Game Playing

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Outline

- Games

- Perfect play
  - minimax decisions
  - α–β pruning

- Resource limits and approximate evaluation

- Games of chance

- Games of imperfect information

games

Games vs. Search Problems

- "Unpredictable" opponent ⇒ solution is a strategy specifying a move for every possible opponent reply

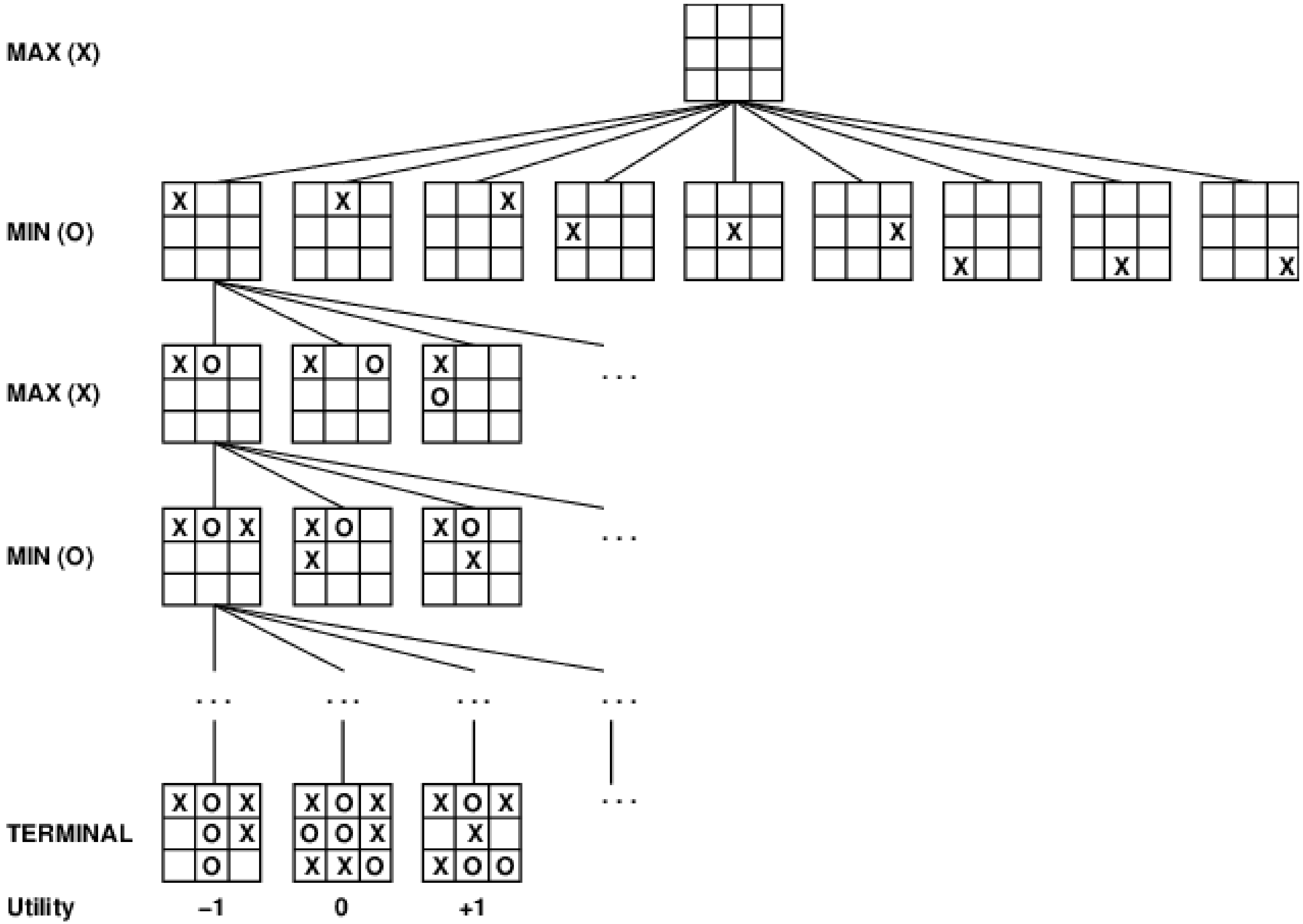
- Time limits ⇒ unlikely to find goal, must approximate

- Plan of attack:
  - computer considers possible lines of play (Babbage, 1846)
  - algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
  - finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
  - first Chess program (Turing, 1951)
  - machine learning to improve evaluation accuracy (Samuel, 1952–57)
  - pruning to allow deeper search (McCarthy, 1956)

Types of Games

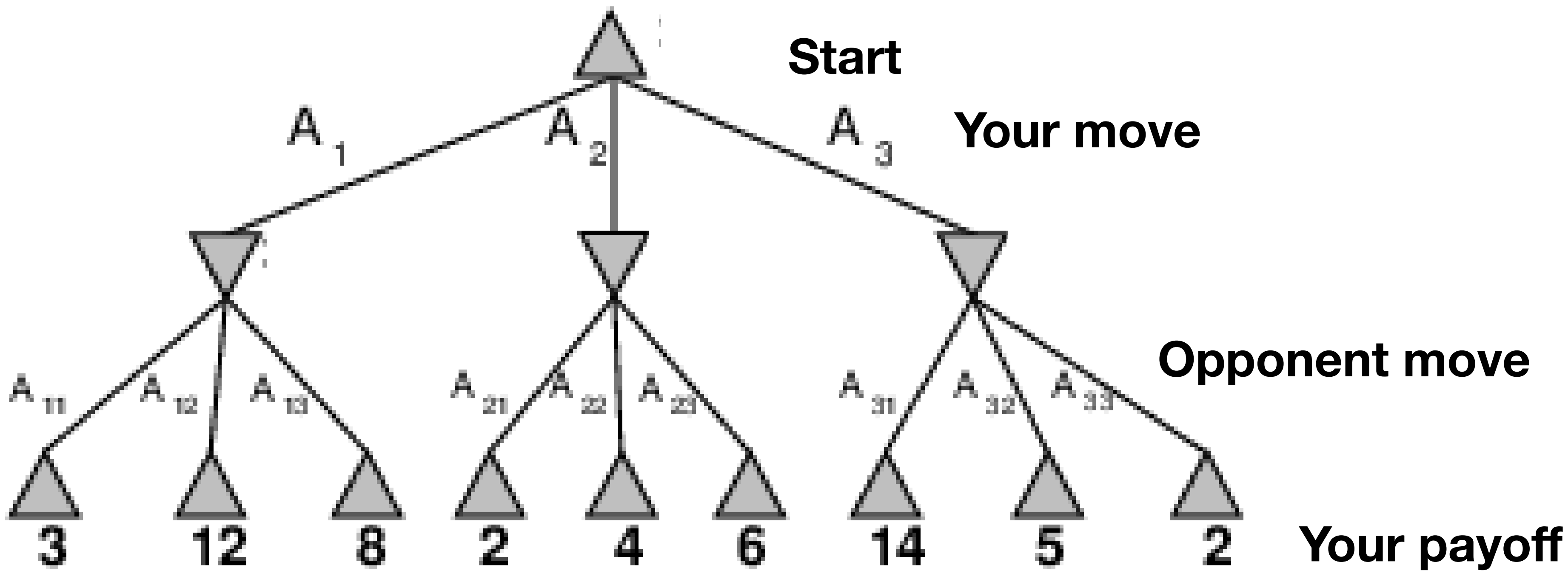
	deterministic	chance
perfect information	Chess Checkers Go Othello	Backgammon Monopoly
imperfect information	battleships Blind Tic Tac Toe	Bridge Poker Scrabble

Game Tree (2-player, Deterministic, Turns)



Simple Game Tree

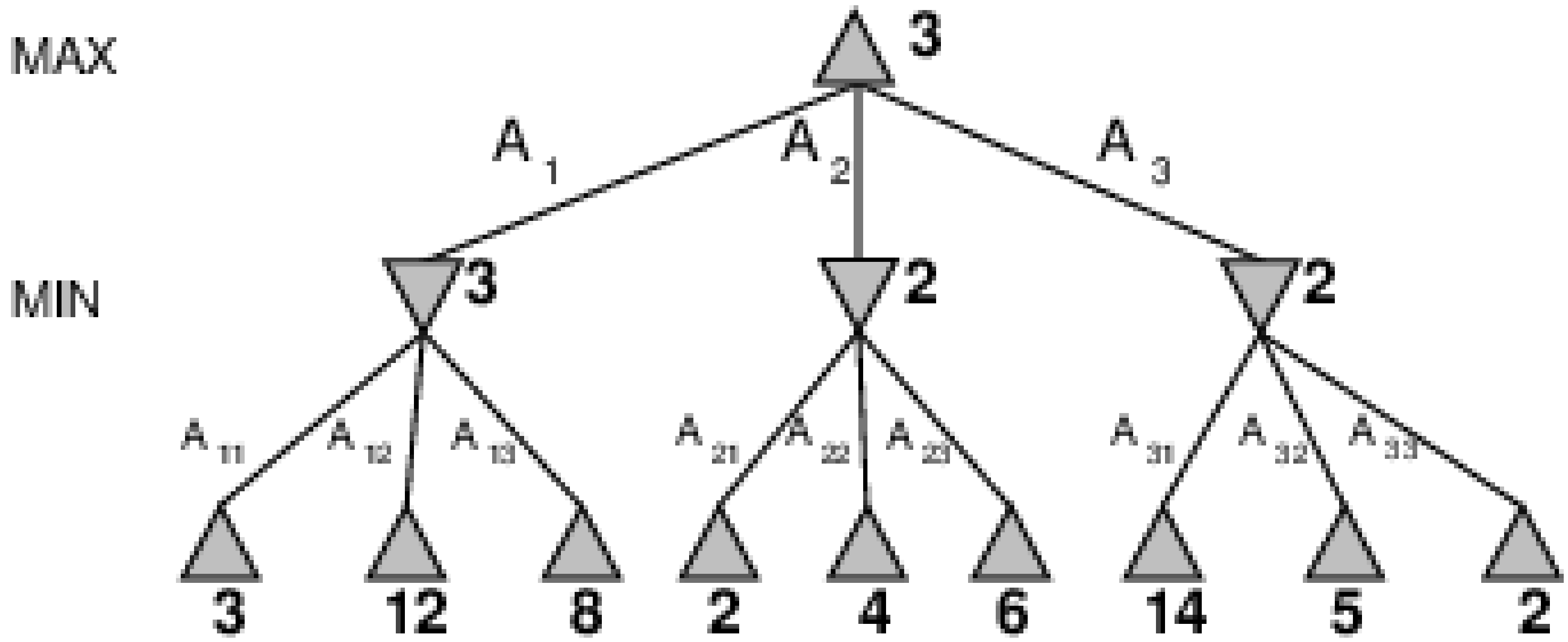
- 2 player game
- Each player has one move
- You move first
- Goal: optimize your payoff (utility)



minimax

Minimax

- Perfect play for deterministic, perfect-information games
- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play
- E.g., 2-player game, one move each:



Minimax Algorithm

```
function MINIMAX-DECISION(state) returns an action
   inputs: state, current state in game
   return the a in ACTIONS(state) maximizing MIN-VALUE(RESULT(a, state))
```

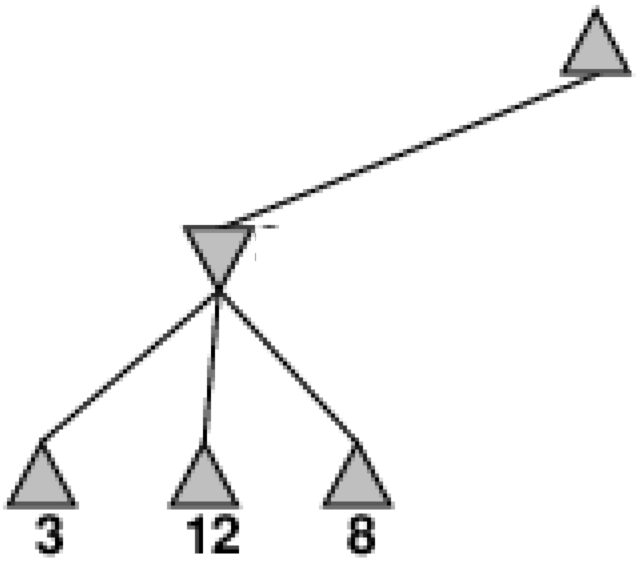
```
function MAX-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for a, s in SUCCESSORS(state) do v ← MAX(v, MIN-VALUE(s))
   return v
```

```
function MIN-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← ∞
   for a, s in SUCCESSORS(state) do v ← MIN(v, MAX-VALUE(s))
   return v
```

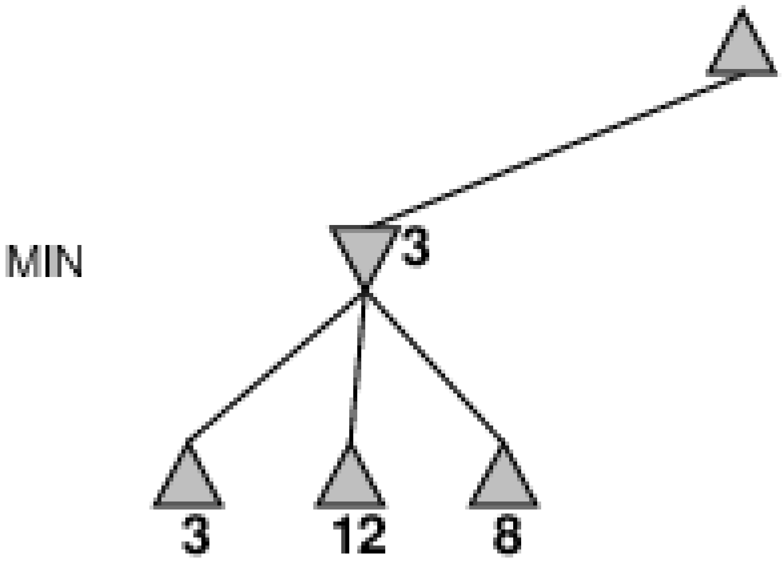
Properties of Minimax

- Complete? Yes, if tree is finite
- Optimal? Yes, against an optimal opponent. Otherwise??
- Time complexity? $O(b^m)$
- Space complexity? $O(bm)$ (depth-first exploration)
- For Chess, $b \approx 35$, $m \approx 100$ for "reasonable" games
  $\Rightarrow$ exact solution completely infeasible
- But do we need to explore every path?

α–β Pruning Example



α–β Pruning Example



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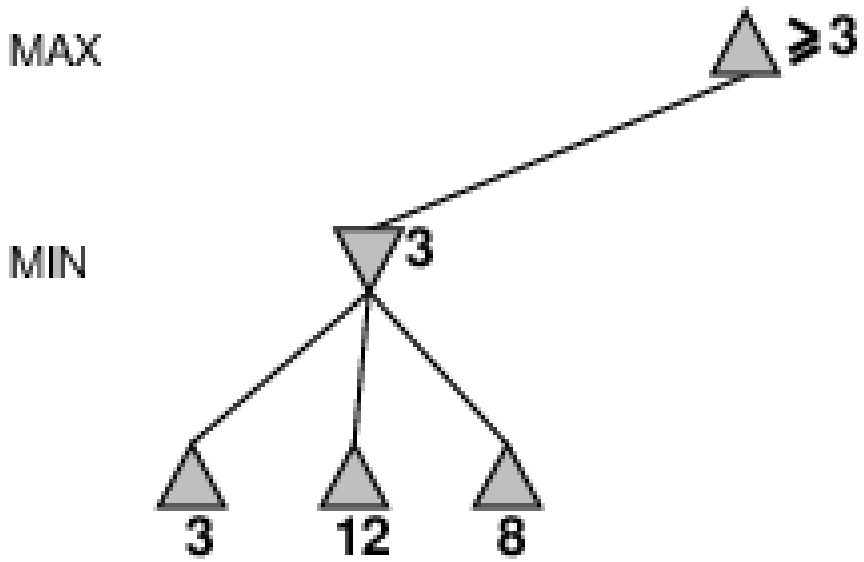

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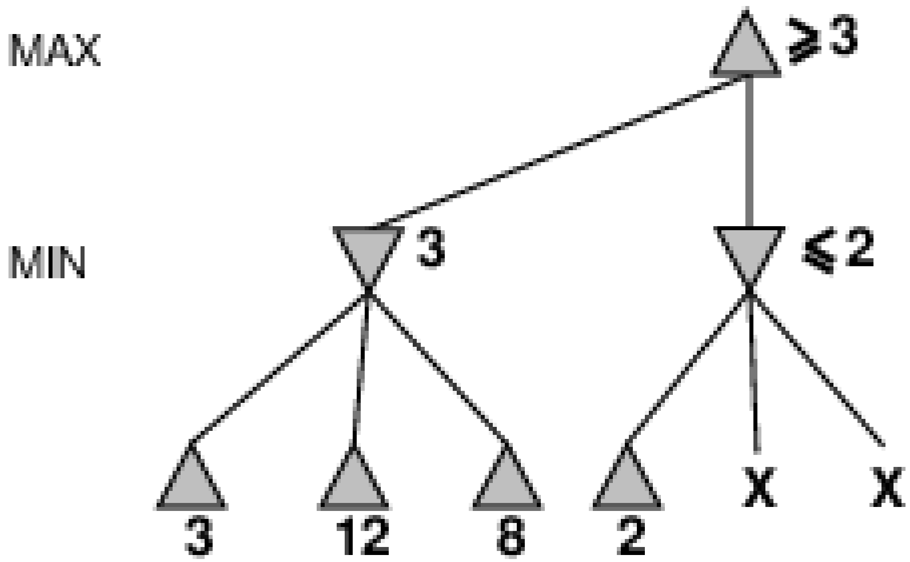

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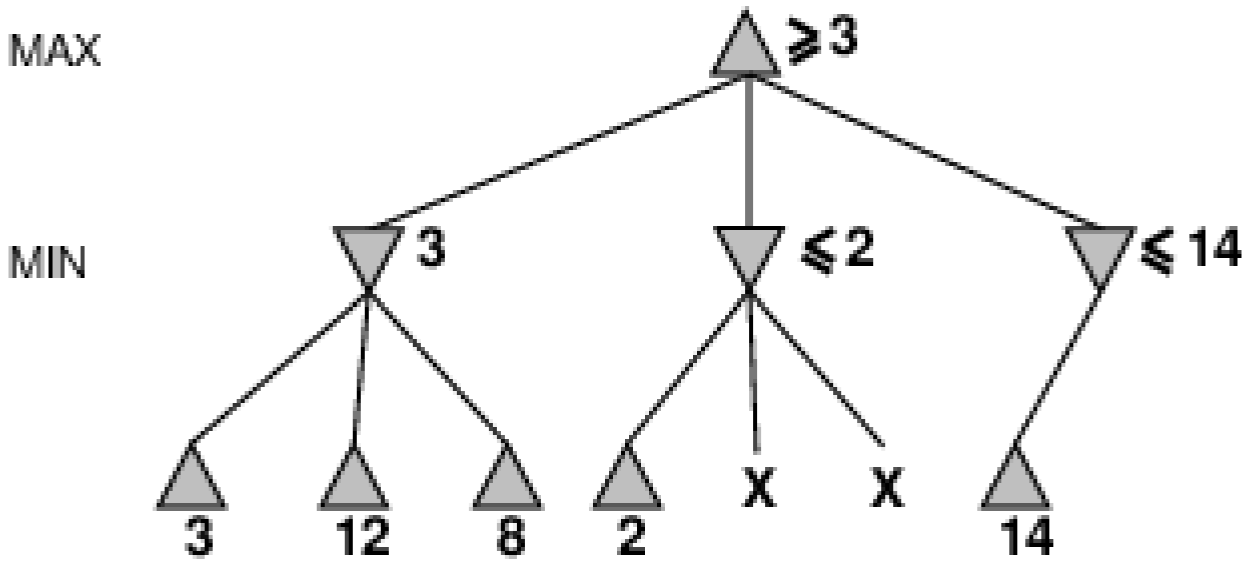

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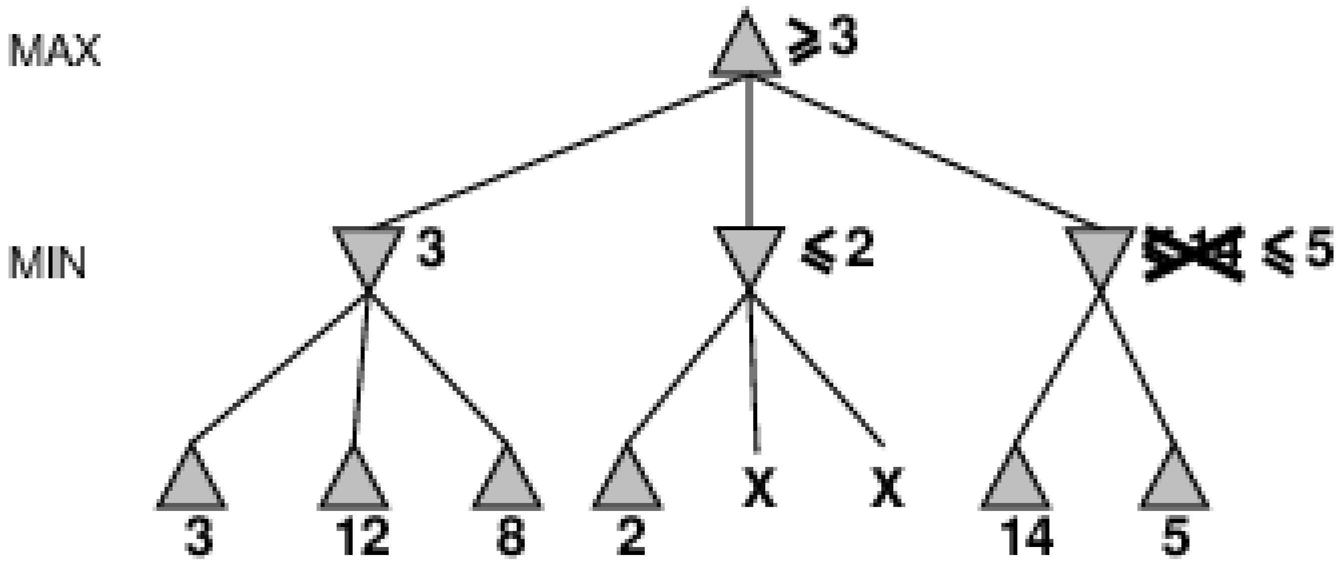

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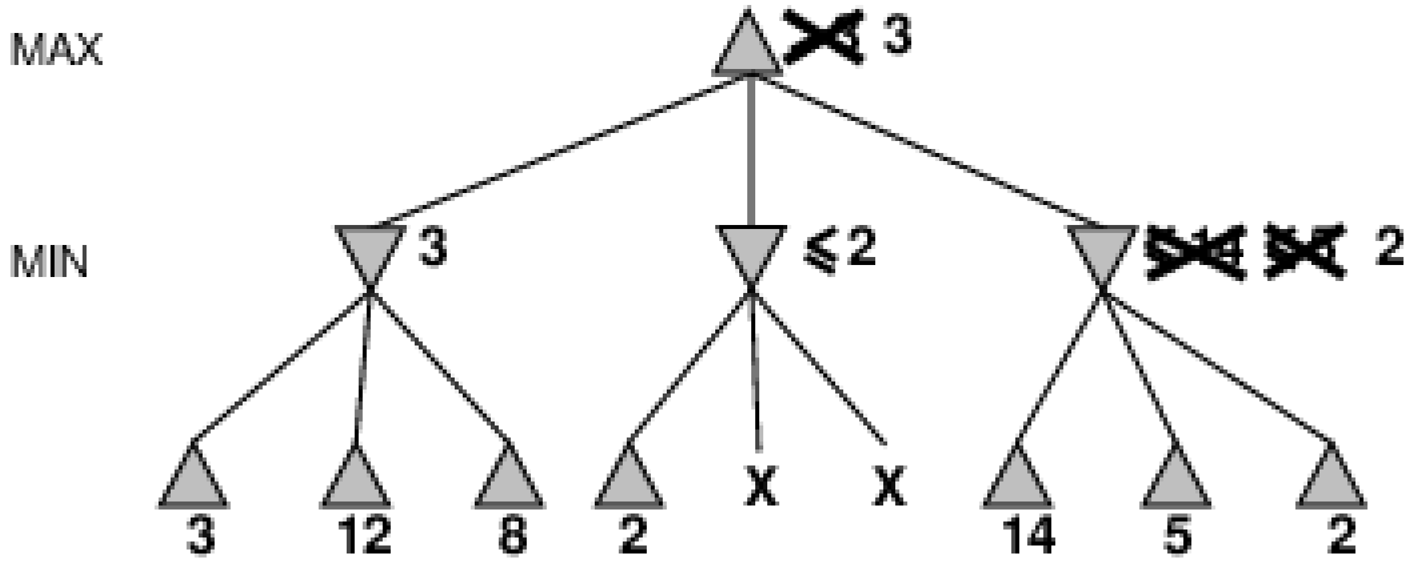
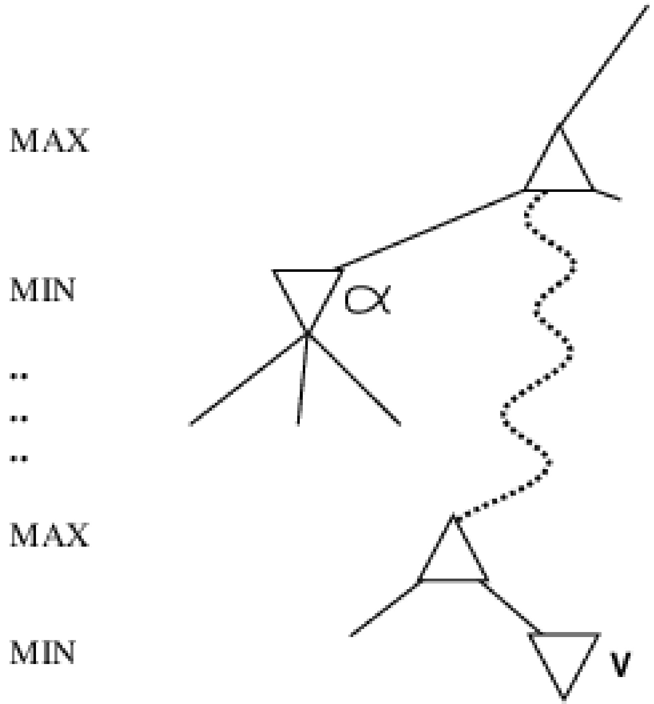


Why is it Called α–β?



- α is the best value (to MAX) found so far off the current path
- If V is worse than α, MAX will avoid it $\Rightarrow$ prune that branch
- Define β similarly for MIN

The α–β Algorithm


```
function ALPHA-BETA-DECISION(state) returns an action
   return the a in ACTIONS(state) maximizing MIN-VALUE(RESULT(a, state))
--------------------------------------------------------------------------
function MAX-VALUE(state, α, β) returns a utility value
   inputs: state, current state in game
           α, the value of the best alternative for MAX along the path to state
           β, the value of the best alternative for MIN along the path to state

   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for a, s in SUCCESSORS(state) do
      v ← MAX(v, MIN-VALUE(s, α, β))
      if v ≥ β then return v
      α ← MAX(α, v)
   return v
--------------------------------------------------------------------------
function MIN-VALUE(state, α, β) returns a utility value
   same as MAX-VALUE but with roles of α, β reversed
```

Properties of α–β

- Safe: Pruning **does not** affect final result
- Good move ordering improves effectiveness of pruning
- With "perfect ordering," time complexity = $O(b^{m/2})$
  $\Rightarrow$ **doubles** solvable depth
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)
- Unfortunately, 35^{50} is still impossible!

Solved Games

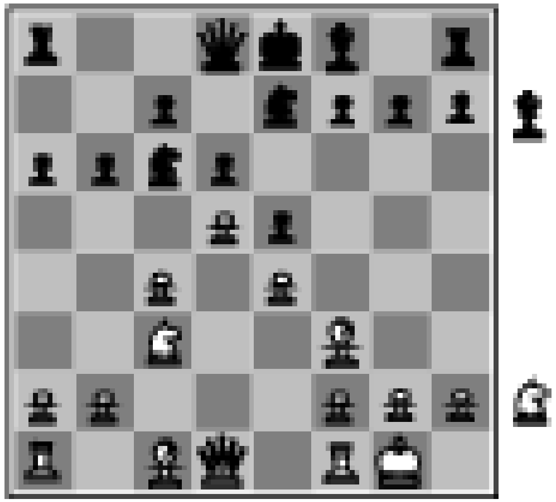
- A game is solved if optimal strategy can be computed
- Tic Tac Toe can be trivially solved
- Biggest solved game: Checkers
  - proof by Schaeffer in 2007
  - both players can force at least a draw
- Most games (Chess, Go, etc.) too complex to be solved

resource limits

Resource Limits

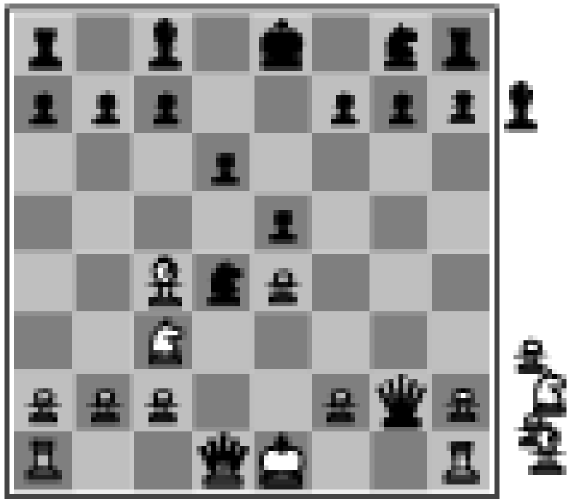
- Standard approach:
  - Use CUTOFF-TEST instead of TERMINAL-TEST
    e.g., depth limit
  - Use EVAL instead of UTILITY
    i.e., evaluation function that estimates desirability of position
- Suppose we have 100 seconds, explore 10^4 nodes/second
  $\Rightarrow$ 10^6 nodes per move $\approx 35^{8/2}$
  $\Rightarrow$ α–β reaches depth 8 $\Rightarrow$ pretty good Chess program

Evaluation Functions



Black to move

White slightly better



White to move

Black winning

- For Chess, typically linear weighted sum of features

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

e.g., $f_1(s)$ = (number of white queens) – (number of black queens)

Evaluation Function for Chess

- Long experience of playing Chess

⇒ Evaluation of positions included in Chess strategy books

  - bishop is worth 3 pawns
  - knight is worth 3 pawns
  - rook is worth 5 pawns
  - good pawn position is worth 0.5 pawns
  - king safety is worth 0.5 pawns
  - etc.

- Pawn count → weight for features

Learning Evaluation Functions

- Designing good evaluation functions requires a lot of expertise

- Machine learning approach
  - collect a large database of games play
  - note for each game who won
  - try to predict game outcome from features of position

  ⇒ learned weights

- May also learn evaluation functions from self-play

Some Concerns

- Quiescence
  - position evaluation not reliable if board is unstable
  - e.g., Chess: queen will be lost in next move
  - → deeper search of game-changing moves required

- Horizon Effect
  - adverse move can be delayed, but not avoided
  - search may prefer to delay, even if costly

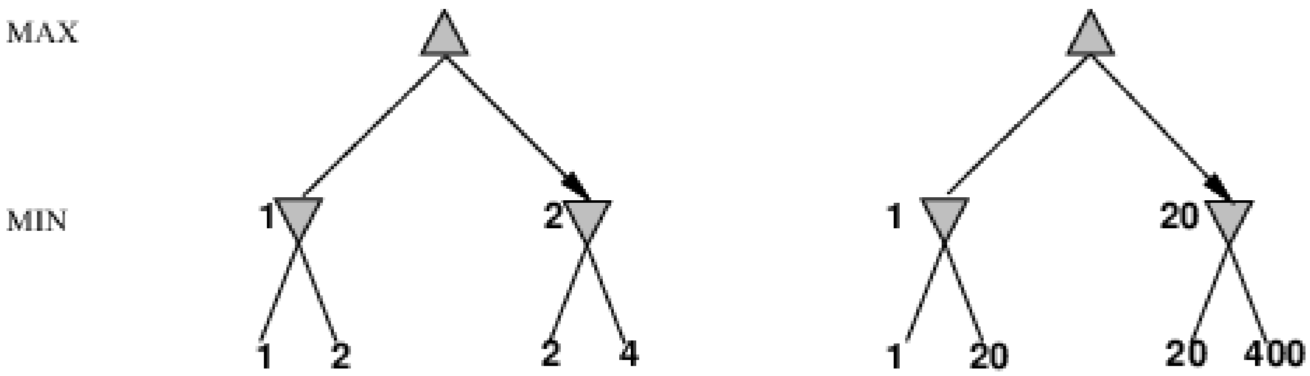
Forward Pruning

- Idea: avoid computation on clearly bad moves
- Cut off searches with bad positions before they reach max-depth
- Risky: initially inferior positions may lead to better positions
- Beam search: explore fixed number of promising moves deeper

Lookup instead of Search

- Library of opening moves
  - even expert Chess players use standard opening moves
  - these can be memorized and followed until divergence
- End game
  - if only few pieces left, optimal final moves may be computed
  - Chess end game with 6 pieces left solved in 2006
  - can be used instead of evaluation function

Digression: Exact Values do not Matter



- Behaviour is preserved under any **monotonic** transformation of EVAL
- Only the order matters:
  payoff in deterministic games acts as an ordinal utility function

Deterministic Games in Practice

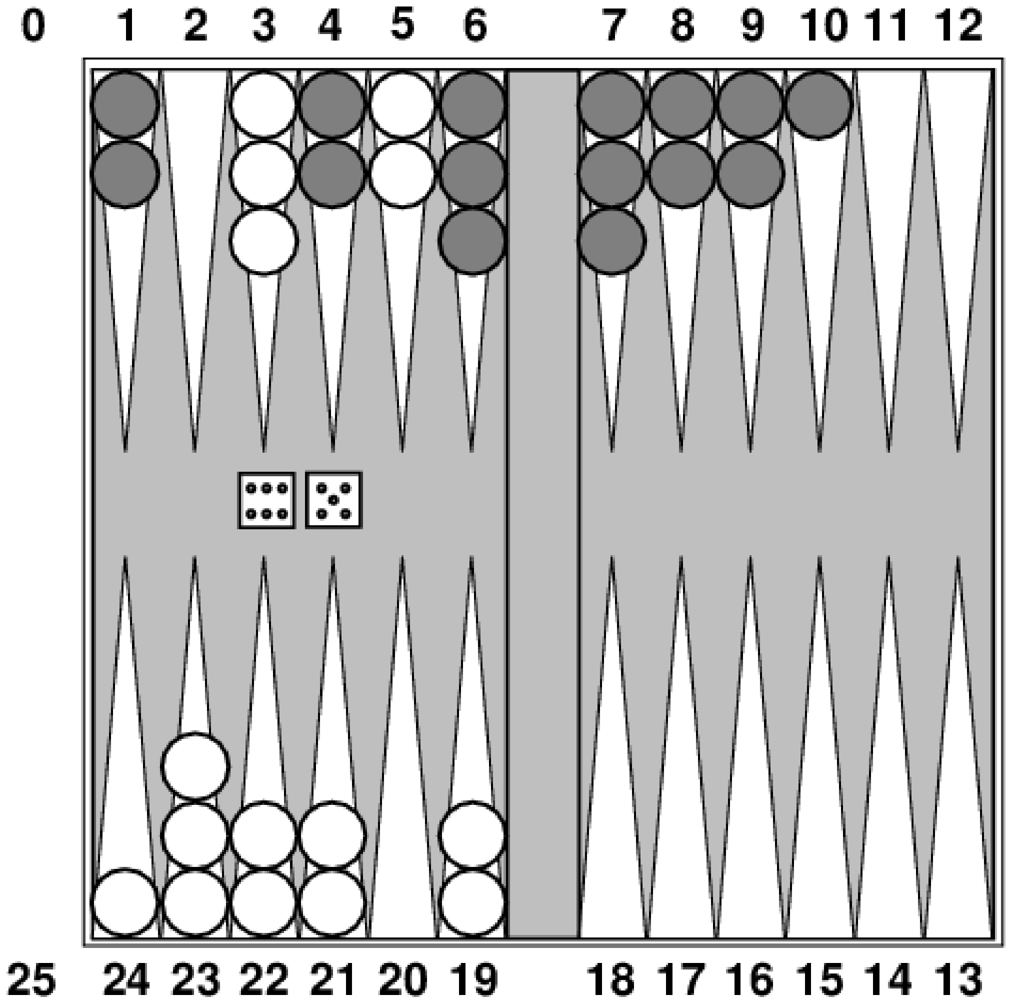
- **Checkers:** Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions. Weakly solved in 2007 by Schaeffer (guaranteed draw).

- **Chess:** Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

- **Go:** In 2016, computer using a neural network for the board evaluation function, was able to beat the human Go champion for the first time. Given the huge branching factor ($b > 300$), Go was long considered too difficult for machines. **We will have a dedicated lecture about this towards the end of the course**

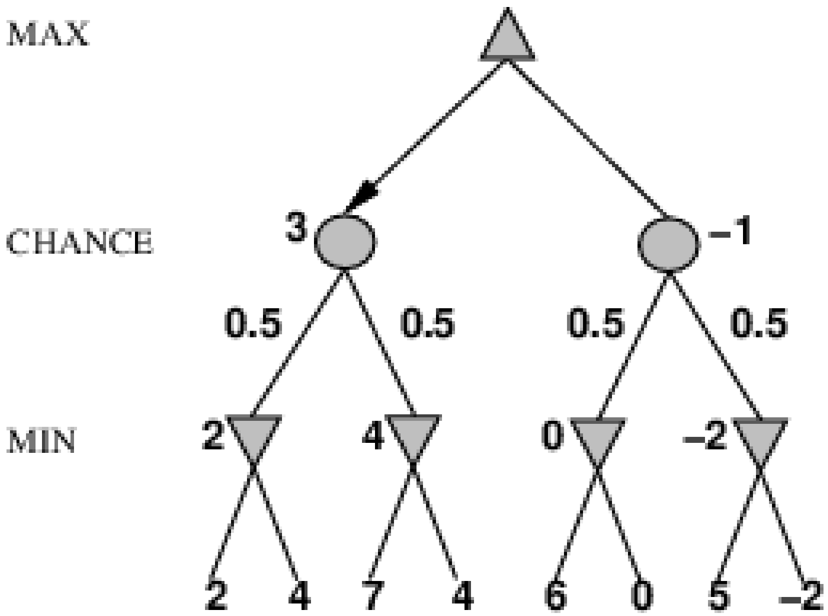
games of chance

Nondeterministic Games: Backgammon



Nondeterministic Games in General

- In nondeterministic games, chance introduced by dice, card-shuffling
- Simplified example with coin-flipping:



Algorithm for Nondeterministic Games

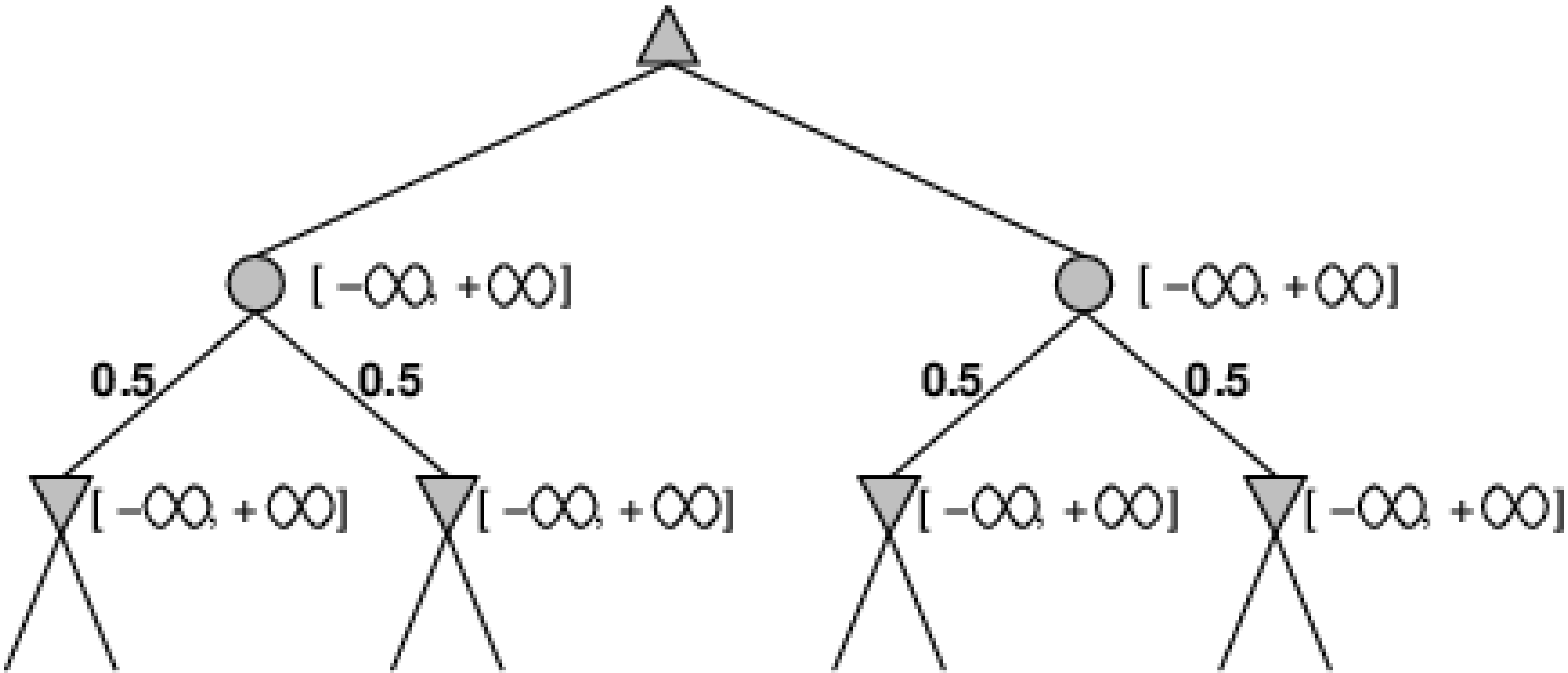
- EXPECTIMINIMAX gives perfect play

- Just like MINIMAX, except we must also handle chance nodes:

```
...
if state is a MAX node then
        return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a MIN node then
        return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a chance node then
        return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
...
```

Pruning in Nondeterministic Game Trees

A version of α-β pruning is possible:



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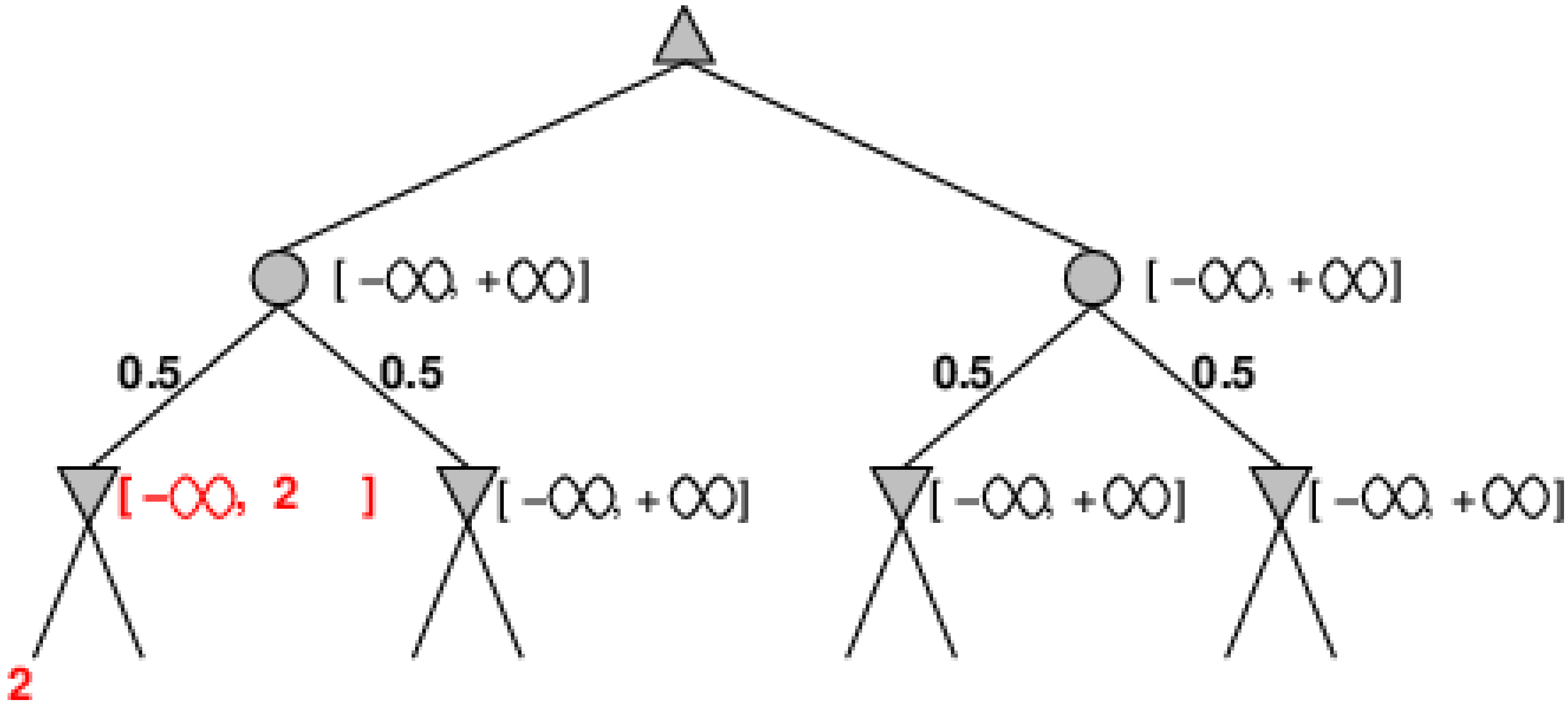

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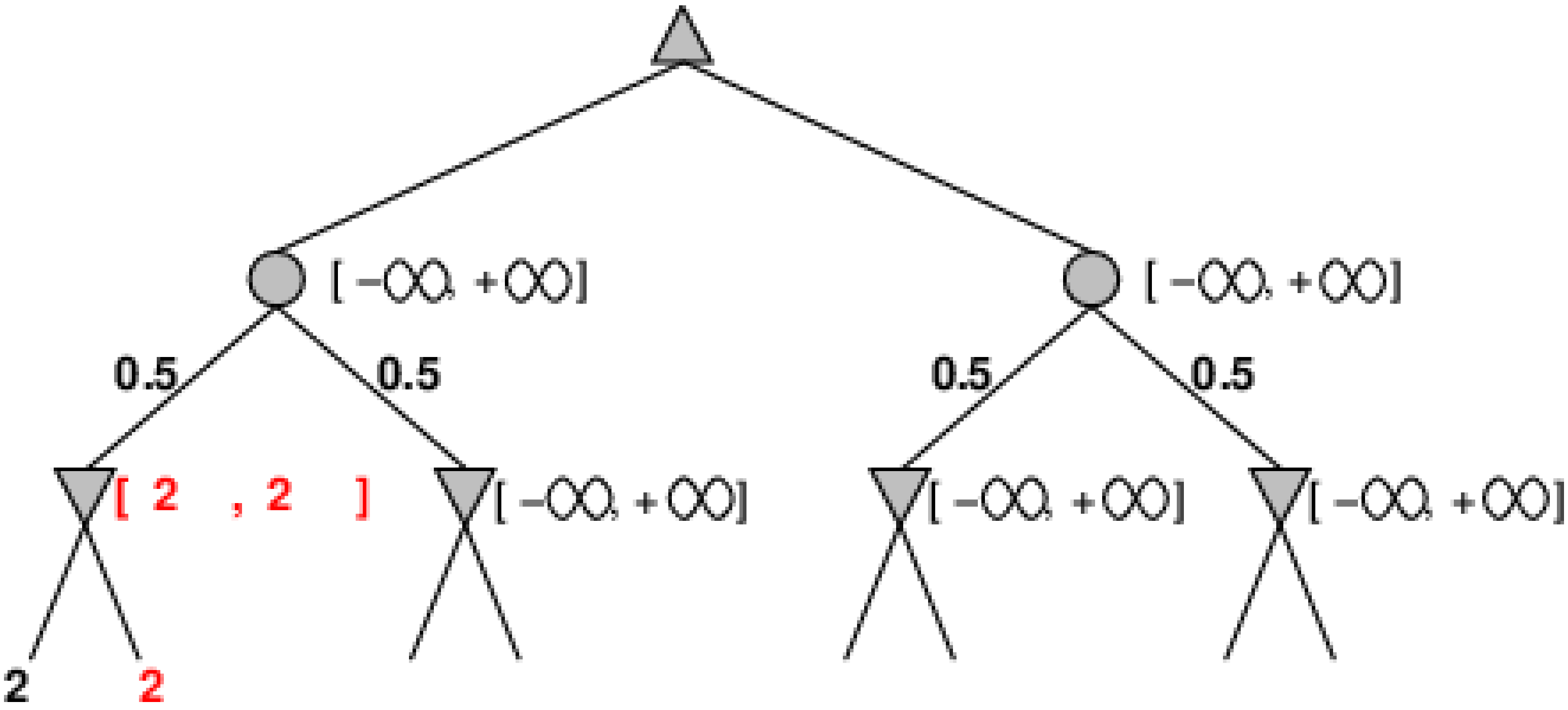

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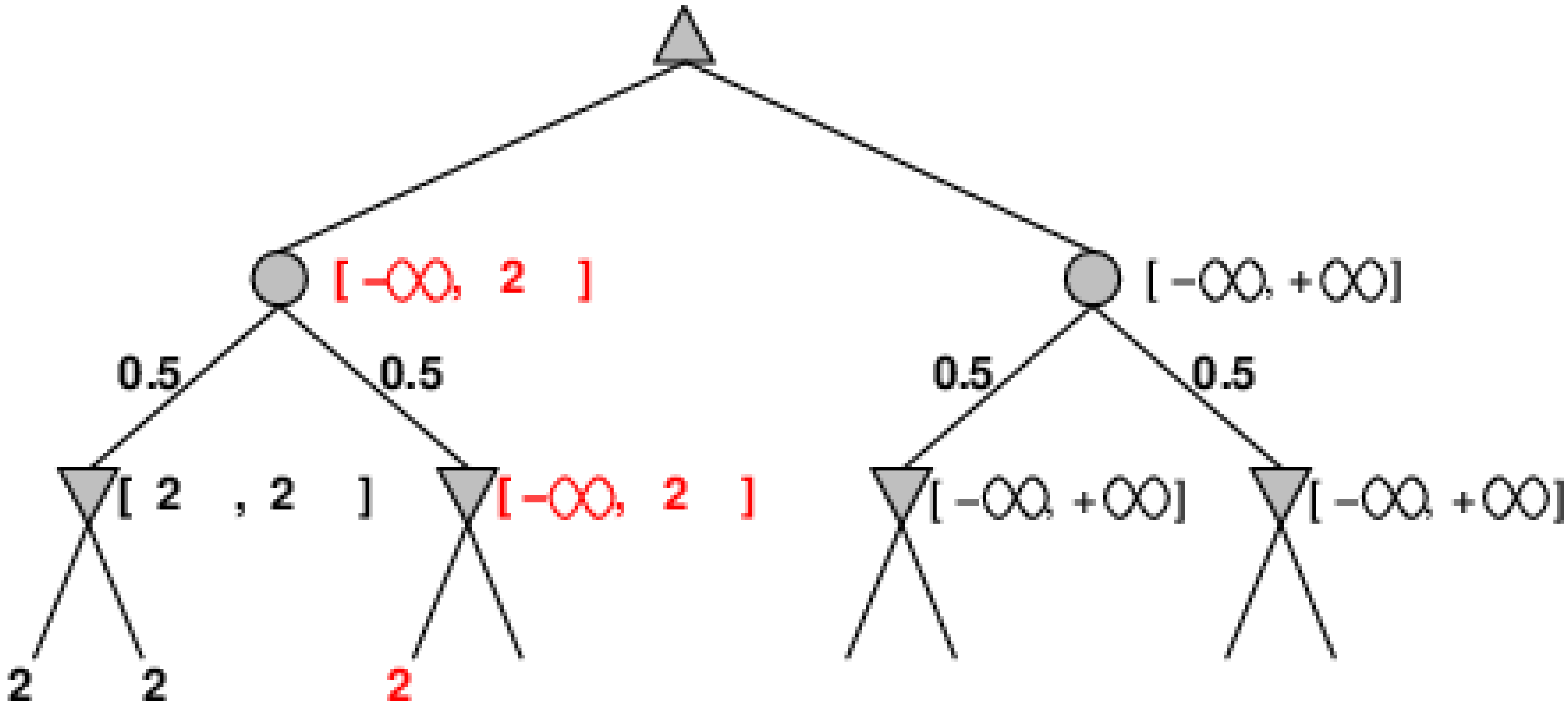

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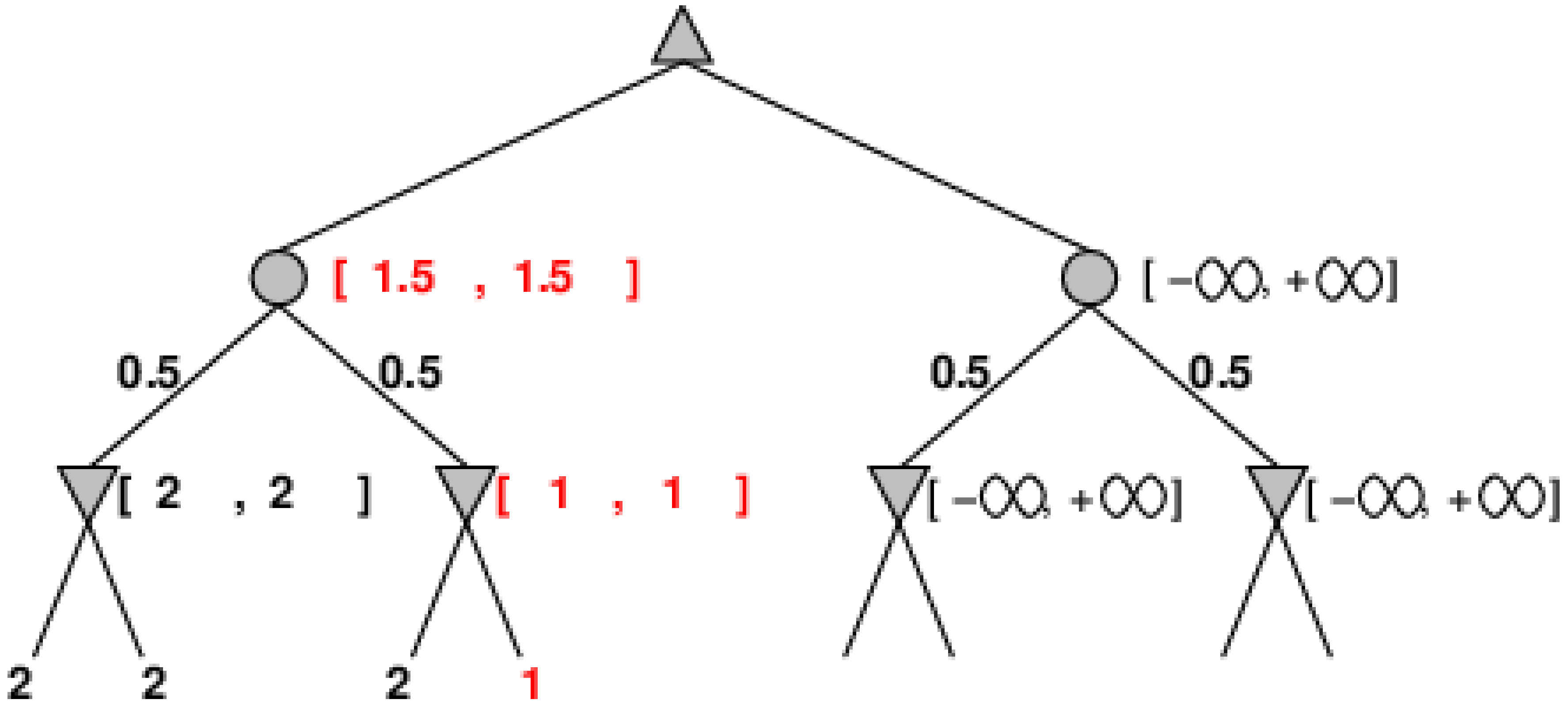

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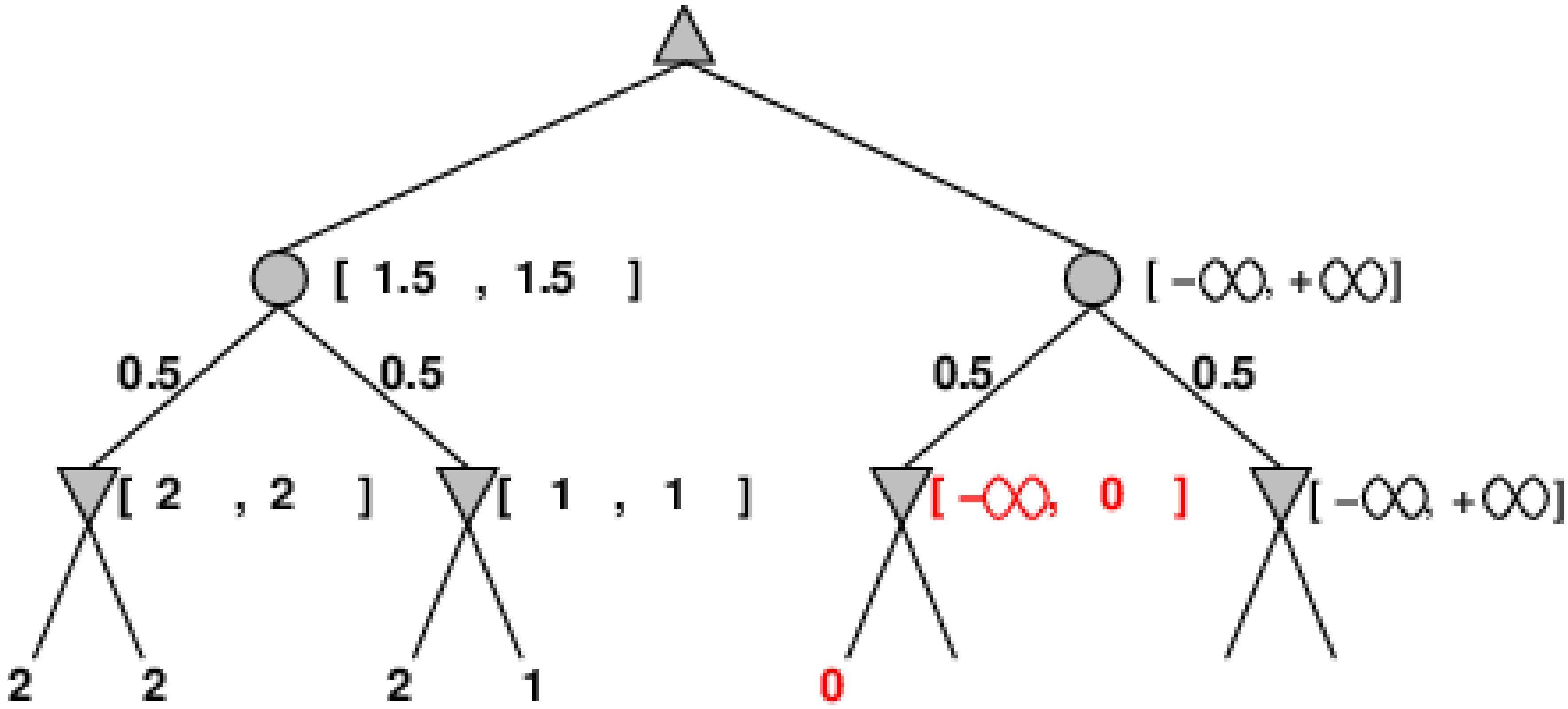

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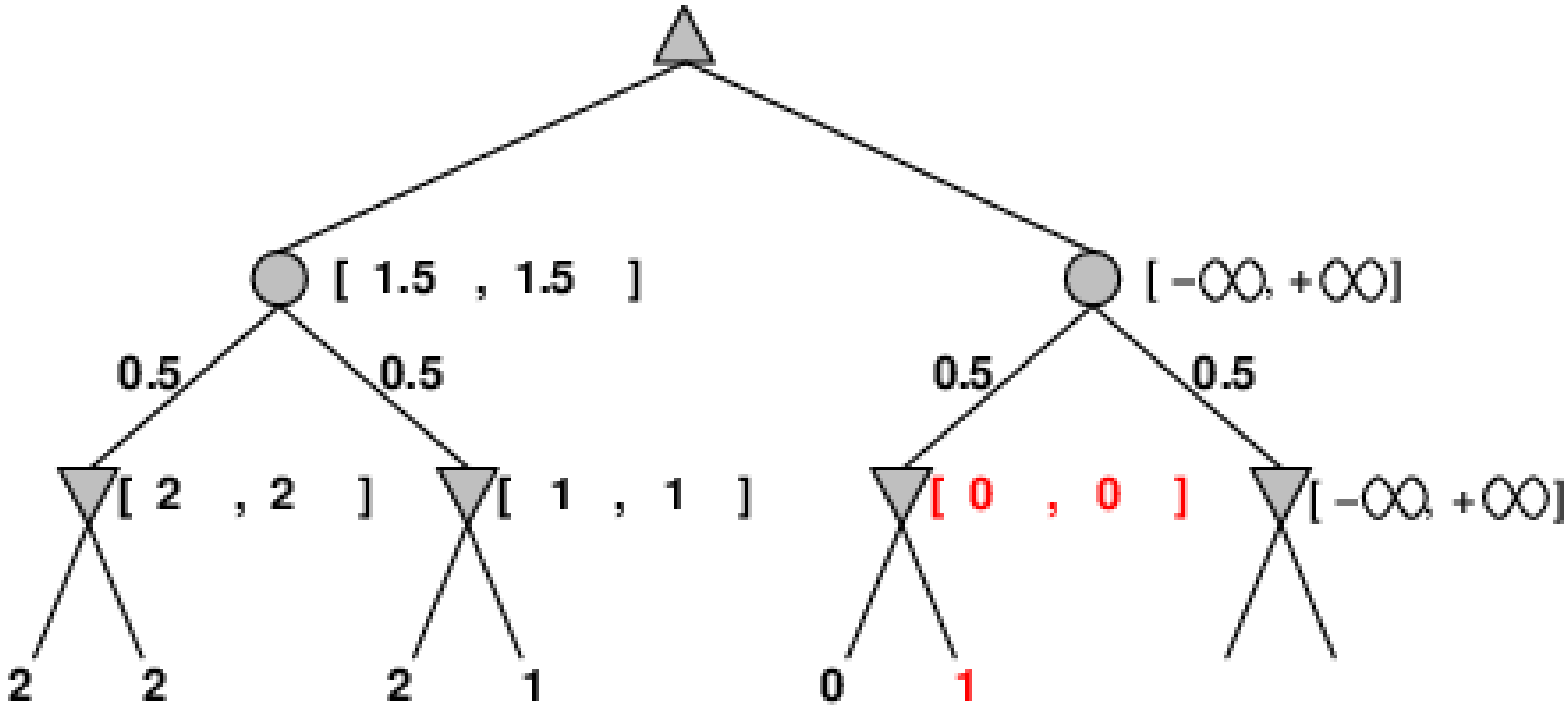
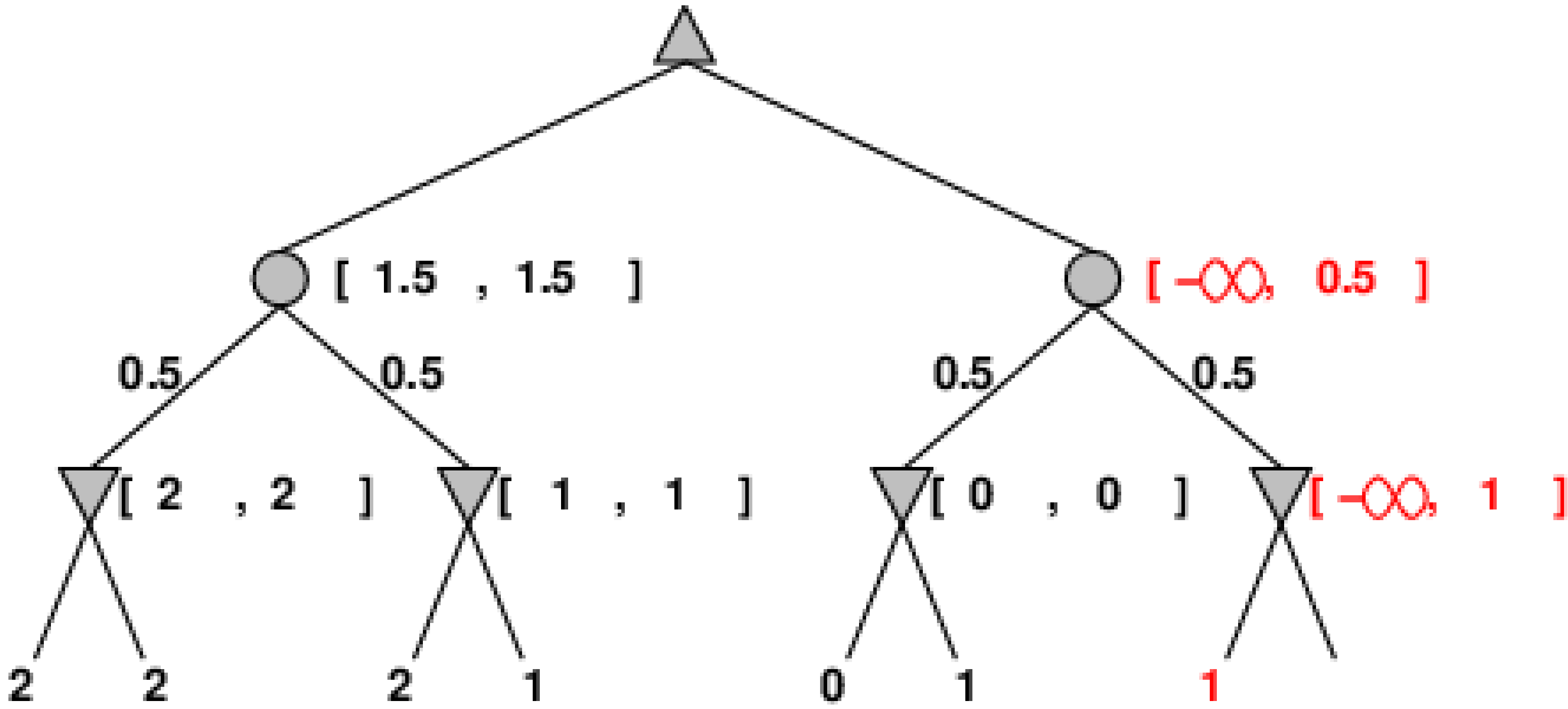


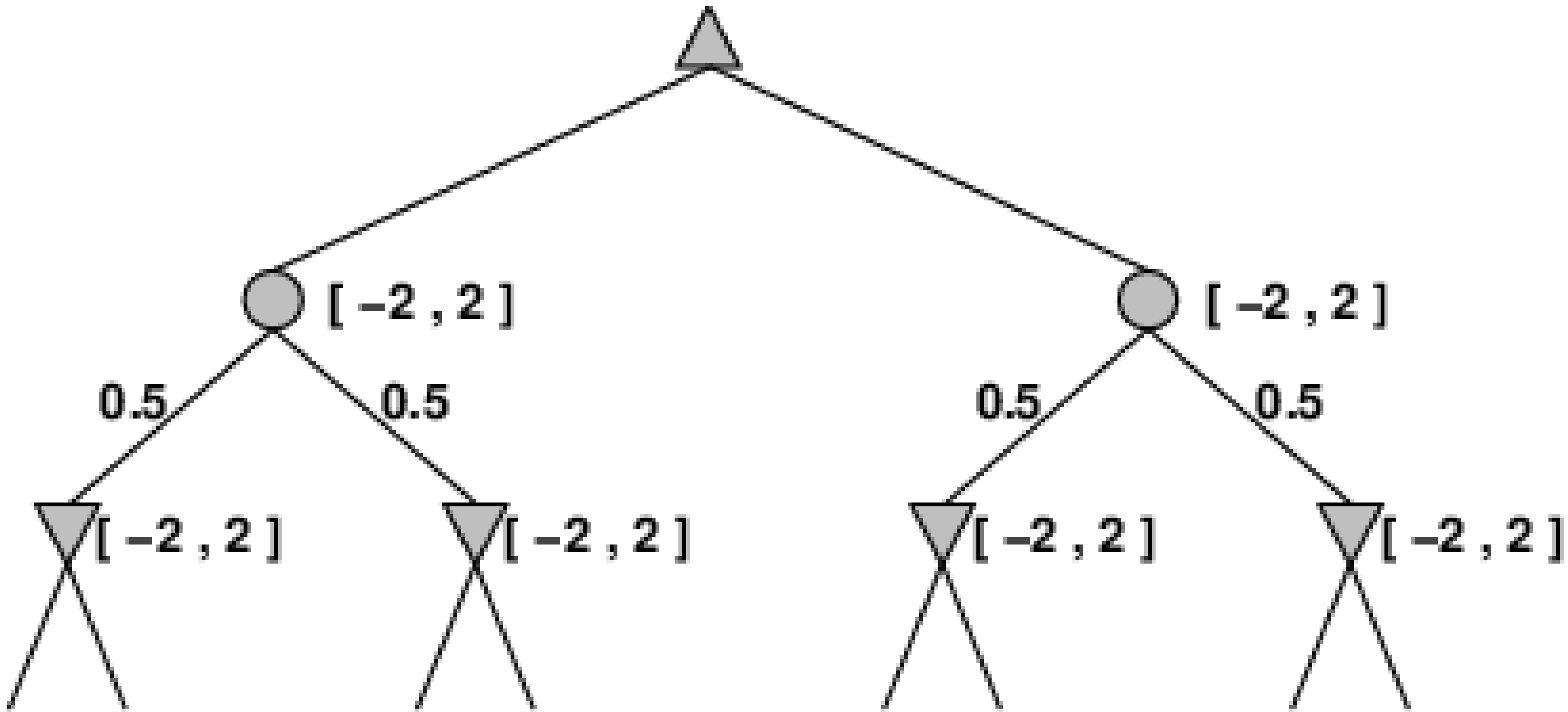
Pruning in Nondeterministic Game Trees

Terminate, since left path will be worth more on average.



Pruning with Bounds

More pruning occurs if we can bound the leaf values (0,1,2)



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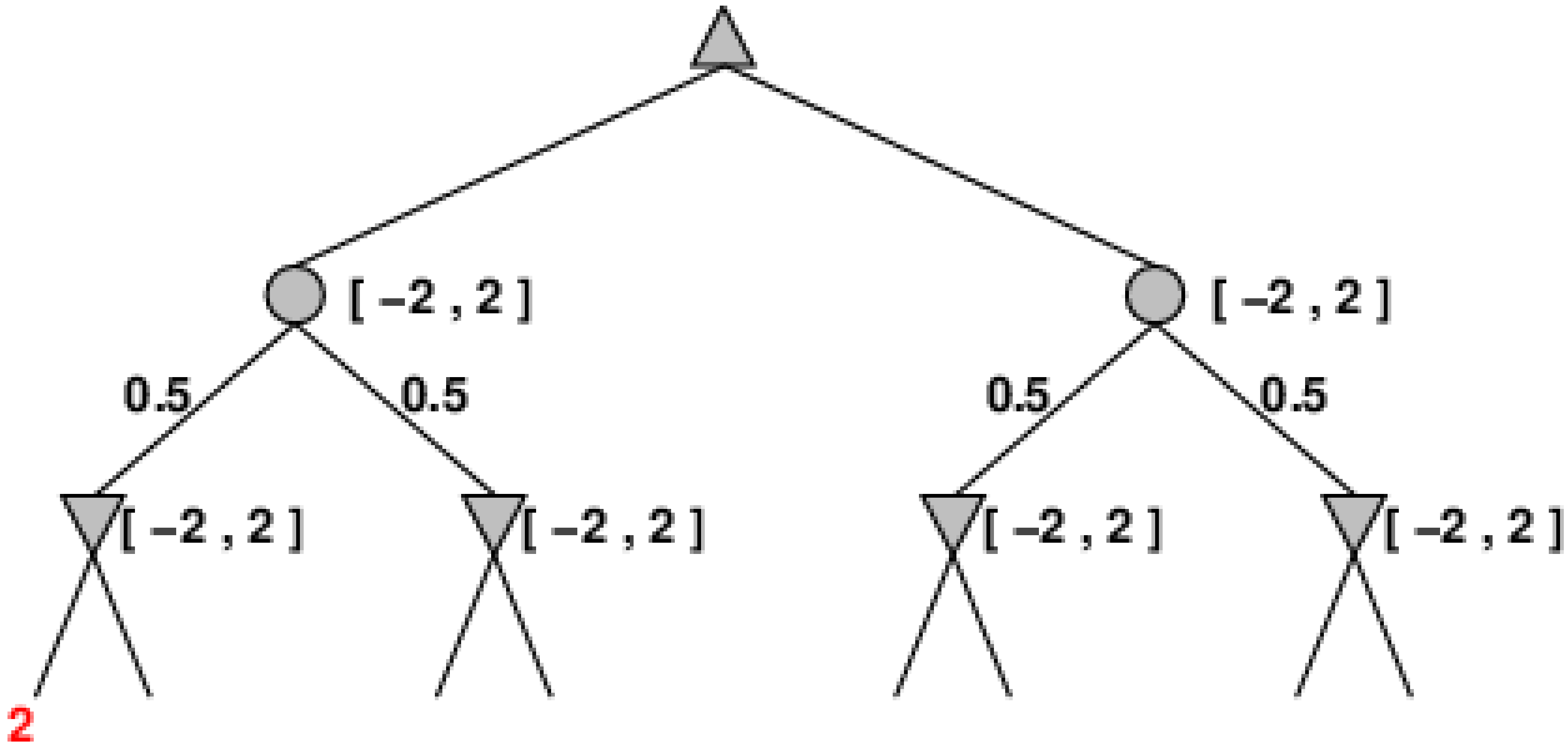

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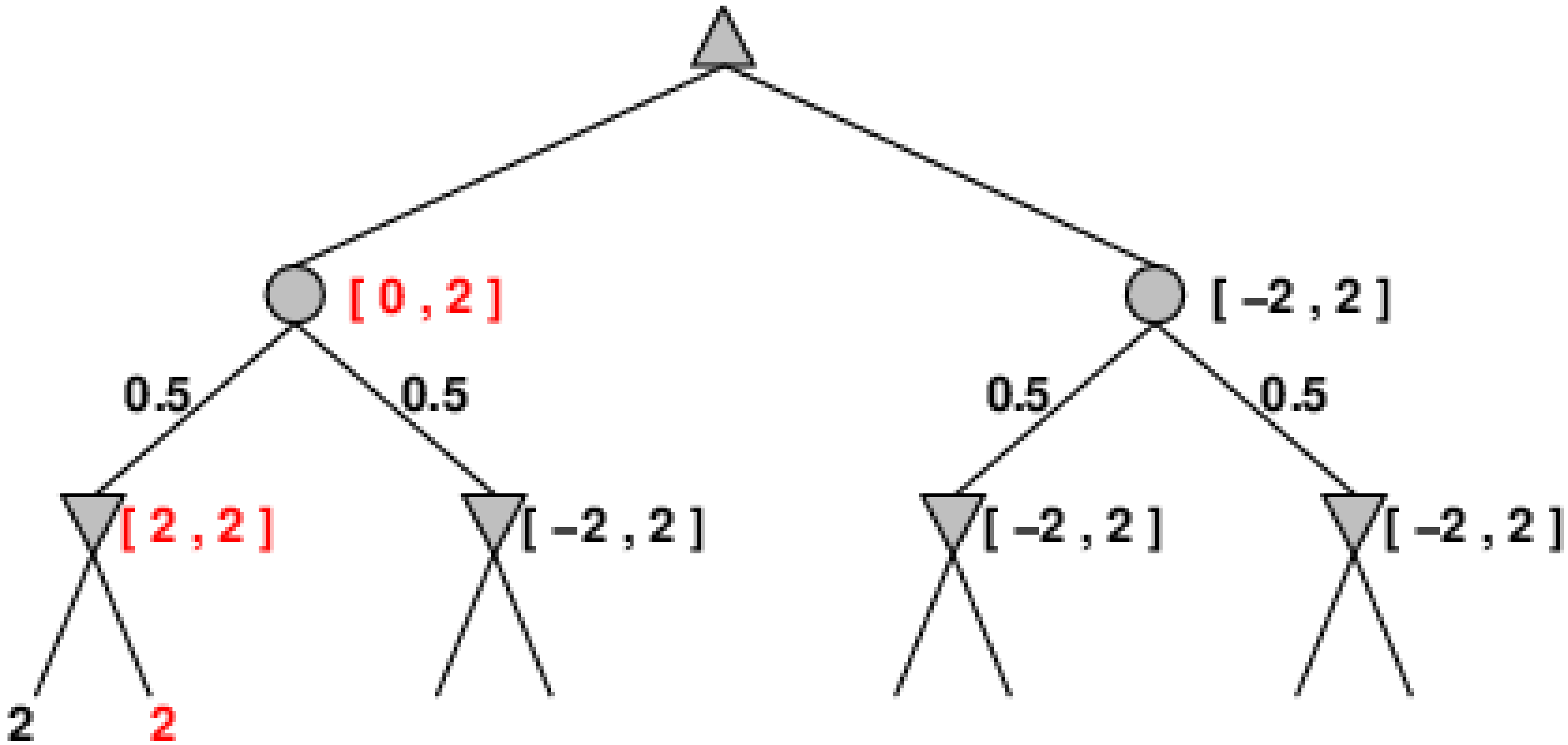

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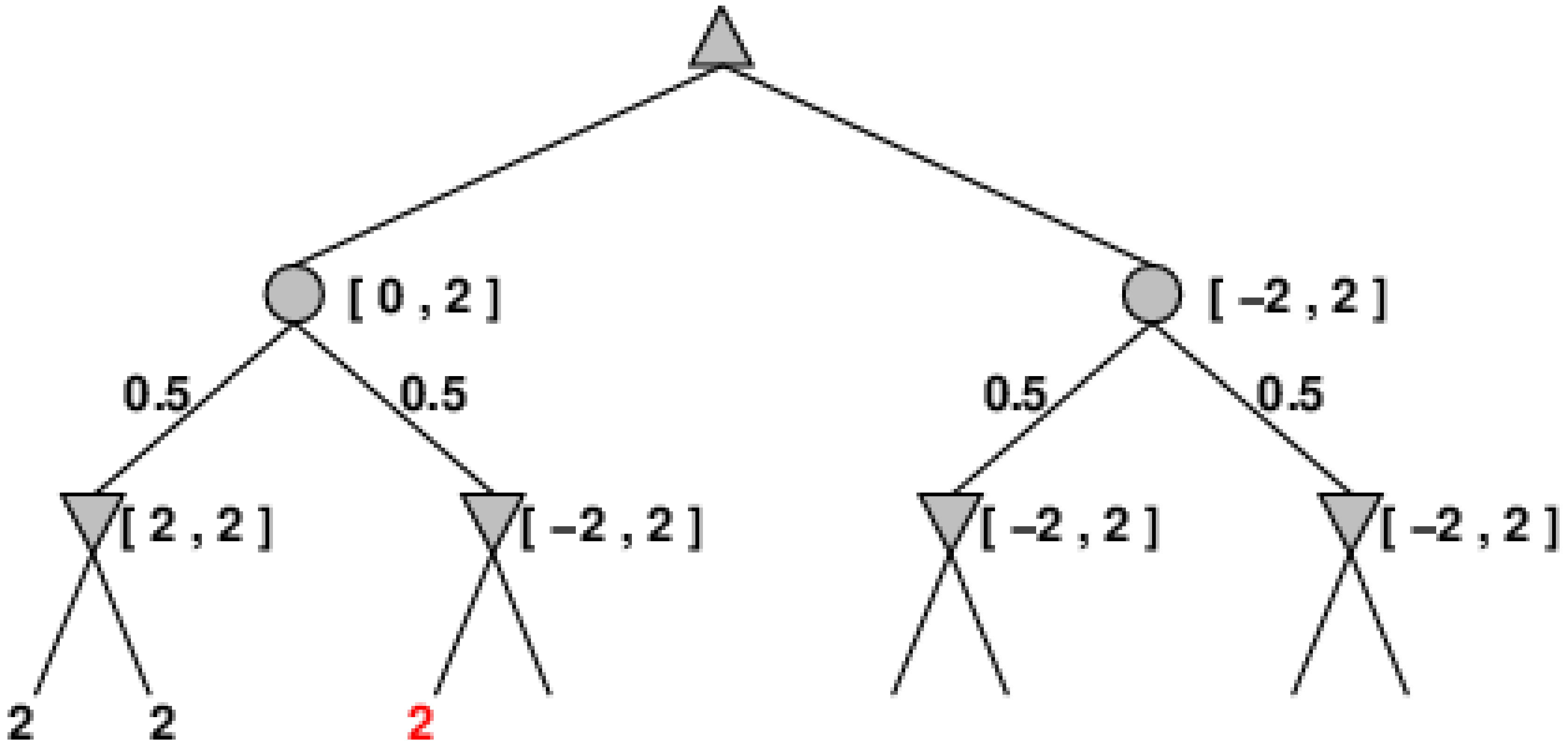

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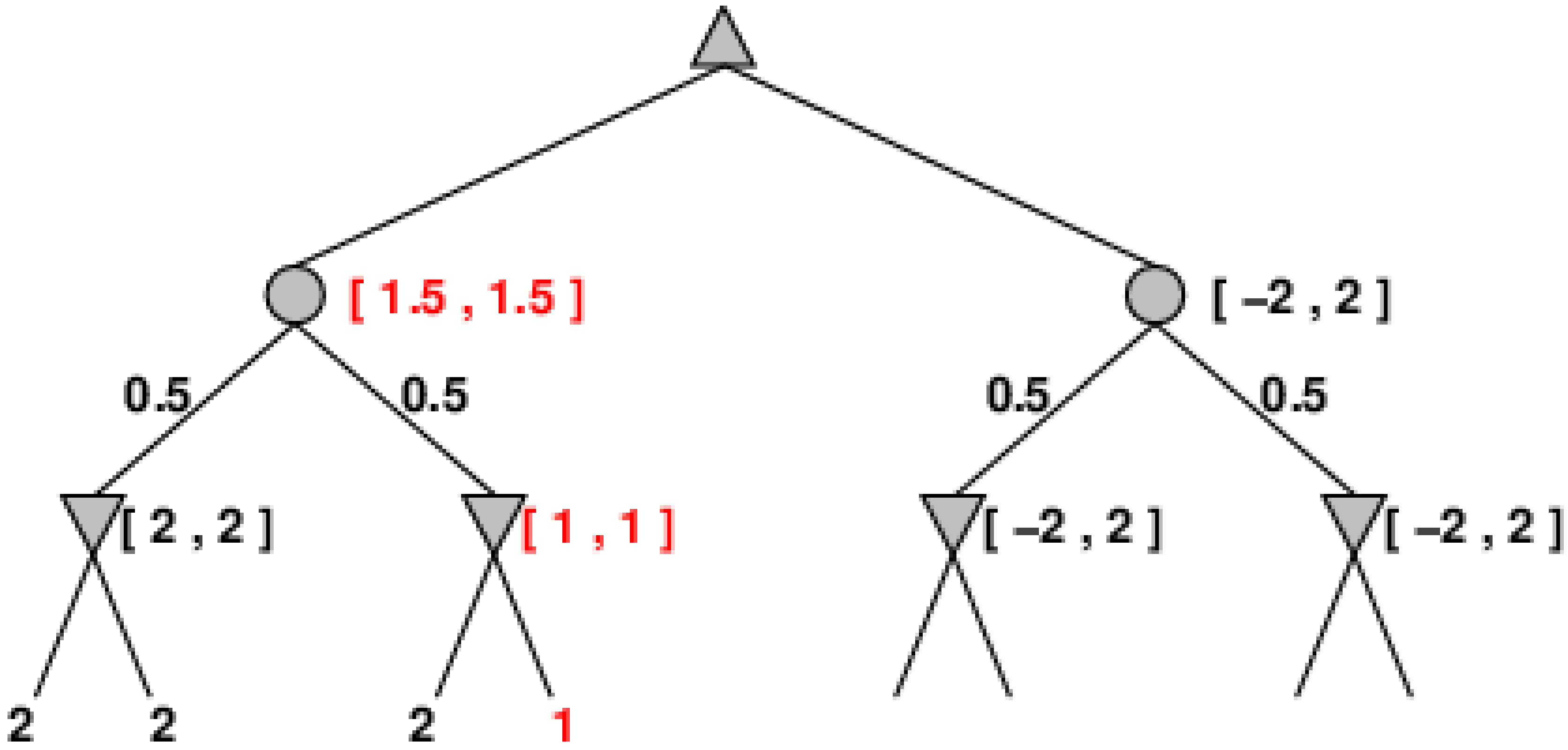
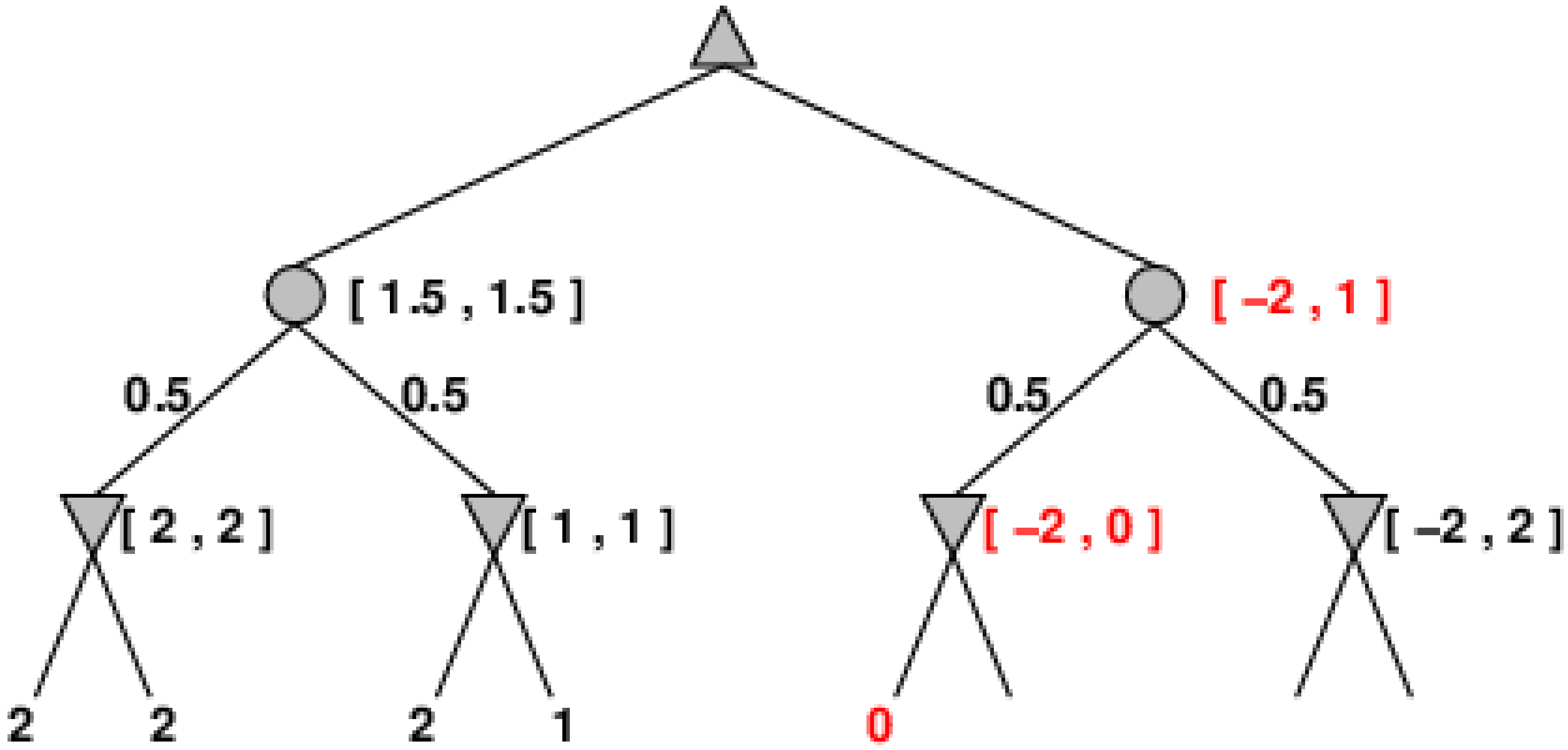


Pruning with Bounds

Can already stop search: best expected value for right branch is 1



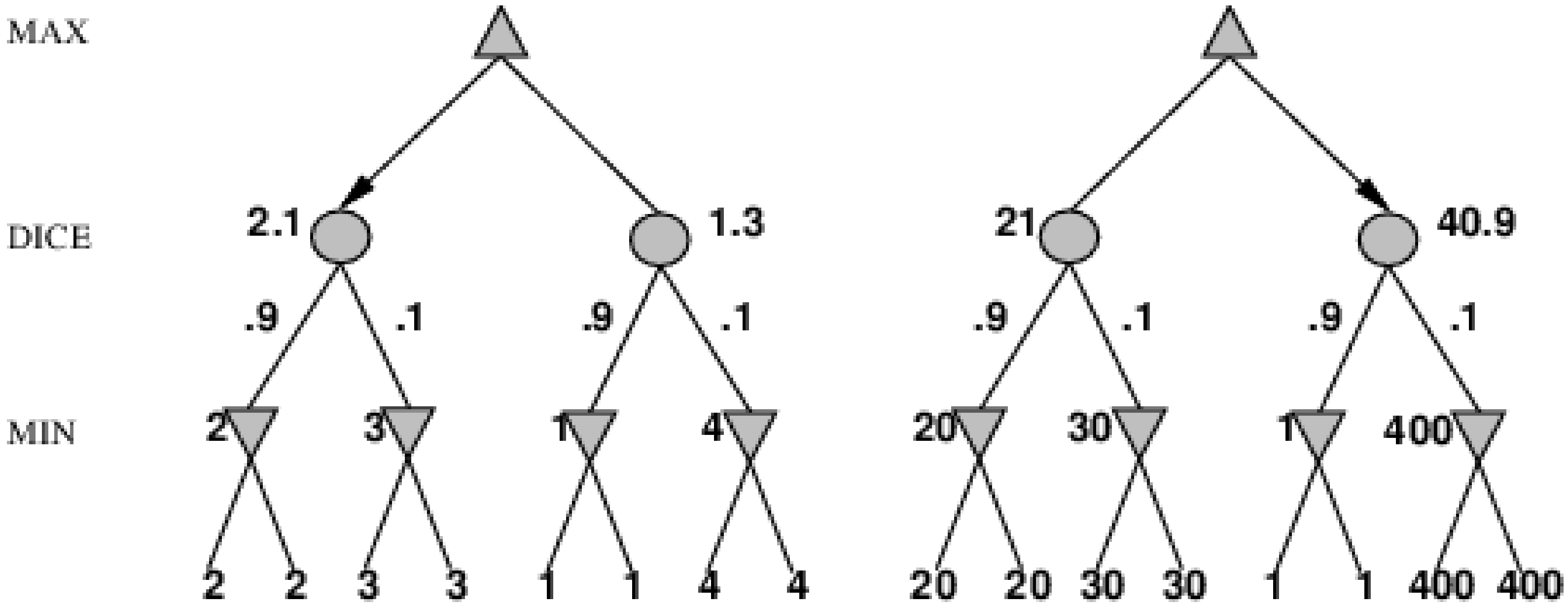
Nondeterministic Games in Practice

- Dice rolls increase b: 21 possible rolls with 2 dice
- Backgammon ≈ 20 legal moves (can be 6,000 with 1-1 roll)

$$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$

- As depth increases, probability of reaching a given node shrinks
  ⇒ value of lookahead is diminished
- α–β pruning is much less effective
- TDGammon uses depth-2 search + very good Eval
  ≈ world-champion level

Digression: Exact Values Do Matter



- Behavior is preserved only by positive linear transformation of EVAL
- Hence EVAL should be proportional to the expected payoff

imperfect information

Games of Imperfect Information

- E.g., card games, where opponent's initial cards are unknown
- Typically we can calculate a probability for each possible deal
- Seems just like having one big dice roll at the beginning of the game
- Idea: compute the minimax value of each action in each deal,
  then choose the action with highest expected value over all deals
- Special case: if an action is optimal for all deals, it's optimal.

Commonsense Counter-Example

- Road A leads to a small heap of gold pieces ($)
  Road B leads to a fork:
  take the left fork and you'll find a mound of jewels ($$$);
  take the right fork and you'll be run over by a bus.

- Road A leads to a small heap of gold pieces ($)
  Road B leads to a fork:
  take the left fork and you'll be run over by a bus;
  take the right fork and you'll find a mound of jewels ($$$);.

$\Rightarrow$ *does not matter if jewels are left or right on road B, it's always better choice*

- Road A leads to a small heap of gold pieces ($);
  Road B leads to a fork:
  guess correctly and you'll find a mound of jewels ($$$);
  guess incorrectly and you'll be run over by a bus.

$\Rightarrow$ *it does matter if we know where forks on road B lead to*

Proper Analysis

- Intuition that the value of an action is the average of its values in all actual states is **WRONG**
- With partial observability, value of an action depends on the information state or belief state the agent is in
- Can generate and search a tree of information states
- Leads to rational behaviors such as
  - acting to obtain information
  - signalling to one's partner
  - acting randomly to minimize information disclosure

Computer Poker

- Hard game
  - imperfect information — including bluffing and trapping
  - stochastic outcomes — cards drawn at random
  - partially observable — may never see other players hand when they fold
  - non-cooperative multi-player — possibility for coalitions

- Few moves (fold, call, raise), but large number of stochastic states

- Relative balance of deception plays very important
  also: when to bluff

⇒ There is no single best move

- Need to model other players (style, collusion, patterns)

- Hard to evaluate (not just win/loss, different types of opponents)

Summary

- Games are fun to work on
- They illustrate several important points about AI
  - perfection is unattainable ⇒ must approximate
  - good idea to think about what to think about
  - uncertainty constrains the assignment of values to states
  - optimal decisions depend on information state, not real state
- Games are to AI as grand prix racing is to automobile design