Basic Search

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Outline

- Problem-solving agents
- Problem types
- Problem formulation
- Example problems
- Basic search algorithms

problem-solving agents

Problem Solving Agents

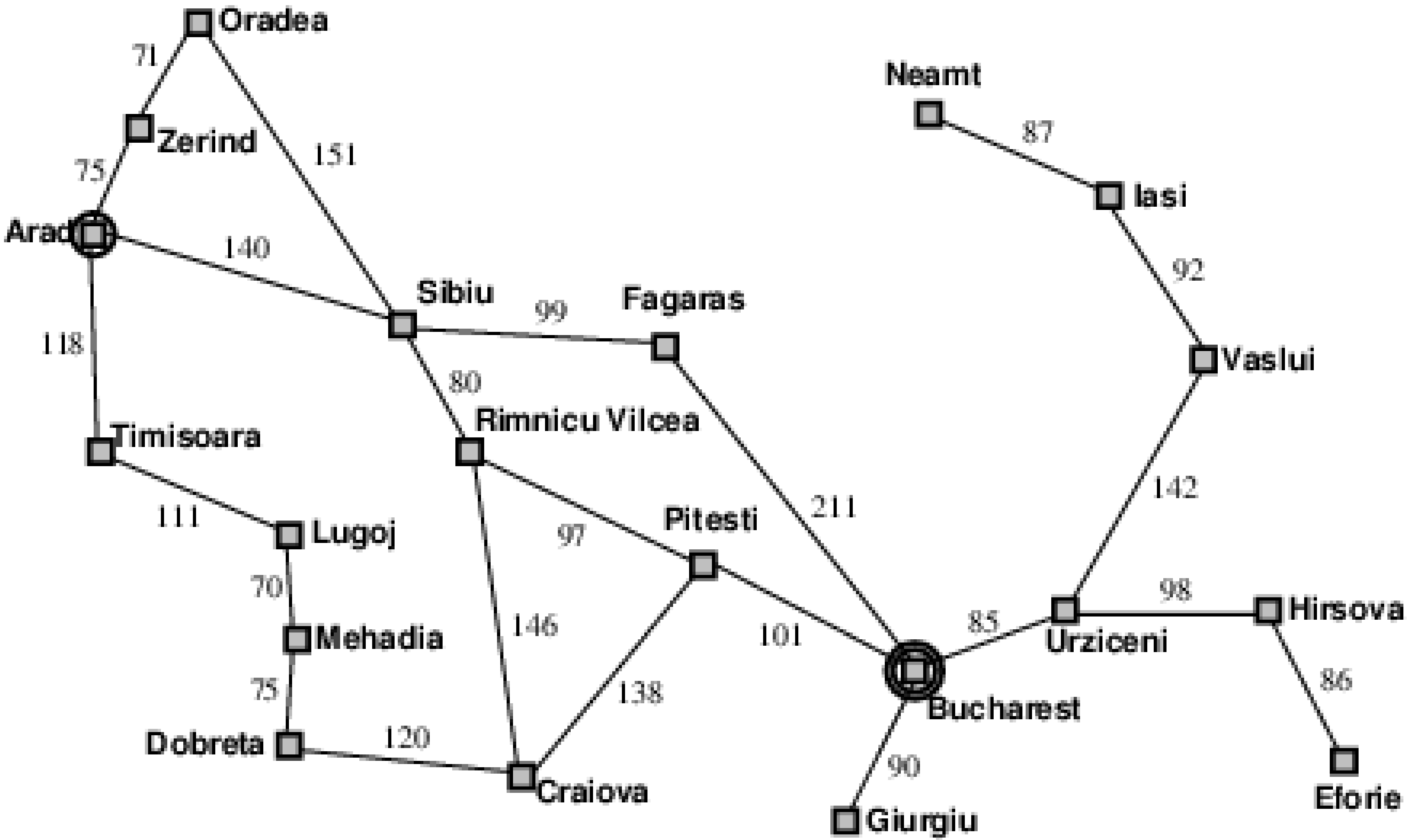
Restricted form of general agent:

```
function SIMPLE-PROBLEM-SOLVING-AGENT( percept) returns an action
   static: seq, an action sequence, initially empty
           state, some description of the current world state
           goal, a goal, initially null
           problem, a problem formulation

   state ← UPDATE-STATE(state, percept)
   if seq is empty then
      goal ← FORMULATE-GOAL(state)
      problem ← FORMULATE-PROBLEM(state, goal)
      seq ← SEARCH( problem)
   action ← RECOMMENDATION(seq, state)
   seq ← REMAINDER(seq, state)
   return action
```

Note: this is **offline** problem solving; solution executed “eyes closed.”
Online problem solving involves acting without complete knowledge.

Example: Romania



Example: Romania

- On holiday in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest
- Formulate goal
  - be in Bucharest
- Formulate problem
  - **states**: various cities
  - **actions**: drive between cities
- Find solution
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

problem types

Problem Types

- **Deterministic, fully observable** $\Longrightarrow$ **single-state problem**
  - agent knows exactly which state it will be in
  - solution is a sequence

- **Non-observable** $\Longrightarrow$ **conformant problem**
  - Agent may have no idea where it is
  - solution (if any) is a sequence

- **Nondeterministic** and/or **partially observable** $\Longrightarrow$ **contingency problem**
  - percepts provide **new** information about current state
  - solution is a **contingent plan** or a **policy**
  - often **interleave** search, execution

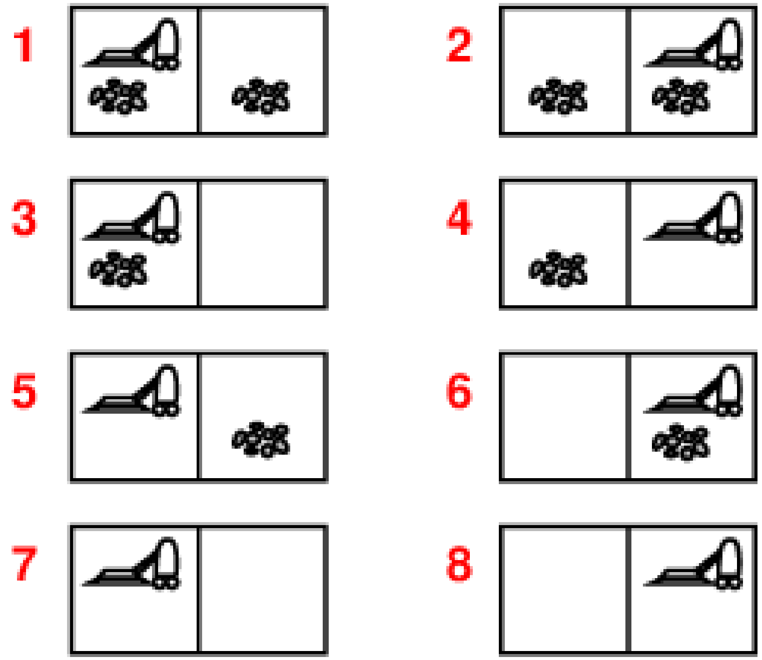
- **Unknown state space** $\Longrightarrow$ **exploration problem** ("online")

Example: Vacuum World

Single-state, start in #5. Solution?
$[Right, Suck]$

Conformant, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., $Right$ goes to $\{2, 4, 6, 8\}$. Solution?
$[Right, Suck, Left, Suck]$

Contingency, start in #5
Murphy's Law: $Suck$ can dirty a clean carpet
Local sensing: dirt, location only.
Solution?
$[Right, \textbf{if } dirt \textbf{ then } Suck]$

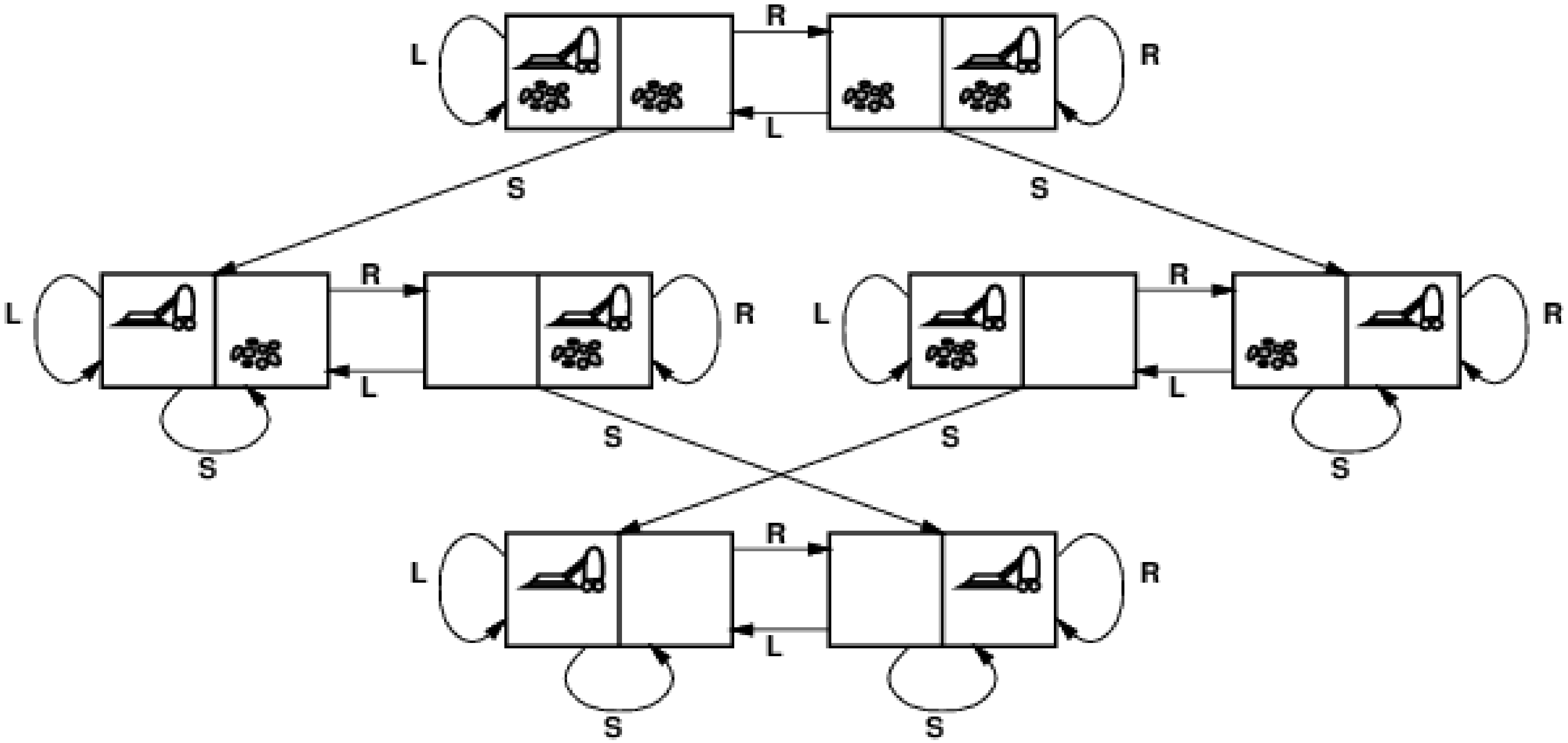


problem formulation

Single-State Problem Formulation

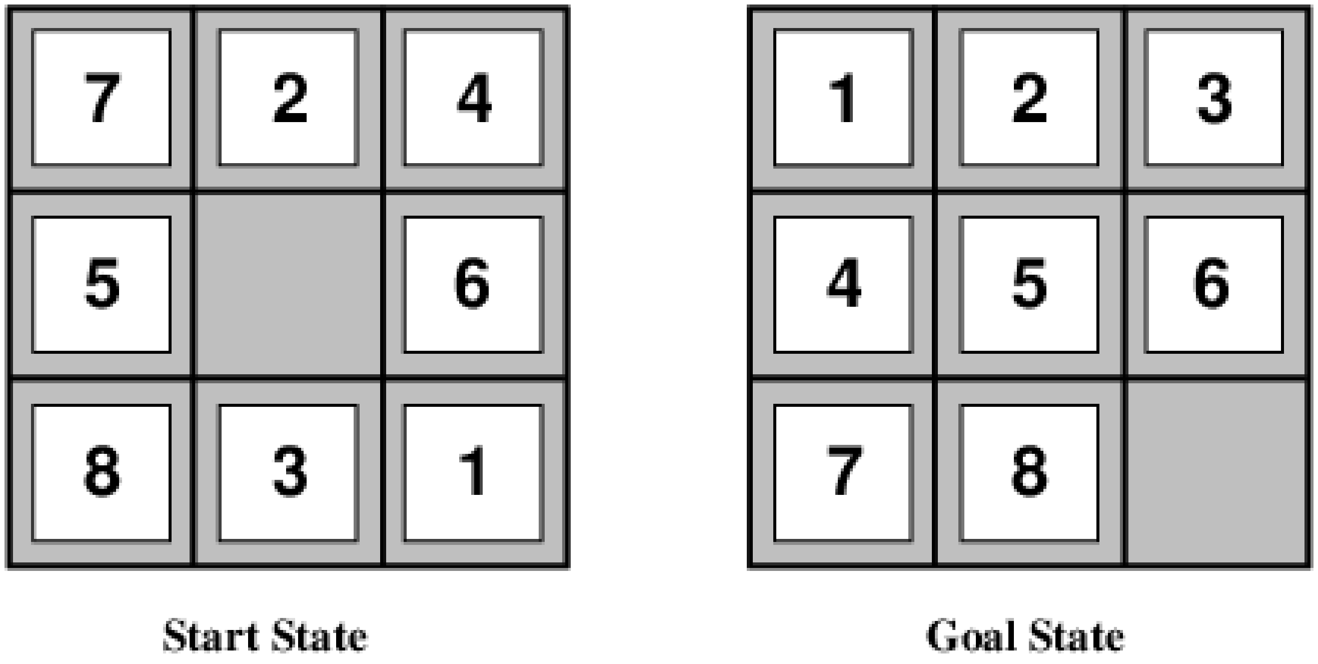
- A **problem** is defined by four items:
  - **initial state** e.g., "at Arad"
  - **successor function** $S(x)$ = set of action–state pairs
    e.g., $S(Arad) = \{\langle Arad \rightarrow Zerind, Zerind\rangle, \ldots\}$
  - **goal test**, can be
    explicit, e.g., x = "at Bucharest"
    implicit, e.g., $NoDirt(x)$
  - **path cost** (additive)
    e.g., sum of distances, number of actions executed, etc.
    $c(x, a, y)$ is the **step cost**, assumed to be ≥ 0
- A **solution** is a sequence of actions
  leading from the initial state to a goal state

Example: Vacuum World State Space Graph



states?: integer dirt and robot locations (ignore dirt **amounts** etc.)
actions?: $Left$, $Right$, $Suck$, $NoOp$
goal test?: no dirt
path cost?: 1 per action (0 for $NoOp$)

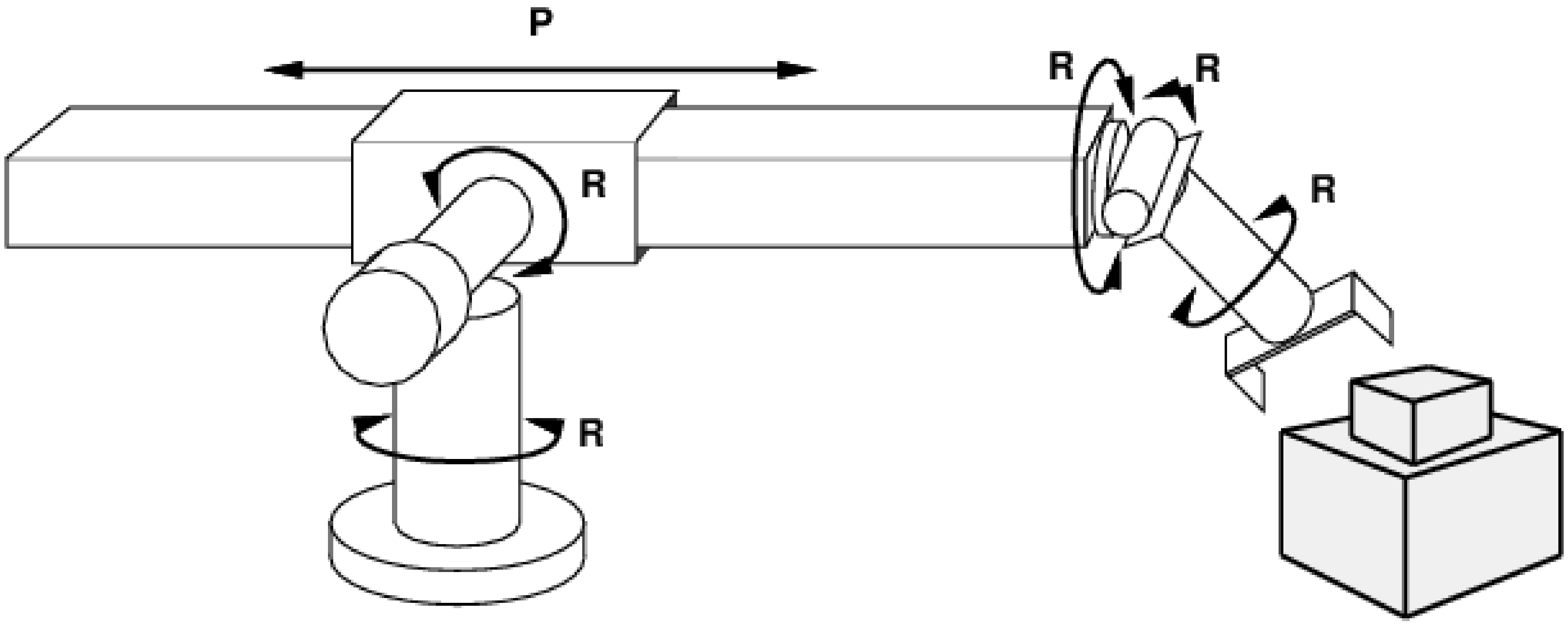
Example: The 8-Puzzle



states?: integer locations of tiles (ignore intermediate positions)
actions?: move blank left, right, up, down (ignore unjamming etc.)
goal test?: = goal state (given)
path cost?: 1 per move

[Note: optimal solution of n-Puzzle family is NP-hard]

Example: Robotic Assembly



states?: real-valued coordinates of robot joint angles
parts of the object to be assembled

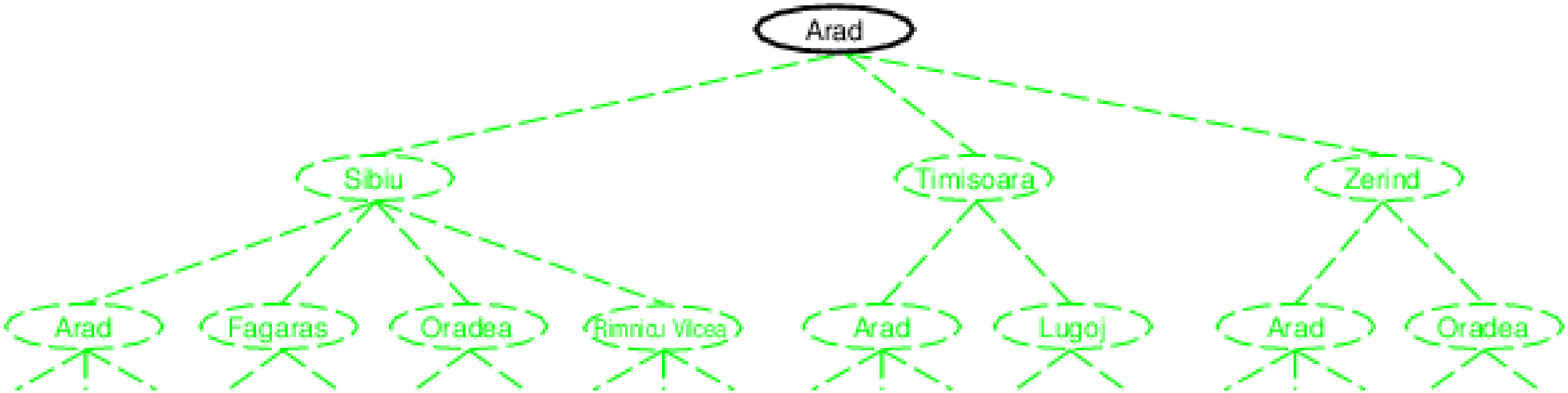
actions?: continuous motions of robot joints

goal test?: complete assembly

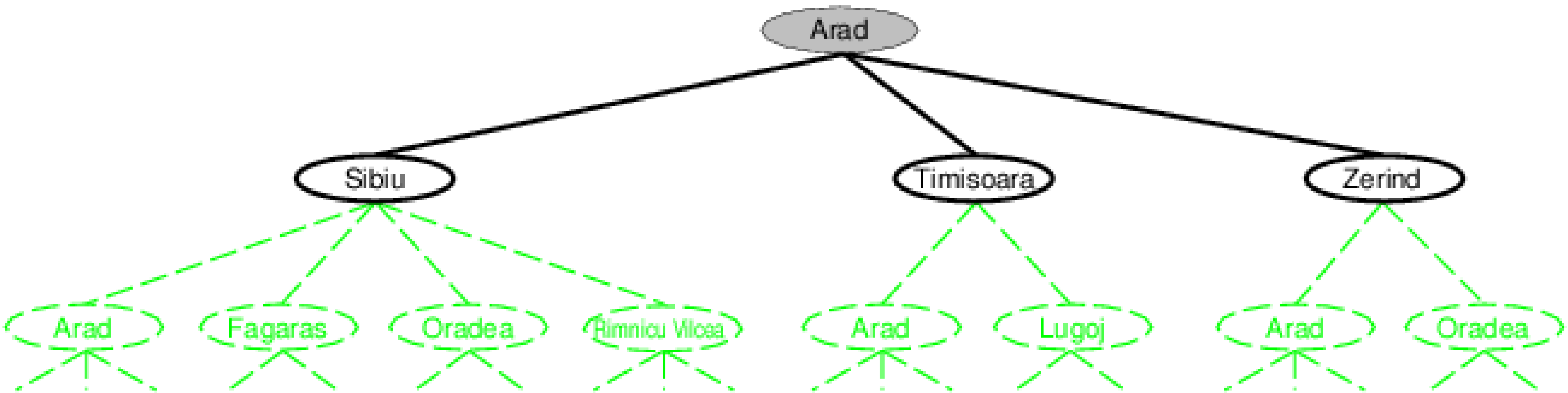
path cost?: time to execute

tree search

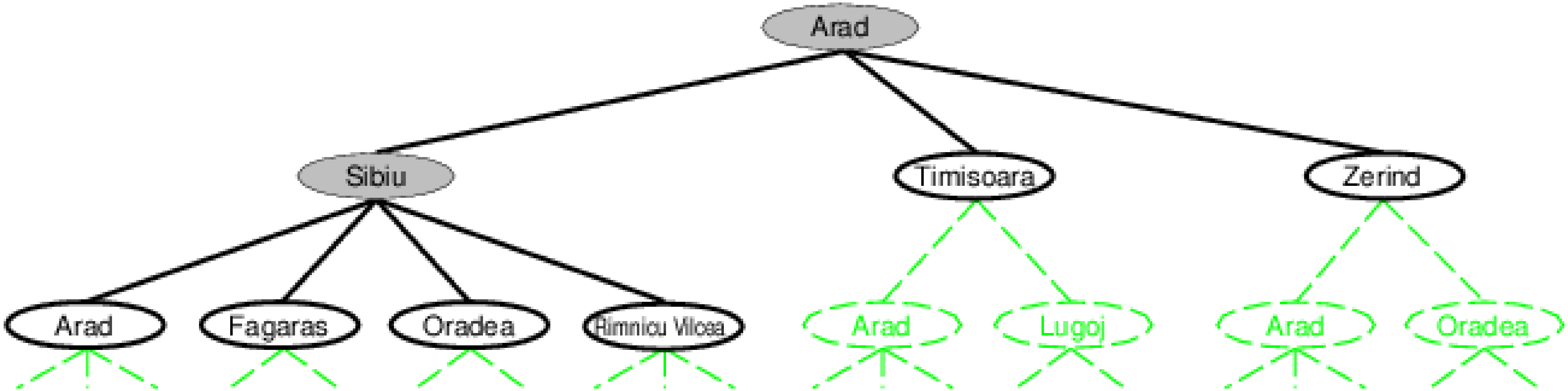
Tree Search Example



Tree Search Example



Tree Search Example



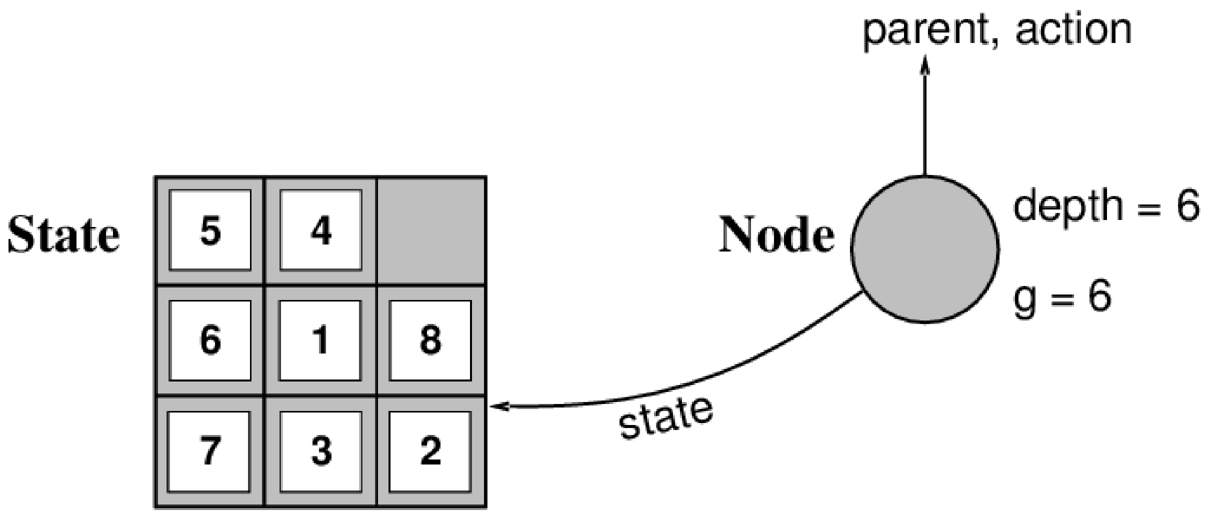
Tree Search Algorithms

- Basic idea: offline, simulated exploration of state space
  by generating successors of already-explored states
  (a.k.a. **expanding** states)

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```

Implementation: States vs. Nodes

- A **state** is a (representation of) a physical configuration
- A **node** is a data structure constituting part of a search tree includes **parent**, **children**, **depth**, **path cost** $g(x)$
- States do not have parents, children, depth, or path cost!



- The EXPAND function creates new nodes, filling in the various fields and using the SUCCESSORFN of the problem to create the corresponding states.

Search Strategies

- A strategy is defined by picking the **order of node expansion**

- Strategies are evaluated along the following dimensions
  - **completeness**—does it always find a solution if one exists?
  - **time complexity**—number of nodes generated/expanded
  - **space complexity**—maximum number of nodes in memory
  - **optimality**—does it always find a least-cost solution?

- Time and space complexity are measured in terms of
  - b — maximum branching factor of the search tree
  - d — depth of the least-cost solution
  - m — maximum depth of the state space (may be ∞)

Uninformed Search Strategies

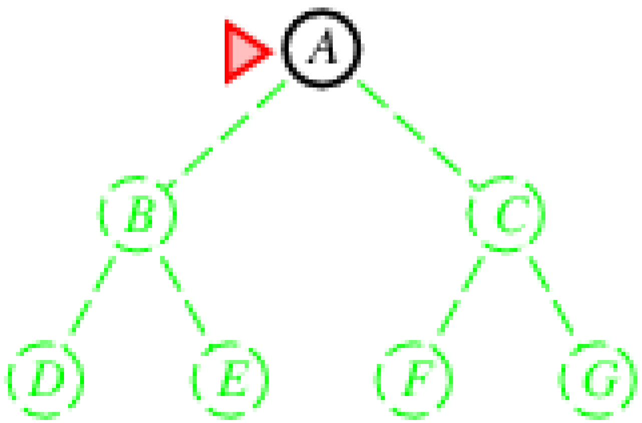
Uninformed strategies use only the information available in the problem definition

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

breadth-first search

Breadth-First Search

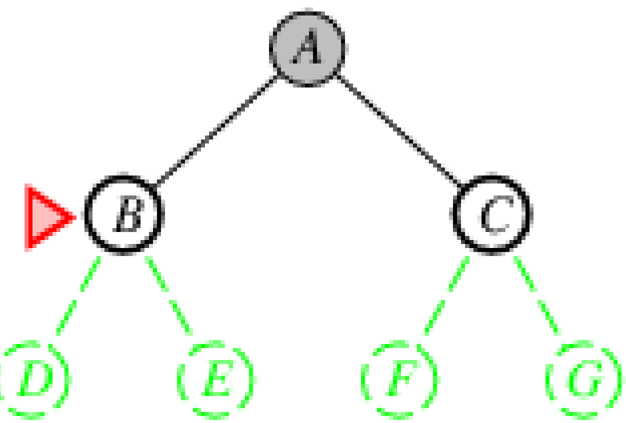
- Expand shallowest unexpanded node
- **Implementation**:
  fringe is a FIFO queue, i.e., new successors go at end



fringe = (A)

Breadth-First Search

- Expand shallowest unexpanded node
- **Implementation**:
  fringe is a FIFO queue, i.e., new successors go at end

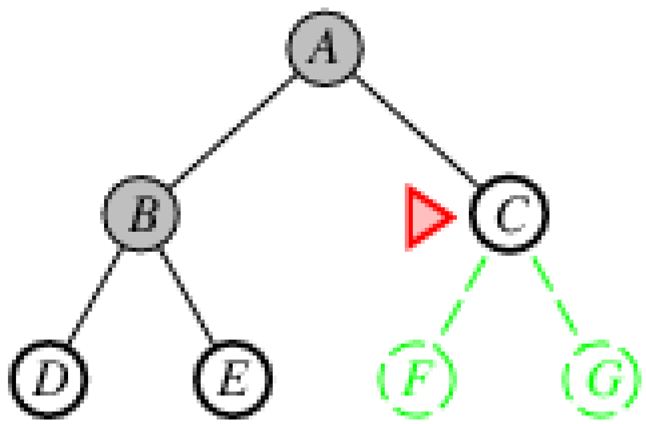


fringe = (B,C)

Breadth-First Search

- Expand shallowest unexpanded node

- **Implementation**:
  fringe is a FIFO queue, i.e., new successors go at end

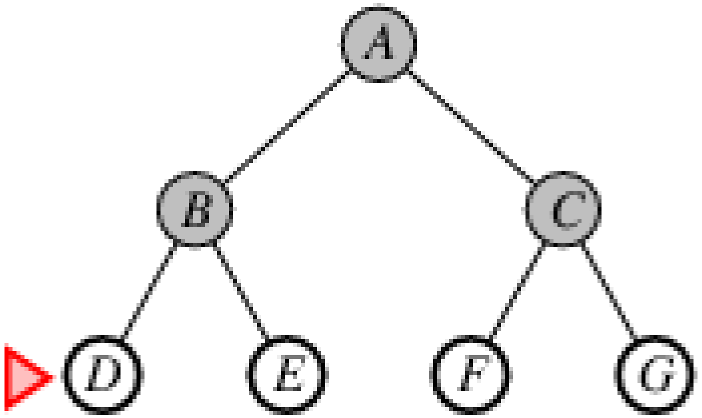


fringe = (C,D,E)

Breadth-First Search

- Expand shallowest unexpanded node

- **Implementation**:
  fringe is a FIFO queue, i.e., new successors go at end



fringe = (D,E,F,G)

Properties of Breadth-First Search

- Complete? Yes (if b is finite)
- Time? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d
- Space? $O(b^{d+1})$ (keeps every node in memory)
- Optimal? Yes (if cost = 1 per step); not optimal in general
- **Space** is the big problem

uniform cost search

Uniform-Cost Search

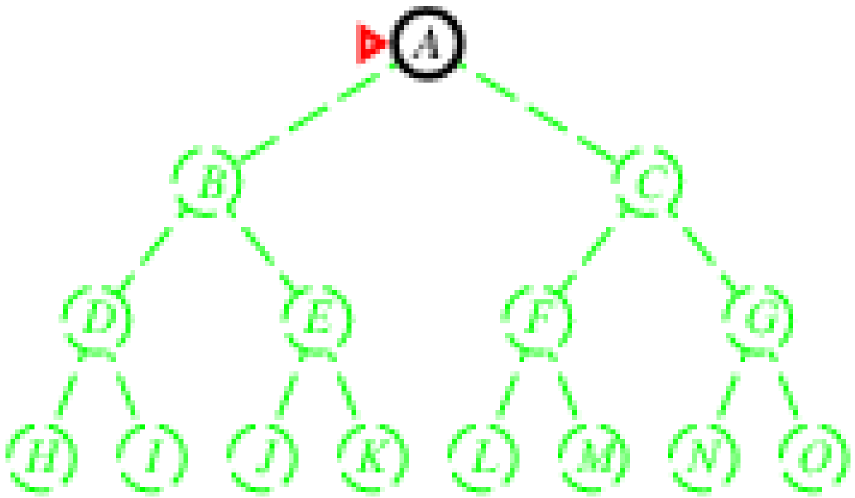
- Expand least-cost unexpanded node
- **Implementation**:
  fringe = queue ordered by path cost, lowest first
- Equivalent to breadth-first if step costs all equal
- Properties
  - Complete? Yes, if step cost $\geq \epsilon$
  - Time? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
    where C^* is the cost of the optimal solution
  - Space? # of nodes with $g \leq$ cost of optimal solution, $O(b^{\lceil C^*/\epsilon \rceil})$
  - Optimal? Yes—nodes expanded in increasing order of $g(n)$

depth first search

Depth-First Search

- Expand deepest unexpanded node

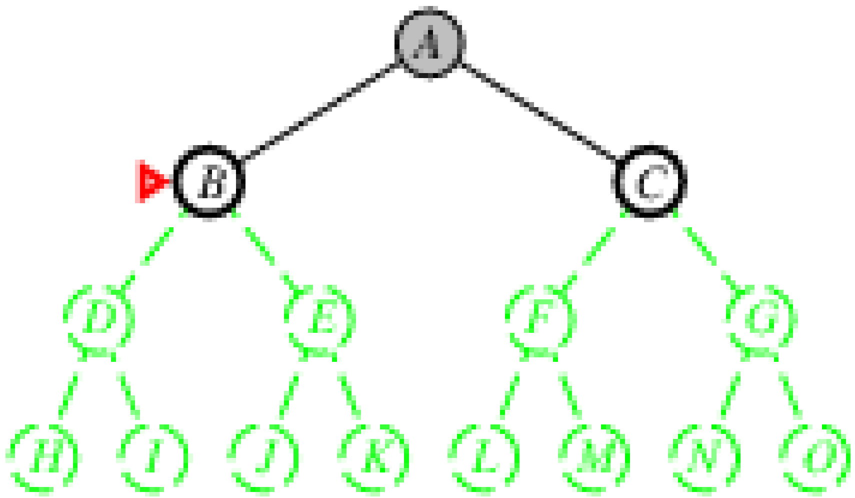
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front



fringe = (A)

Depth-First Search

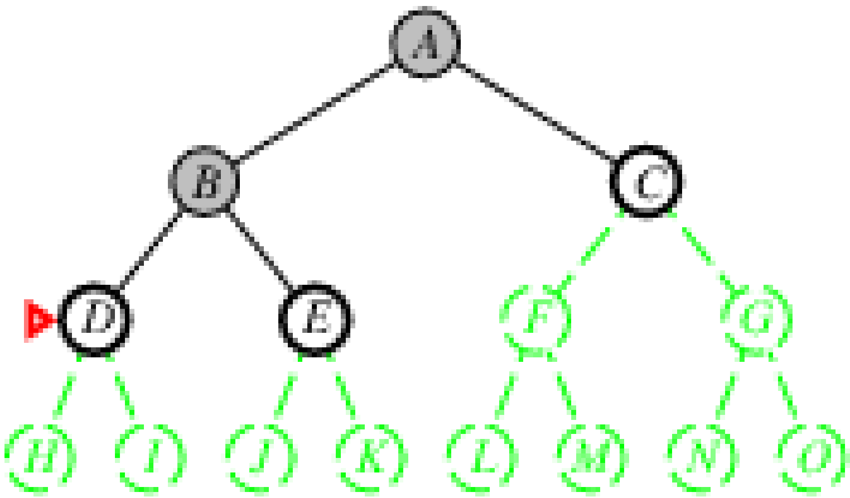
- Expand deepest unexpanded node
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front



fringe = (B,C)

- Expand deepest unexpanded node

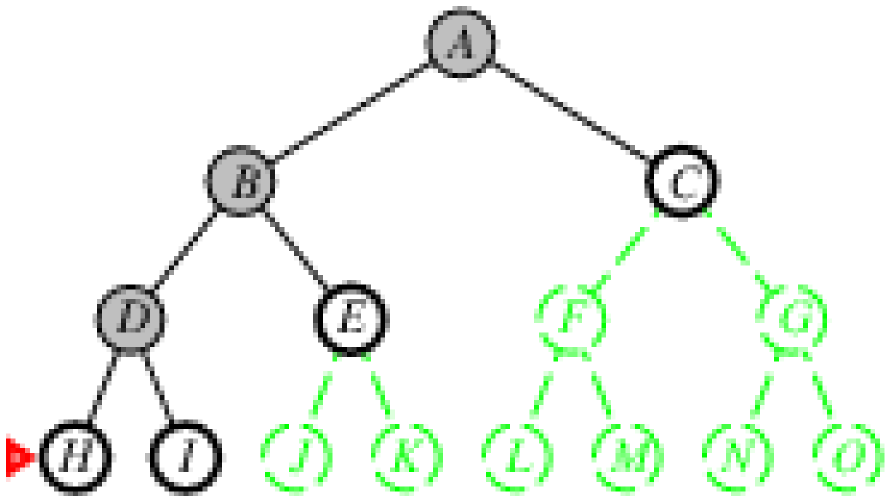
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front



fringe = (D,E,C)

Depth-First Search

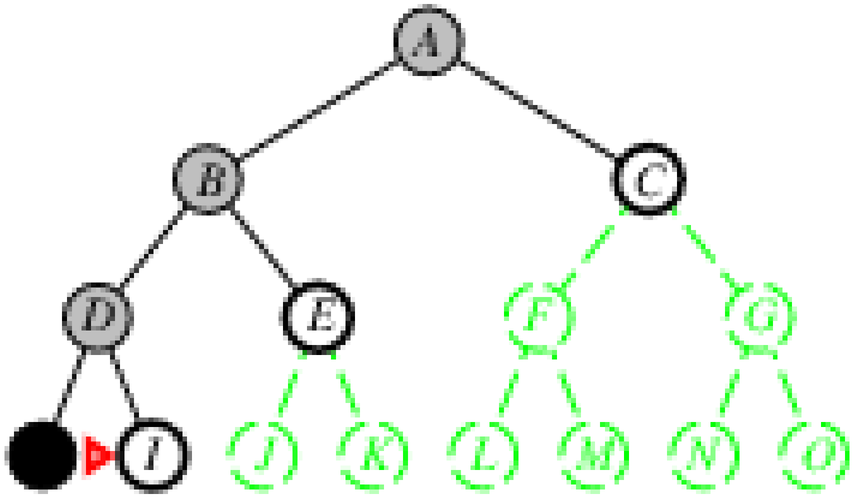
- Expand deepest unexpanded node
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front



fringe = (H,I,E,C)

Depth-First Search

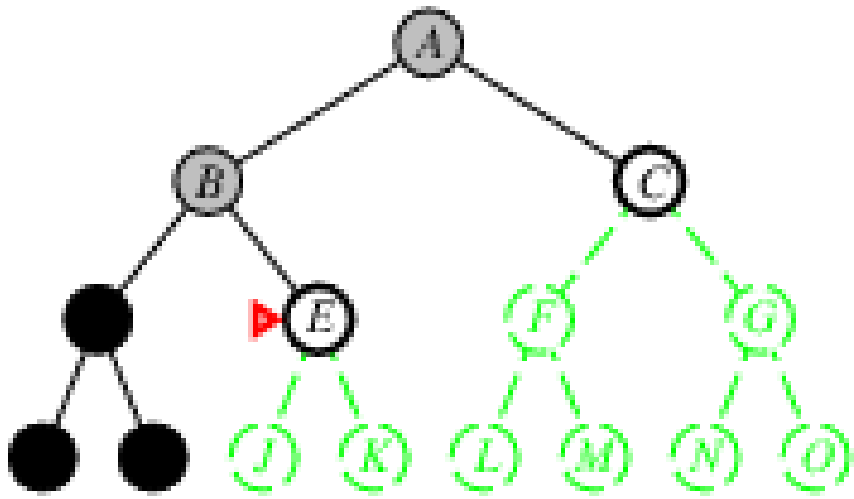
- Expand deepest unexpanded node
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front



fringe = (I,E,C)

Depth-First Search

- Expand deepest unexpanded node
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front

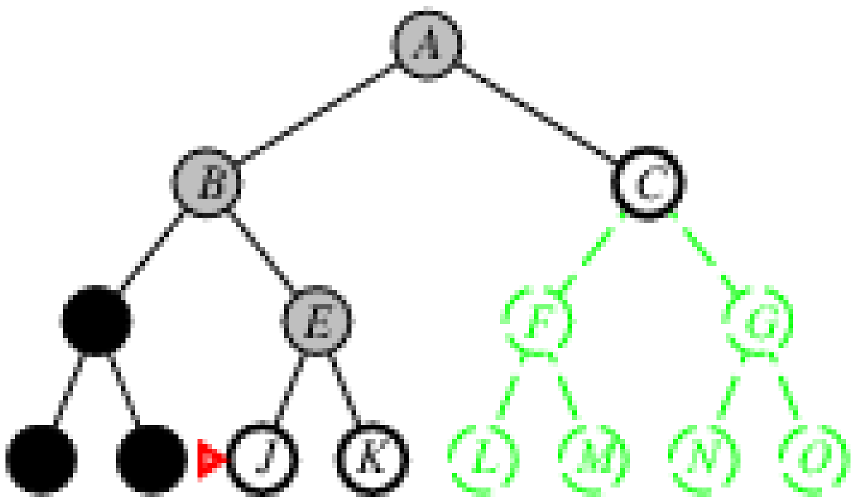


fringe = (E,C)

Depth-First Search

- Expand deepest unexpanded node

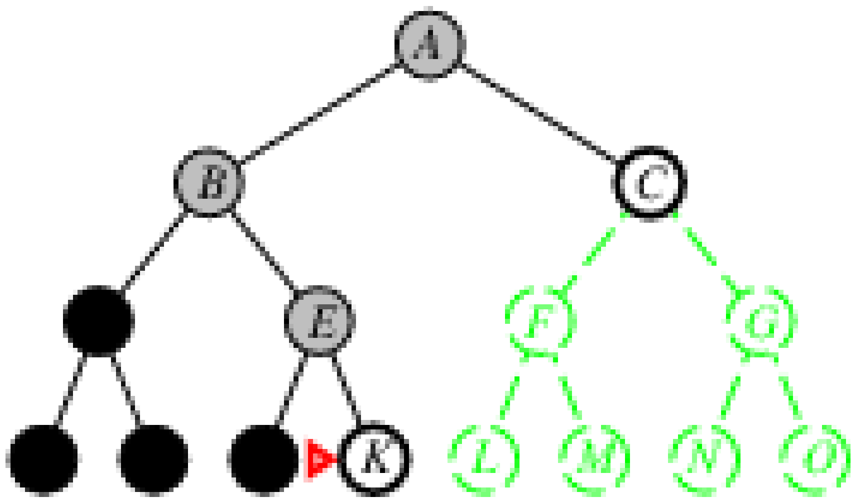
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front



fringe = (J,K,C)

Depth-First Search

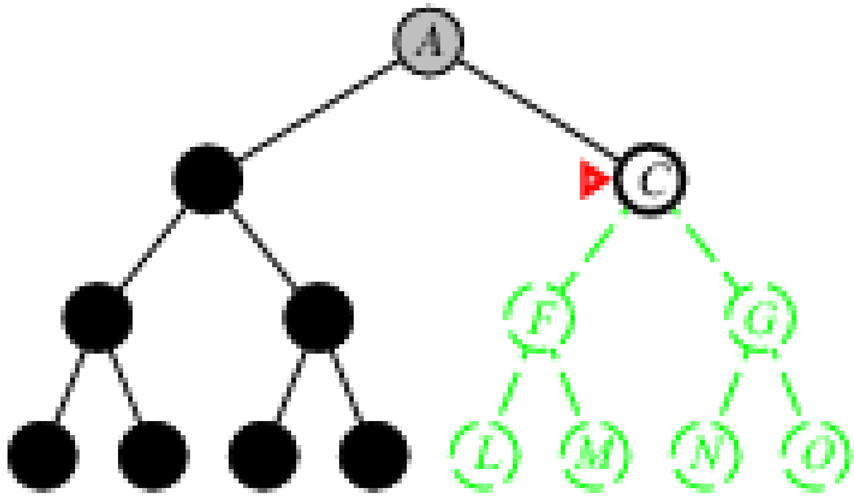
- Expand deepest unexpanded node
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front



fringe = (K,C)

Depth-First Search

- Expand deepest unexpanded node
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front

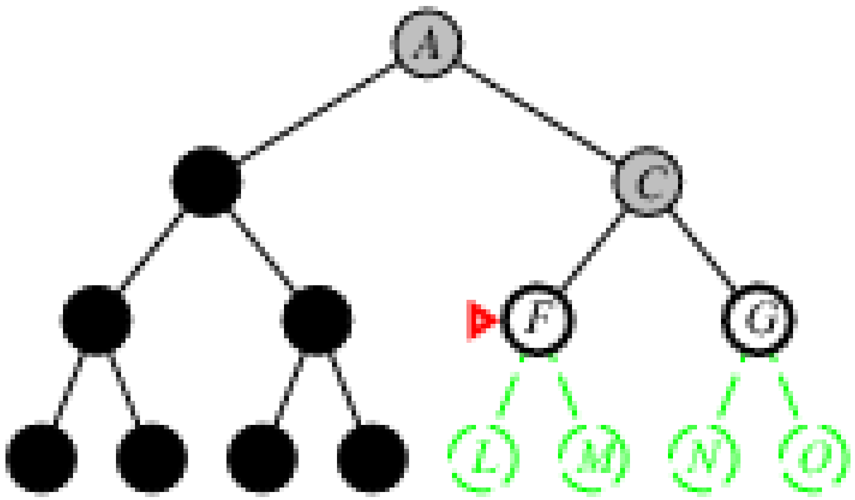


fringe = (C)

Depth-First Search

- Expand deepest unexpanded node

- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front

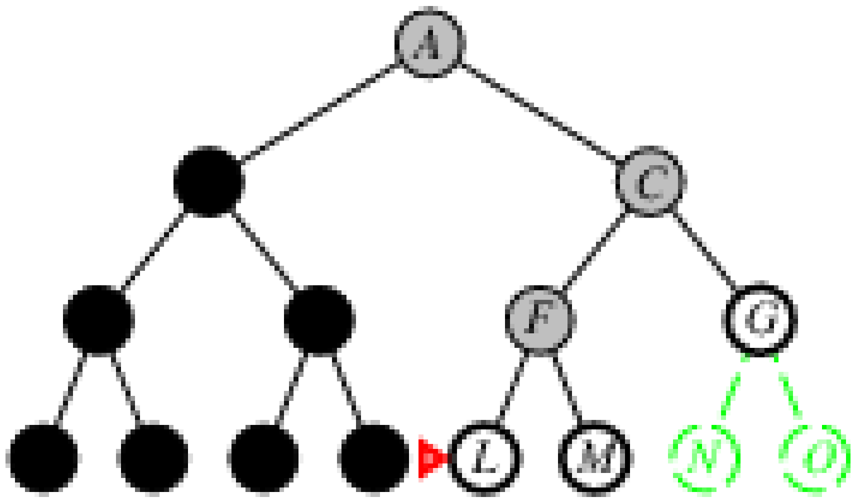


fringe = (F,G)

Depth-First Search

- Expand deepest unexpanded node

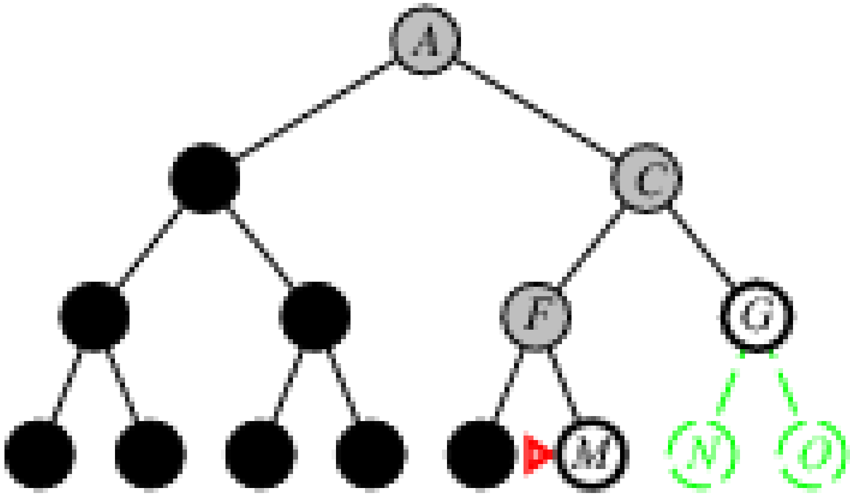
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front



fringe = (L,M,G)

Depth-First Search

- Expand deepest unexpanded node
- **Implementation**:
  fringe = LIFO queue, i.e., put successors at front



fringe = (M,G)

Properties of Depth-First Search

- Complete?
  - no: fails in infinite-depth spaces, spaces with loops
  - modify to avoid repeated states along path
    $\Rightarrow$ complete in finite spaces
- Time? $O(b^m)$
  - terrible if m is much larger than d
  - but if solutions are dense, may be much faster than breadth-first
- Space? $O(bm)$, i.e., linear space!
- Optimal? No

iterative deepening

Depth-Limited Search

- Depth-first search with depth limit l, i.e., nodes at depth l have no successors
- **Recursive implementation**:

```
function DEPTH-LIMITED-SEARCH( problem, limit) returns soln/fail/cutoff
   RECURSIVE-DLS(MAKE-NODE(INITIAL-STATE[problem]), problem, limit)

function RECURSIVE-DLS(node, problem, limit) returns soln/fail/cutoff
   cutoff-occurred? ← false
   if GOAL-TEST(problem, STATE[node]) then return node
   else if DEPTH[node] = limit then return cutoff
   else for each successor in EXPAND(node, problem) do
      result ← RECURSIVE-DLS(successor, problem, limit)
      if result = cutoff then cutoff-occurred? ← true
      else if result ≠ failure then return result
   if cutoff-occurred? then return cutoff else return failure
```

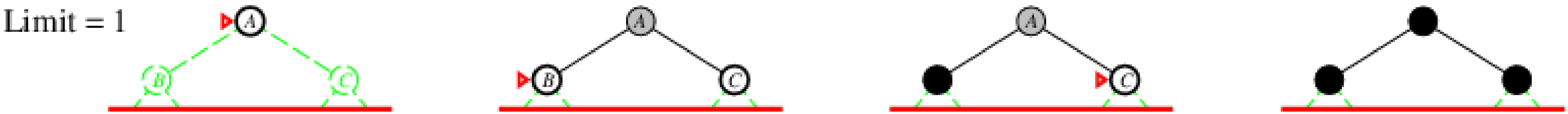
Iterative Deepening Search


```
function ITERATIVE-DEEPENING-SEARCH( problem) returns a solution
    inputs: problem, a problem
    for depth ← 0 to ∞ do
        result ← DEPTH-LIMITED-SEARCH( problem, depth)
        if result ≠ cutoff then return result
    end
```

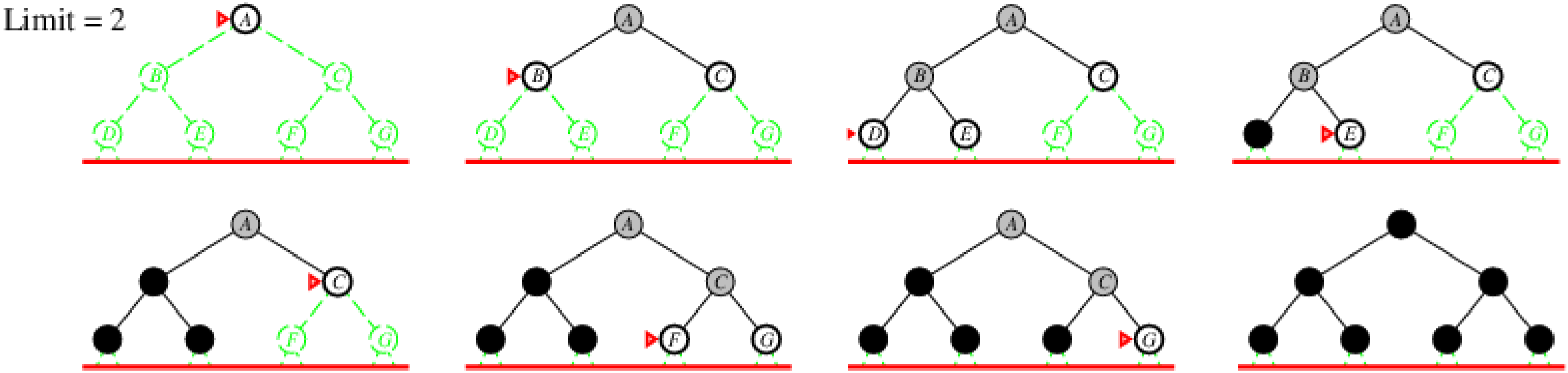
Iterative Deepening Search $l = 0$

Limit = 0

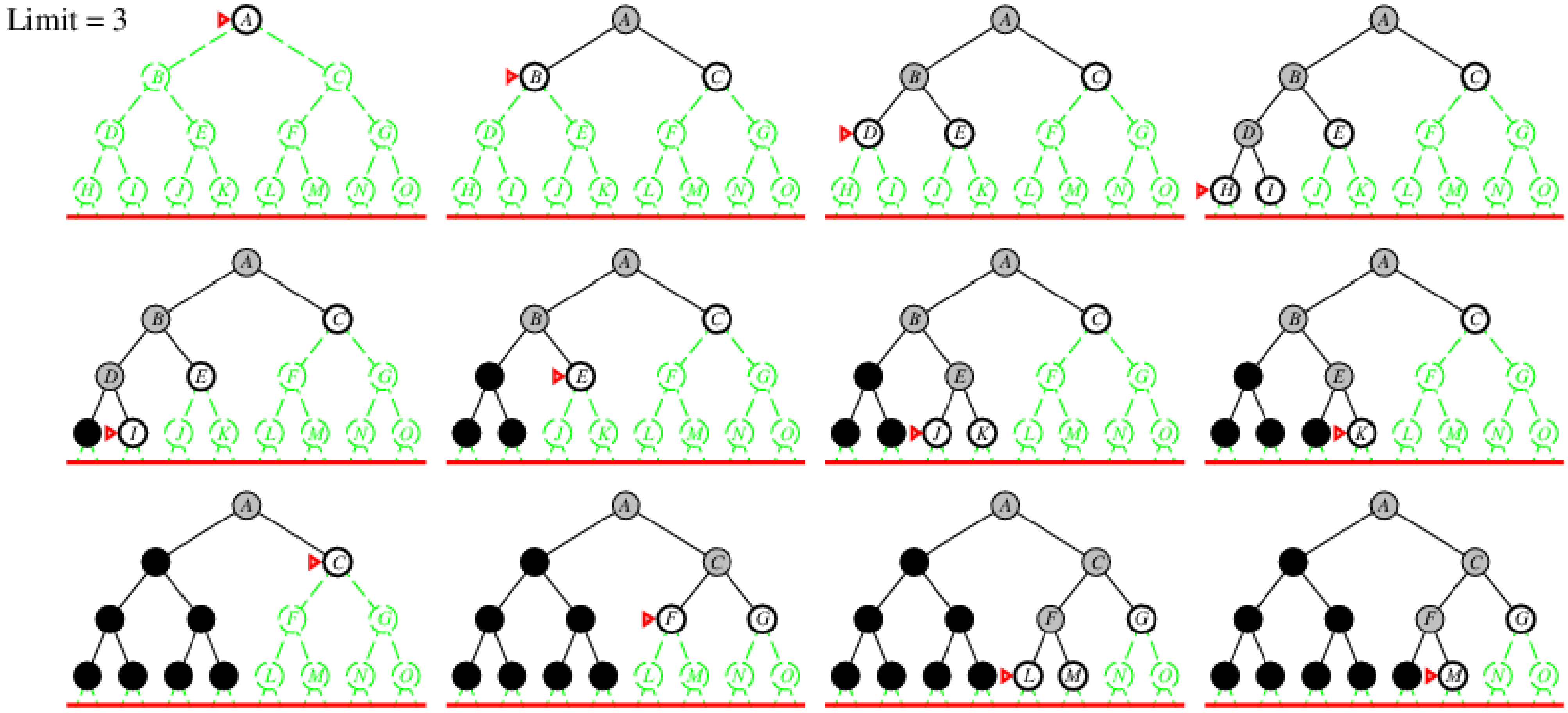
Iterative Deepening Search $l = 1$



Iterative Deepening Search $l = 2$



Iterative Deepening Search $l = 3$



Properties of Iterative Deepening Search

- Complete? Yes
- Time? $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
- Space? $O(bd)$
- Optimal? Yes, if step cost = 1
  Can be modified to explore uniform-cost tree
- Numerical comparison for $b = 10$ and $d = 5$, solution at far right leaf:

$$N(\text{IDS}) = 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450$$
$$N(\text{BFS}) = 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100$$

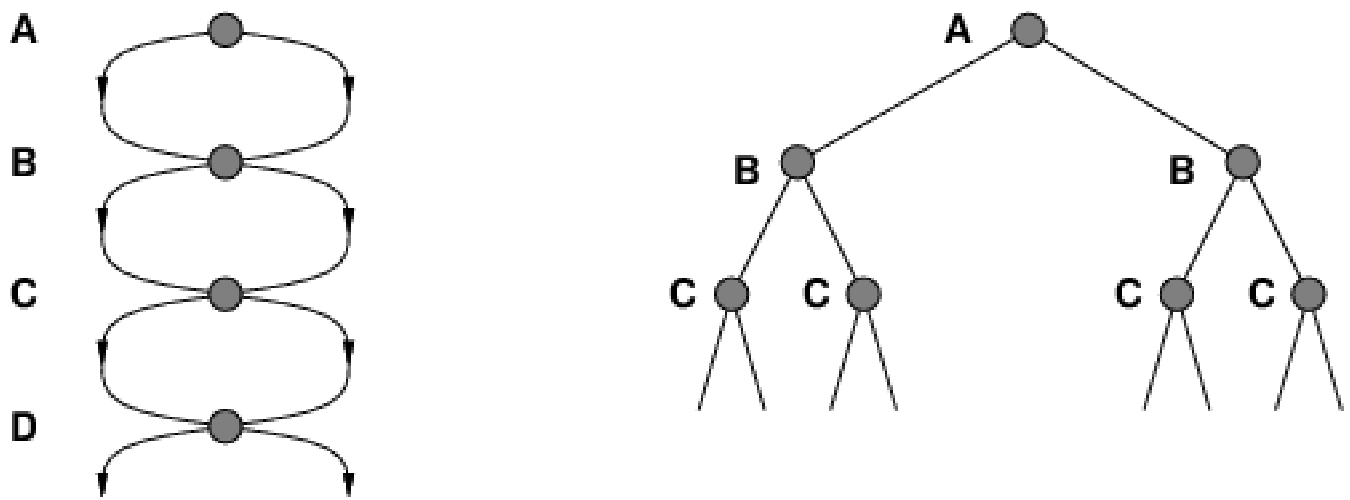
- IDS does better because other nodes at depth d are not expanded
- BFS can be modified to apply goal test when a node is **generated**

Summary of Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes*	Yes*	No	Yes, if $l \geq d$	Yes
Time	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	b^m	b^l	b^d
Space	b^{d+1}	$b^{\lceil C^*/\epsilon \rceil}$	bm	bl	bd
Optimal?	Yes*	Yes	No	No	Yes*

Repeated States

Failure to detect repeated states can turn a linear problem into an exponential one



Graph Search


```
function GRAPH-SEARCH( problem, fringe) returns a solution, or failure
   closed ← an empty set
   fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← REMOVE-FRONT(fringe)
      if GOAL-TEST(problem, STATE[node]) then return node
      if STATE[node] is not in closed then
         add STATE[node] to closed
         fringe ← INSERTALL(EXPAND(node, problem), fringe)
   end
```

Summary

- Problem formulation usually requires abstracting away real-world details to define a state space that can feasibly be explored

- Variety of uninformed search strategies

- Iterative deepening search uses only linear space
  and not much more time than other uninformed algorithms

- Graph search can be exponentially more efficient than tree search