Simple simulation model in GTNets.

- Background is hyper-exponential arrival with deterministic service, modeling UDP packets
- Foreground is exponential arrival with deterministic service, modeling UDP packets
- Foreground is later modeled as TCP stream
- Investigate foreground packet delay (UDP and TCP), loss (UDP and TCP), and goodput (TCP)

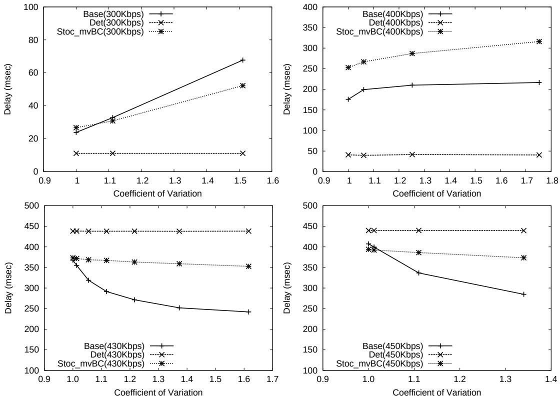
UDP Delay and Loss Results



UDP delay and loss versus rate in mixed traffic-type queue.

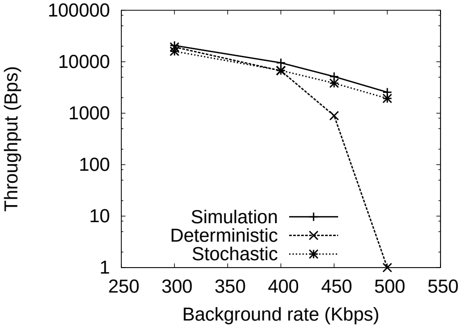
- Deterministic fluid model has no mechanism to allow foreground traffic buffer access at high utilization.
- Stochastic model allows foreground traffic access even in overload situations.

UDP Delay Results versus C_v



UDP delay versus C_v in mixed traffic-type queue.

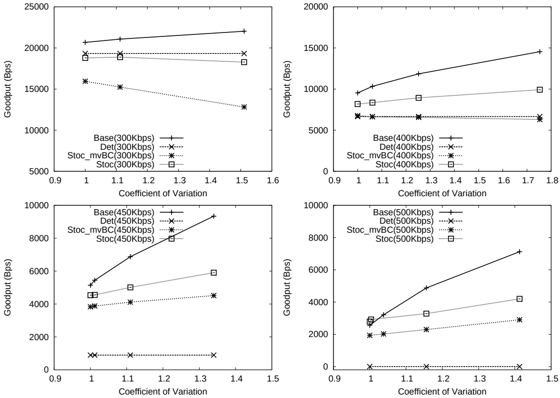
TCP Goodput Results



TCP goodput versus rate in mixed traffic-type queue.

- Stochastic model matches well with simulation results for TCP dynamics.

TCP Goodput versus C_v



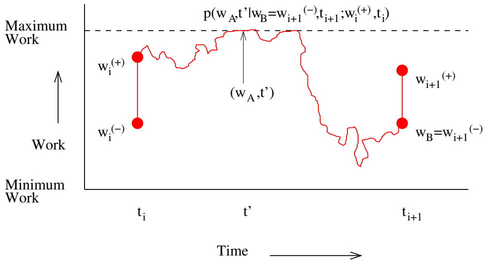
TCP goodput versus C_v in mixed traffic-type queue.

Research Challenges

- Improve Mixed Queue Models:
  - Not happy with $C_v \neq 1$ results.
    Possible solution: discretize distribution [Kobayashi, 1974b] or investigate scaling laws for diffusional drift and variance.
  - Develop the background loss models.
    Possible approach is based upon applications of Bayes Theorem (see below).
- Develop Network Flow Models:
  - Only investigated mixing at single node models to date.
  - Leverage literature of network queueing, e.g., [Whitt, 1995], [Kobayashi and Mark, 2002], others.

Loss Models of Fluid

Application of Bayes Theorem



- Fluid loss (t $\longrightarrow$ t') $= \lambda \int_t^{t'} p(w_{max}, x)dx$
- Bayes Equation: P(A|B) = P(B|A)P(A)/P(B)

$$p(w_A, t'|w_B = w_{i+1}^{(-)}, t_{i+1}; w_i^{(+)}, t_i) = \frac{p(w_B = w_{i+1}^{(-)}, t_{i+1}|w_A, t')p(w_A, t'|w_i^{(+)}, t_i)}{p(w_B = w_{i+1}^{(-)}, t_{i+1}|w_i^{(+)}, t_i)} \quad (8)$$

Summary

- Initial investigations into mixed, hybrid stochastic simulation models
- Much work to be done:
  - Improve $C_v \neq 1$ results
  - Develop time-dependent fluid loss models
  - Develop network flow models, i.e. time-dependent network calculus
- Outlook
  - Initial simulation results are encouraging
  - Need much more development and simulation studies results of time-dependent dynamics

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