Markov Decision Processes

Philipp Koehn presented by Shuoyang Ding

11 April 2017



Outline



- Hidden Markov models
- Inference: filtering, smoothing, best sequence
- Kalman filters (a brief mention)
- Dynamic Bayesian networks
- Speech recognition

Time and Uncertainty

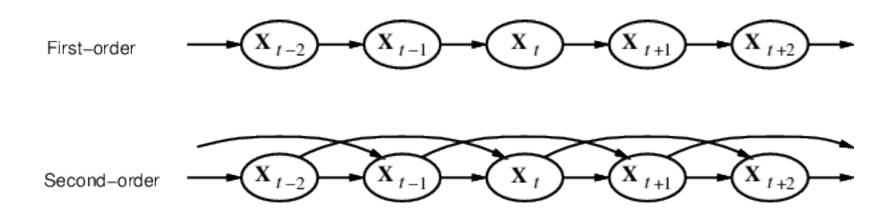


- The world changes; we need to track and predict it
- Diabetes management vs vehicle diagnosis
- Basic idea: sequence of state and evidence variables
- X_t = set of unobservable state variables at time t e.g., BloodSugar_t, StomachContents_t, etc.
- **E**_t = set of observable evidence variables at time t e.g., MeasuredBloodSugar_t, PulseRate_t, FoodEaten_t
- This assumes **discrete time**; step size depends on problem
- Notation: $\mathbf{X}_{a:b} = \mathbf{X}_a, \mathbf{X}_{a+1}, \dots, \mathbf{X}_{b-1}, \mathbf{X}_b$

Markov Processes (Markov Chains)



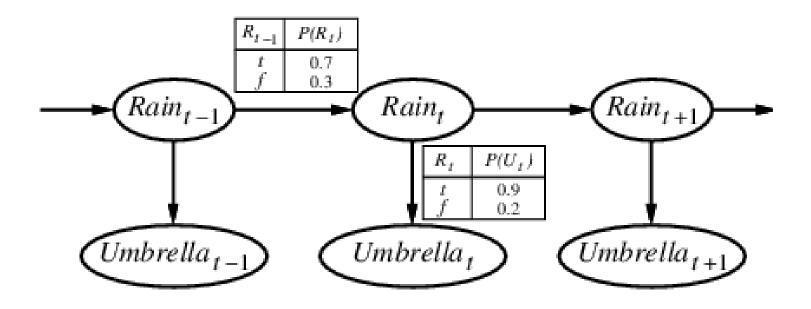
- Construct a Bayes net from these variables: parents?
- Markov assumption: X_t depends on **bounded** subset of $X_{0:t-1}$
- First-order Markov process: $P(X_t|X_{0:t-1}) = P(X_t|X_{t-1})$ Second-order Markov process: $P(X_t|X_{0:t-1}) = P(X_t|X_{t-2}, X_{t-1})$



- Sensor Markov assumption: $P(E_t|X_{0:t}, E_{0:t-1}) = P(E_t|X_t)$
- Stationary process: transition model P(X_t|X_{t-1}) and sensor model P(E_t|X_t) fixed for all t

Example





- First-order Markov assumption not exactly true in real world!
- Possible fixes:
 - 1. Increase order of Markov process
 - 2. Augment state, e.g., add $Temp_t$, $Pressure_t$



inference

Inference Tasks



- Filtering: P(X_t|e_{1:t}) belief state—input to the decision process of a rational agent
- Smoothing: P(X_k|e_{1:t}) for 0 ≤ k < t better estimate of past states, essential for learning
- Most likely explanation: $\arg \max_{\mathbf{x}_{1:t}} P(\mathbf{x}_{1:t} | \mathbf{e}_{1:t})$ speech recognition, decoding with a noisy channel

Filtering



• Aim: devise a **recursive** state estimation algorithm

 $P(X_{t+1}|e_{1:t+1}) = P(X_{t+1}|e_{1:t}, e_{t+1})$

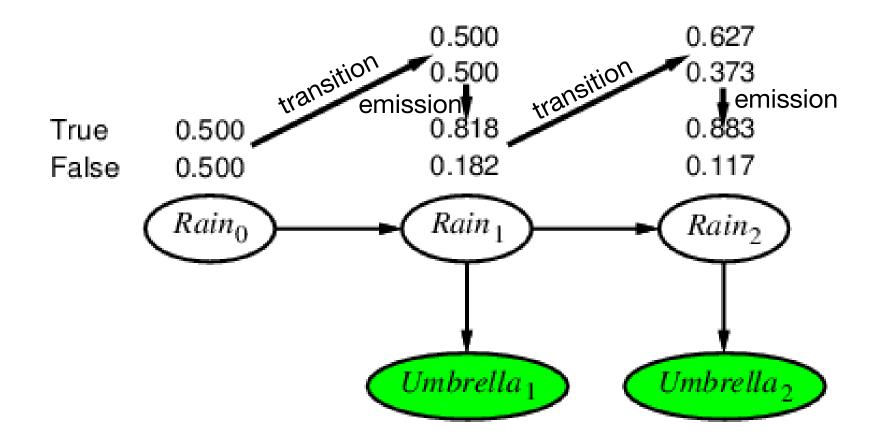
- $= \alpha \mathbf{P}(\mathbf{e}_{t+1} | \mathbf{X}_{t+1}, \mathbf{e}_{1:t}) \mathbf{P}(\mathbf{X}_{t+1} | \mathbf{e}_{1:t}) \quad (Bayes \ rule)$
- $= \alpha \mathsf{P}(\mathbf{e}_{t+1}|\mathbf{X}_{t+1}) \mathsf{P}(\mathbf{X}_{t+1}|\mathbf{e}_{1:t})$ (Sensor Markov assumption)
- $= \alpha \mathbf{P}(\mathbf{e}_{t+1}|\mathbf{X}_{t+1}) \sum_{\mathbf{x}_t} \mathbf{P}(\mathbf{X}_{t+1}|\mathbf{x}_t, \mathbf{e}_{1:t}) P(\mathbf{x}_t|\mathbf{e}_{1:t})$ (multiplying out)

 $= \alpha \mathbf{P}(\mathbf{e}_{t+1}|\mathbf{X}_{t+1}) \sum_{\mathbf{x}_t} \mathbf{P}(\mathbf{X}_{t+1}|\mathbf{x}_t) P(\mathbf{x}_t|\mathbf{e}_{1:t}) \quad \text{(first order Markov model)}$

- Summary: $P(\mathbf{X}_{t+1}|\mathbf{e}_{1:t+1}) = \alpha \underbrace{P(\mathbf{e}_{t+1}|\mathbf{X}_{t+1})}_{\text{emission}} \sum_{\mathbf{x}_t} \underbrace{P(\mathbf{X}_{t+1}|\mathbf{x}_t)}_{\text{transition}} \underbrace{P(\mathbf{x}_t|\mathbf{e}_{1:t})}_{\text{recursive call}}$
- $\mathbf{f}_{1:t+1} = \text{FORWARD}(\mathbf{f}_{1:t}, \mathbf{e}_{t+1})$ where $\mathbf{f}_{1:t} = \mathbf{P}(\mathbf{X}_t | \mathbf{e}_{1:t})$ Time and space **constant** (independent of *t*)

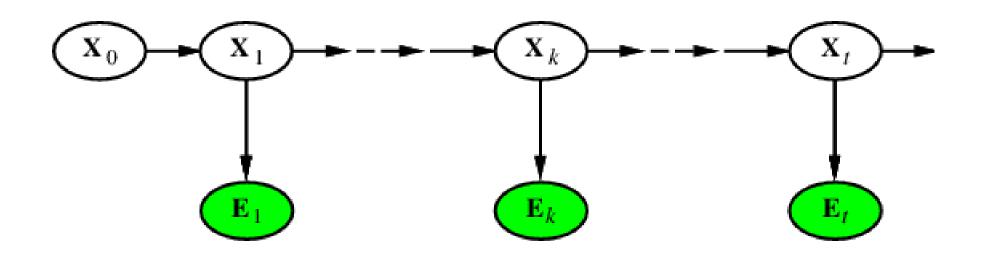
Filtering Example





Smoothing

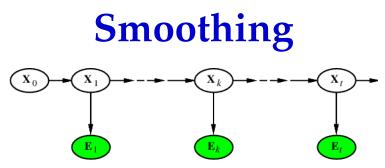




• If full sequence is known

 \Rightarrow what is the state probability $P(X_k | e_{1:t})$ including future evidence?

• Smoothing: sum over all paths



• Divide evidence $\mathbf{e}_{1:t}$ into $\mathbf{e}_{1:k}$, $\mathbf{e}_{k+1:t}$:

$$\mathsf{P}(\mathsf{X}_k|\mathsf{e}_{1:t}) = \mathsf{P}(\mathsf{X}_k|\mathsf{e}_{1:k},\mathsf{e}_{k+1:t})$$

 $= \alpha \mathbf{P}(\mathbf{X}_k | \mathbf{e}_{1:k}) \mathbf{P}(\mathbf{e}_{k+1:t} | \mathbf{X}_k, \mathbf{e}_{1:k})$

$$= \alpha \mathbf{P}(\mathbf{X}_k | \mathbf{e}_{1:k}) \mathbf{P}(\mathbf{e}_{k+1:t} | \mathbf{X}_k)$$

 $= \alpha \mathbf{f}_{1:k} \mathbf{b}_{k+1:t}$

• Backward message $\mathbf{b}_{k+1:t}$ computed by a backwards recursion

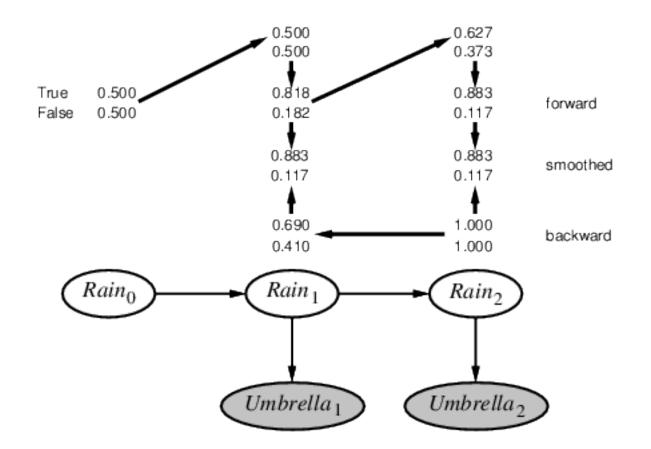
$$\mathbf{P}(\mathbf{e}_{k+1:t}|\mathbf{X}_k) = \sum_{\mathbf{x}_{k+1}} \mathbf{P}(\mathbf{e}_{k+1:t}|\mathbf{X}_k, \mathbf{x}_{k+1}) \mathbf{P}(\mathbf{x}_{k+1}|\mathbf{X}_k)$$

$$= \sum_{\mathbf{x}_{k+1}} P(\mathbf{e}_{k+1:t}|\mathbf{x}_{k+1}) \mathbf{P}(\mathbf{x}_{k+1}|\mathbf{X}_k)$$

$$= \sum_{\mathbf{x}_{k+1}} P(\mathbf{e}_{k+1}|\mathbf{x}_{k+1}) P(\mathbf{e}_{k+2:t}|\mathbf{x}_{k+1}) \mathbf{P}(\mathbf{x}_{k+1}|\mathbf{X}_k)$$

Smoothing Example





Forward–backward algorithm: cache forward messages along the way Time linear in t (polytree inference), space $O(t|\mathbf{f}|)$

Most Likely Explanation



- Most likely sequence *≠* sequence of most likely states
- Most likely path to each x_{t+1}
 = most likely path to some x_t plus one more step

$$\max_{\mathbf{x}_{1}...\mathbf{x}_{t}} \mathbf{P}(\mathbf{x}_{1},...,\mathbf{x}_{t},\mathbf{X}_{t+1}|\mathbf{e}_{1:t+1})$$

= $\mathbf{P}(\mathbf{e}_{t+1}|\mathbf{X}_{t+1}) \max_{\mathbf{x}_{t}} \left(\mathbf{P}(\mathbf{X}_{t+1}|\mathbf{x}_{t}) \max_{\mathbf{x}_{1}...\mathbf{x}_{t-1}} P(\mathbf{x}_{1},...,\mathbf{x}_{t-1},\mathbf{x}_{t}|\mathbf{e}_{1:t}) \right) \mathbf{I}$

• Identical to filtering, except $f_{1:t}$ replaced by

$$\mathbf{m}_{1:t} = \max_{\mathbf{x}_1...\mathbf{x}_{t-1}} \mathbf{P}(\mathbf{x}_1,\ldots,\mathbf{x}_{t-1},\mathbf{X}_t | \mathbf{e}_{1:t})$$

i.e., $\mathbf{m}_{1:t}(i)$ gives the probability of the most likely path to state *i*.

• Update has sum replaced by max, giving the Viterbi algorithm:

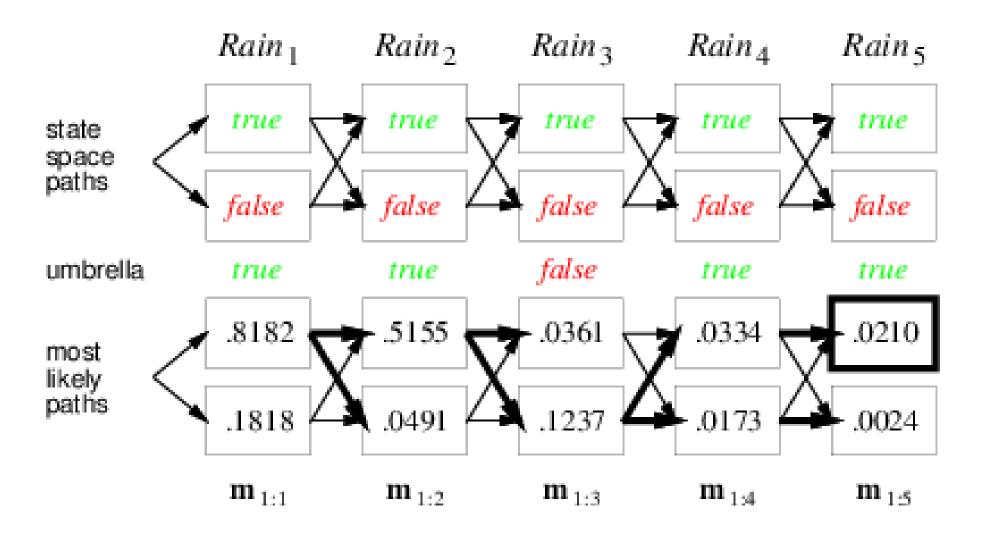
$$\mathbf{m}_{1:t+1} = \mathbf{P}(\mathbf{e}_{t+1}|\mathbf{X}_{t+1}) \max_{\mathbf{x}_t} (\mathbf{P}(\mathbf{X}_{t+1}|\mathbf{x}_t)\mathbf{m}_{1:t})$$

Also requires back-pointers for backward pass to retrieve best sequence

 $\mathbf{b}_{\mathbf{X}_{t+1},t+1} = \operatorname{argmax}_{\mathbf{x}_t} (\mathbf{P}(\mathbf{X}_{t+1}|\mathbf{x}_t)\mathbf{m}_{1:t})$

Viterbi Example





Hidden Markov Models



- X_t is a single, discrete variable (usually E_t is too) Domain of X_t is $\{1, \dots, S\}$
- Transition matrix $\mathbf{T}_{ij} = P(X_t = j | X_{t-1} = i)$, e.g., $\begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix}$
- Sensor matrix \mathbf{O}_t for each time step, diagonal elements $P(e_t|X_t = i)$ e.g., with $U_1 = true$, $\mathbf{O}_1 = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.2 \end{pmatrix}$
- Forward and backward messages as column vectors:

$$\mathbf{f}_{1:t+1} = \alpha \mathbf{O}_{t+1} \mathbf{T}^{\mathsf{T}} \mathbf{f}_{1:t}$$
$$\mathbf{b}_{k+1:t} = \mathbf{T} \mathbf{O}_{k+1} \mathbf{b}_{k+2:t}$$

• Forward-backward algorithm needs time $O(S^2t)$ and space O(St)

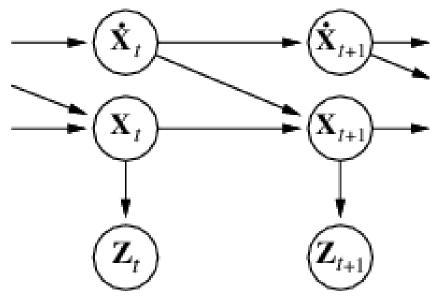


kalman filters

Kalman Filters



Modelling systems described by a set of continuous variables,
 e.g., tracking a bird flying—X_t = X, Y, Z, X, Y, Z.
 Airplanes, robots, ecosystems, economies, chemical plants, planets, ...



 $(Z_t = observed position)$

• Gaussian prior, linear Gaussian transition model and sensor model

Updating Gaussian Distributions



• Prediction step: if $P(X_t | e_{1:t})$ is Gaussian, then prediction

$$\mathbf{P}(\mathbf{X}_{t+1}|\mathbf{e}_{1:t}) = \int_{\mathbf{X}_t} \mathbf{P}(\mathbf{X}_{t+1}|\mathbf{x}_t) P(\mathbf{x}_t|\mathbf{e}_{1:t}) d\mathbf{x}_t$$

is Gaussian. If $P(X_{t+1}|e_{1:t})$ is Gaussian, then the updated distribution

$$P(X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1}) P(X_{t+1}|e_{1:t})$$

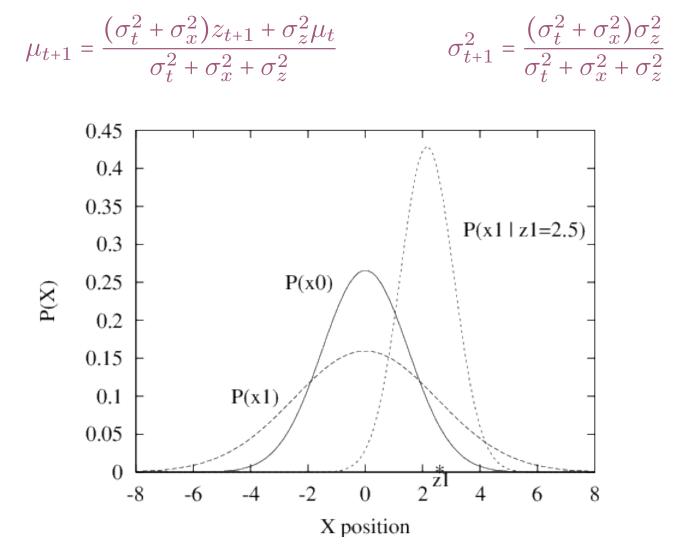
is Gaussian

- Hence $P(X_t | e_{1:t})$ is multivariate Gaussian $N(\mu_t, \Sigma_t)$ for all t
- General (nonlinear, non-Gaussian) process: description of posterior grows **unboundedly** as $t \to \infty$

Simple 1-D Example



• Gaussian random walk on X-axis, s.d. σ_x , sensor s.d. σ_z



General Kalman Update



• Transition and sensor models:

$$P(\mathbf{x}_{t+1}|\mathbf{x}_t) = N(\mathbf{F}\mathbf{x}_t, \mathbf{\Sigma}_x)(\mathbf{x}_{t+1})$$
$$P(\mathbf{z}_t|\mathbf{x}_t) = N(\mathbf{H}\mathbf{x}_t, \mathbf{\Sigma}_z)(\mathbf{z}_t)$$

F is the matrix for the transition; Σ_x the transition noise covariance **H** is the matrix for the sensors; Σ_z the sensor noise covariance

• Filter computes the following update:

$$\mu_{t+1} = \mathbf{F}\mu_t + \mathbf{K}_{t+1}(\mathbf{z}_{t+1} - \mathbf{H}\mathbf{F}\mu_t)$$

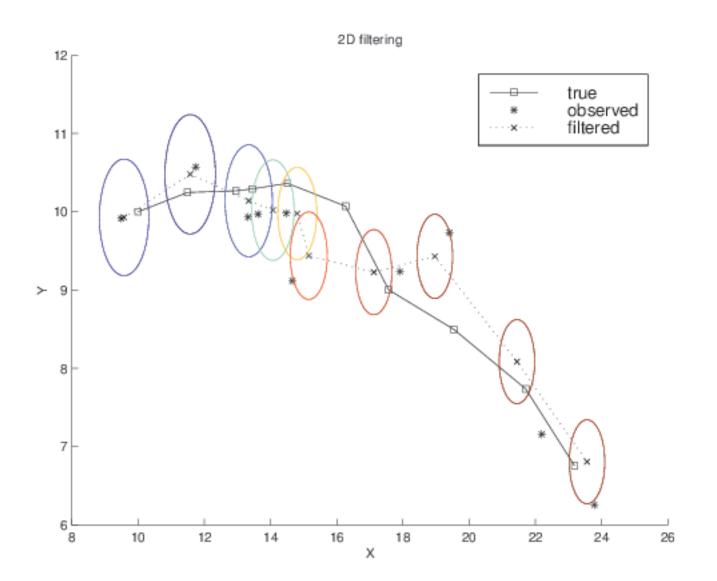
$$\Sigma_{t+1} = (\mathbf{I} - \mathbf{K}_{t+1})(\mathbf{F}\Sigma_t\mathbf{F}^{\mathsf{T}} + \Sigma_x)$$

where $\mathbf{K}_{t+1} = (\mathbf{F} \boldsymbol{\Sigma}_t \mathbf{F}^{\mathsf{T}} + \boldsymbol{\Sigma}_x) \mathbf{H}^{\mathsf{T}} (\mathbf{H} (\mathbf{F} \boldsymbol{\Sigma}_t \mathbf{F}^{\mathsf{T}} + \boldsymbol{\Sigma}_x) \mathbf{H}^{\mathsf{T}} + \boldsymbol{\Sigma}_z)^{-1}$ is the Kalman gain matrix

• Σ_t and K_t are independent of observation sequence, so compute offline

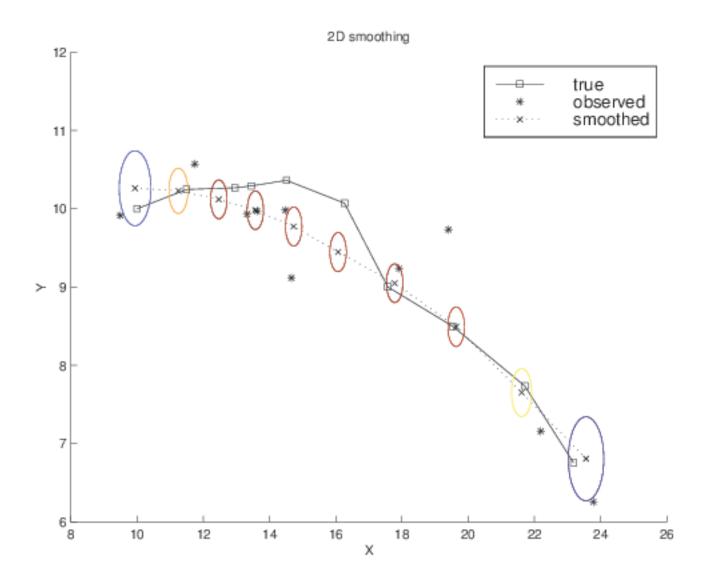
2-D Tracking Example: Filtering





2-D Tracking Example: Smoothing





Philipp Koehn

Artificial Intelligence: Markov Decision Processes

11 April 2017

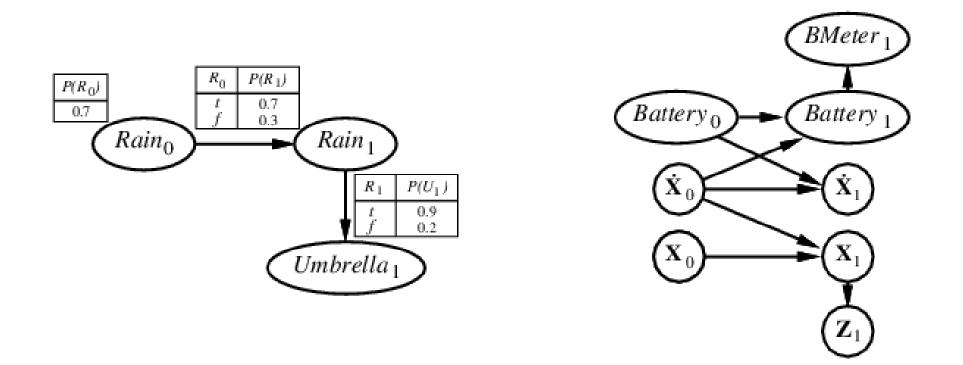


dynamic baysian networks

Dynamic Bayesian Networks



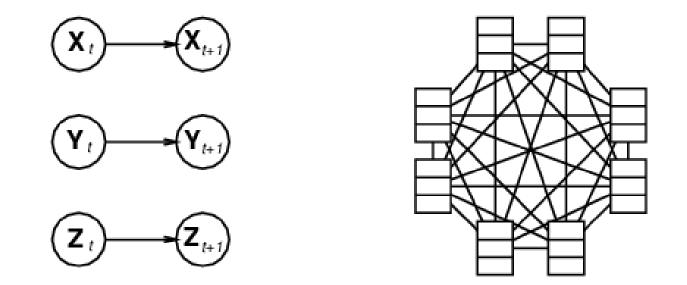
• X_t , E_t contain arbitrarily many variables in a sequentialized Bayes net



DBNs vs. HMMs



• Every HMM is a single-variable DBN; every discrete DBN is an HMM

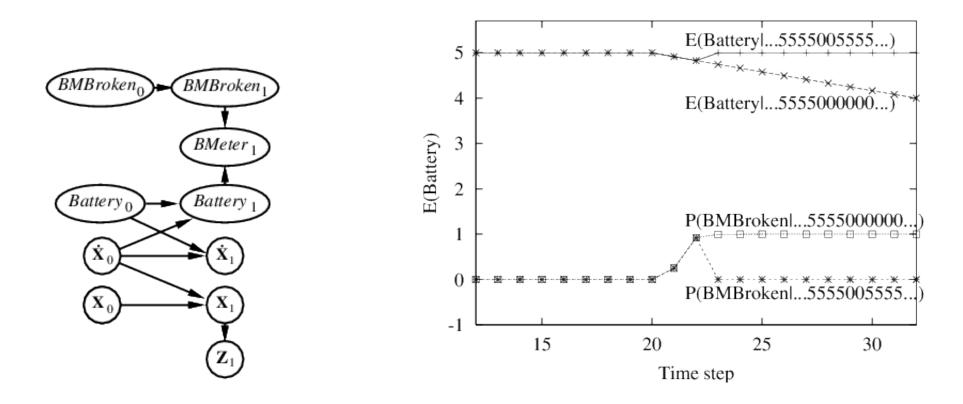


Sparse dependencies ⇒ exponentially fewer parameters;
 e.g., 20 state variables, three parents each
 DBN has 20 × 2³ = 160 parameters, HMM has 2²⁰ × 2²⁰ ≈ 10¹²

DBNs vs Kalman Filters



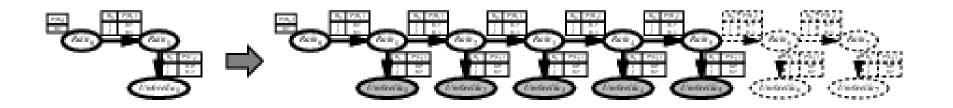
- Every Kalman filter model is a DBN, but few DBNs are KFs; real world requires non-Gaussian posteriors
- E.g., where my keys? What's the battery charge?



Exact Inference in DBNs



• Naive method: unroll the network and run any exact algorithm

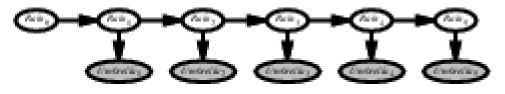


- Problem: inference cost for each update grows with *t*
- Rollup filtering: add slice t + 1, "sum out" slice t using variable elimination
- Largest factor is $O(d^{n+1})$, update cost $O(d^{n+2})$ (cf. HMM update cost $O(d^{2n})$)

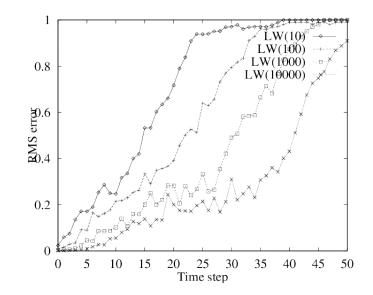


Likelihood Weighting for DBNs

• Set of weighted samples approximates the belief state



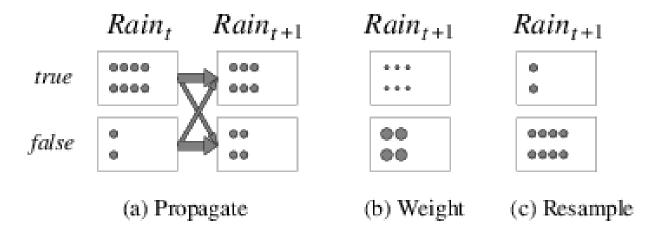
- LW samples pay no attention to the evidence!
 - \Rightarrow fraction "agreeing" falls exponentially with t
 - \Rightarrow number of samples required grows exponentially with t



Particle Filtering



- Basic idea: ensure that the population of samples ("particles") tracks the high-likelihood regions of the state-space
- Replicate particles proportional to likelihood for \mathbf{e}_t



- Widely used for tracking nonlinear systems, esp. in vision
- Also used for simultaneous localization and mapping in mobile robots 10^5 -dimensional state space



speech recognition

Speech as Probabilistic Inference



It's not easy to wreck a nice beach

- Speech signals are noisy, variable, ambiguous
- What is the **most likely** word sequence, given the speech signal? I.e., choose *Words* to maximize P(Words|signal)
- Use Bayes' rule:

 $P(Words|signal) = \alpha P(signal|Words)P(Words)$ i.e., decomposes into acoustic model + language model

• *Words* are the hidden state sequence, *signal* is the observation sequence

Phones



- All human speech is composed from 40-50 phones, determined by the configuration of articulators (lips, teeth, tongue, vocal cords, air flow)
- Form an intermediate level of hidden states between words and signal
 acoustic model = pronunciation model + phone model
- ARPAbet designed for American English

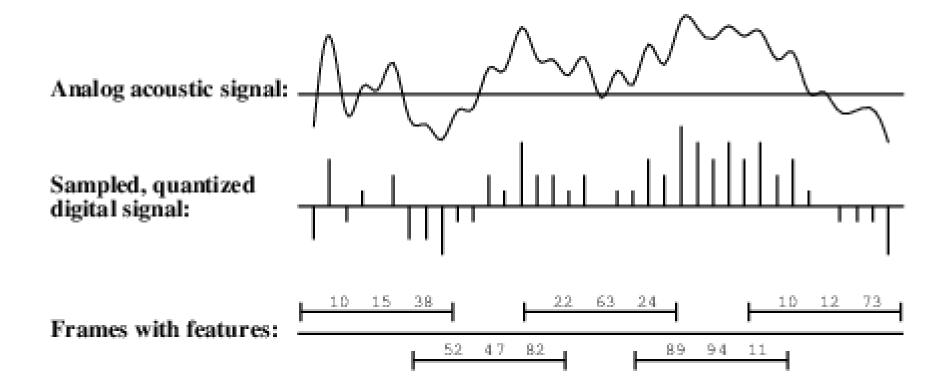
| [iy] | b <u>ea</u> t | [b] | <u>b</u> et | [p] | p et |
|------|---------------------|------|--------------------|------|----------------|
| [ih] | b <u>i</u> t | [ch] | <u>Ch</u> et | [r] | |
| [ey] | b e t | [d] | <u>d</u> ebt | [s] | <u>s</u> et |
| [ao] | b ough t | [hh] | <u>h</u> at | [th] | <u>th</u> ick |
| [ow] | b <mark>oa</mark> t | [hv] | <u>h</u> igh | [dh] | <u>th</u> at |
| [er] | B <u>er</u> t | [1] | let | [w] | <u>w</u> et |
| [ix] | ros e s | [ng] | si ng | [en] | butt <u>on</u> |
| : | : | : | : | • | : |

e.g., "ceiling" is [s iy l ih ng] / [s iy l ix ng] / [s iy l en]

Speech Sounds



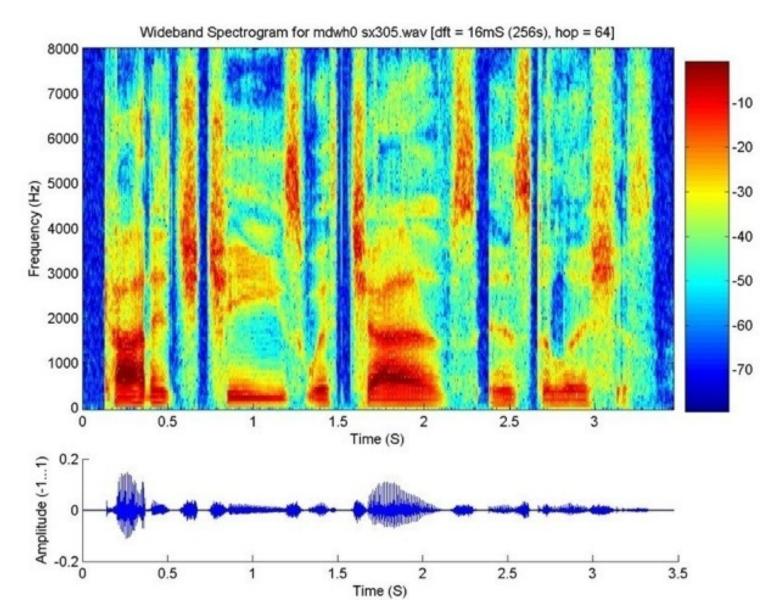
• Raw signal is the microphone displacement as a function of time; processed into overlapping 30ms frames, each described by features



• Frame features are typically formants—peaks in the power spectrum

Speech Spectrogram





Phone Models

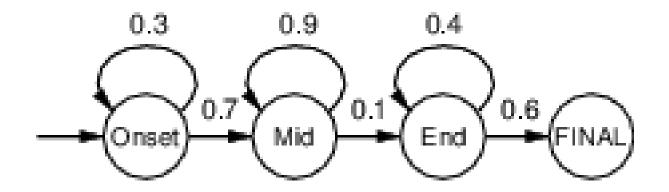


- Frame features in *P*(*features*|*phone*) summarized by
 - an integer in $[0 \dots 255]$ (using vector quantization); or
 - the parameters of a mixture of Gaussians
- Three-state phones: each phone has three phases (Onset, Mid, End)
 E.g., [t] has silent Onset, explosive Mid, hissing End
 ⇒ P(features|phone, phase)
- Triphone context: each phone becomes n² distinct phones, depending on the phones to its left and right
 E.g., [t] in "star" is written [t(s,aa)] (different from "tar"!)
- Triphones useful for handling coarticulation effects: the articulators have inertia and cannot switch instantaneously between positions E.g., [t] in "eighth" has tongue against front teeth

Phone Model Example



Phone HMM for [m]:



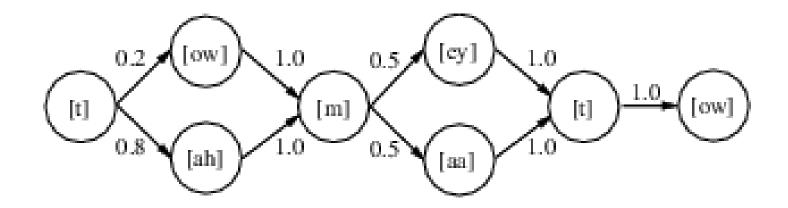
Output probabilities for the phone HMM:

| Onset: | Mid: | End: |
|---------|--------|---------|
| C1:0.5 | C3:0.2 | C4: 0.1 |
| C2: 0.2 | C4:0.7 | C6: 0.5 |
| C3: 0.3 | C5:0.1 | C7: 0.4 |

Word Pronunciation Models



- Each word is described as a distribution over phone sequences
- Distribution represented as an HMM transition model



P([towmeytow]|"tomato") = P([towmaatow]|"tomato") = 0.1P([tahmeytow]|"tomato") = P([tahmaatow]|"tomato") = 0.4

• Structure is created manually, transition probabilities learned from data

Recognition of Isolated Words



• Phone models + word models fix likelihood $P(e_{1:t}|word)$ for isolated word

 $P(word|e_{1:t}) = \alpha P(e_{1:t}|word)P(word)$

• Prior probability P(word) obtained simply by counting word frequencies $P(e_{1:t}|word)$ can be computed recursively: define

 $\mathbf{A}_{1:t} = \mathbf{P}(\mathbf{X}_t, \mathbf{e}_{1:t})$

and use the recursive update

 $\mathbf{A}_{1:t+1} = \mathsf{FORWARD}(\ell_{1:t}, \mathbf{e}_{t+1})$

and then $P(e_{1:t}|word) = \sum_{\mathbf{x}_t} \mathbf{A}_{1:t}(\mathbf{x}_t)$

• Isolated-word dictation systems with training reach 95–99% accuracy

Continuous Speech



- Not just a sequence of isolated-word recognition problems!
 - adjacent words highly correlated
 - sequence of most likely words ≠ most likely sequence of words
 - segmentation: there are few gaps in speech
 - cross-word coarticulation—e.g., "next thing"
- Complications
 - mismatch between speaker in training and test
 - noise
 - crosstalk
 - bad microphone position
- Continuous speech systems manage over 90% accuracy on a good day

Language Model



• Prior probability of a word sequence is given by chain rule:

$$P(w_1 \cdots w_n) = \prod_{i=1}^n P(w_i | w_1 \cdots w_{i-1})$$

• Bigram model:

 $P(w_i|w_1\cdots w_{i-1}) \approx P(w_i|w_{i-1})$

- Train by counting all word pairs in a large text corpus
- More sophisticated models (trigrams, grammars, etc.) help a little bit

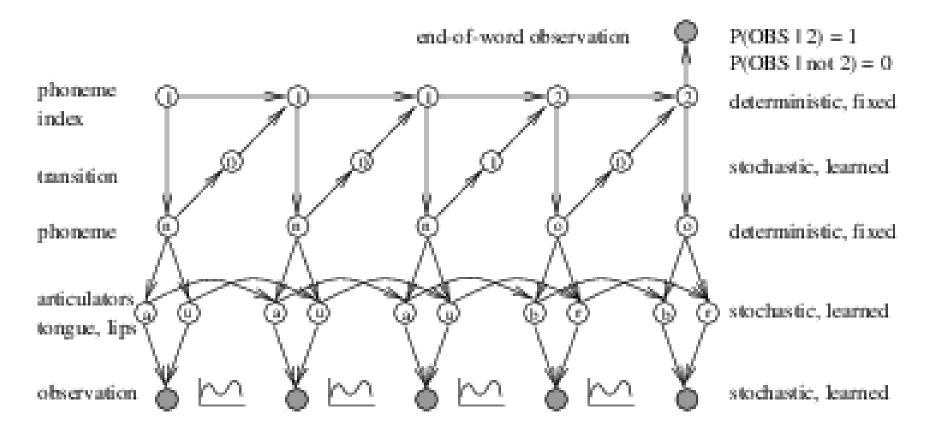
Combined HMM



- States of the combined language+word+phone model are labelled by the word we're in + the phone in that word + the phone state in that phone
- Viterbi algorithm finds the most likely **phone state** sequence
- Does segmentation by considering all possible word sequences and boundaries
- Doesn't always give the most likely word sequence because each word sequence is the sum over many state sequences
- Jelinek invented A^{*} in 1969 a way to find most likely word sequence where "step cost" is $-\log P(w_i|w_{i-1})$

DBNs for Speech Recognition





- Also easy to add variables for, e.g., gender, accent, speed
- Zweig and Russell (1998) show up to 40% error reduction over HMMs

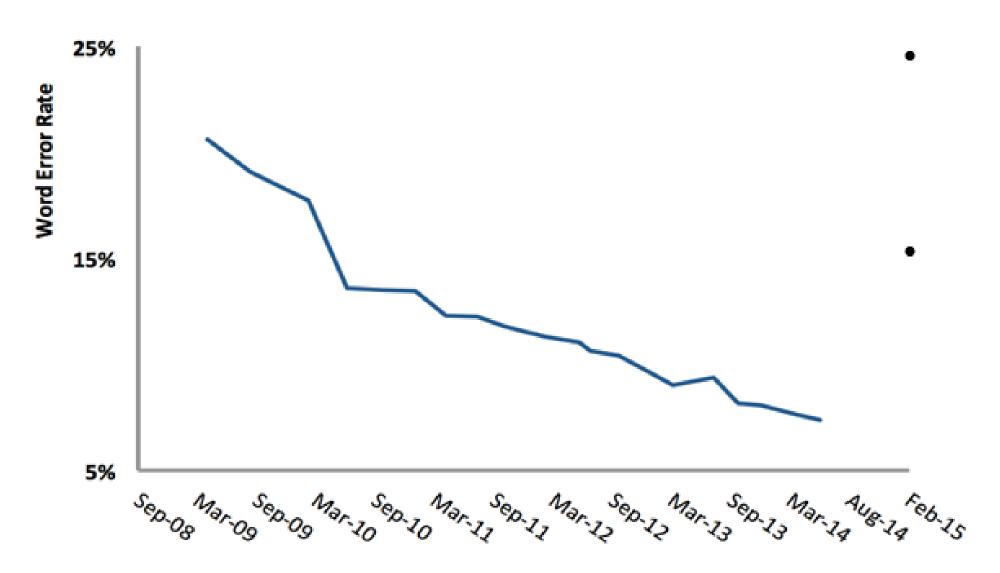
Progress



NIST STT Benchmark Test History – May. '09 100% Switchboard **Conversational Speech Meeting Speech** (Non-English) Meeting – SDM OV4 Read eeting – MD M O V4 Speech Switchboard II Broadcast Speech CTS Arabic (UL) itchboard Cellular CTS Mandarin (UL)0 Meeting - IH N Air Travel Planning Kiosk Speech News Mandarin 10X Varied Microphones (Non-English News Arabic 10X CTS Fisher (UL) 20k News English 1X WER(%) News English unlimited 10% Noisy = News English 10X 5k 1k 4% Range of Human Error In Transcription 2% 1% 1988 1989 1990 1991 1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011

Progress





Summary



- Temporal models use state and sensor variables replicated over time
- Markov assumptions and stationarity assumption, so we need
 - transition model $P(X_t | X_{t-1})$
 - sensor model $\mathbf{P}(\mathbf{E}_t | \mathbf{X}_t)$
- Tasks are filtering, smoothing, most likely sequence;
 all done recursively with constant cost per time step
- Hidden Markov models have a single discrete state variable; used for speech recognition
- Kalman filters allow n state variables, linear Gaussian, $O(n^3)$ update
- Dynamic Bayes nets subsume HMMs, Kalman filters; exact update intractable
- Speech recognition