Logical Agents

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The world is everything that is the case.

Wittgenstein, Tractatus

Outline



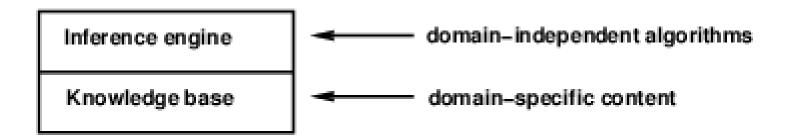
- Knowledge-based agents
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
 - forward chaining
 - backward chaining
 - resolution



knowledge-based agents

Knowledge-Based Agent





- Knowledge base = set of sentences in a **formal** language
- Declarative approach to building an agent (or other system): TELL it what it needs to know
- Then it can ASK itself what to do—answers should follow from the KB
- Agents can be viewed at the knowledge level i.e., what they know, regardless of how implemented
- Or at the implementation level

i.e., data structures in KB and algorithms that manipulate them

A Simple Knowledge-Based Agent



```
function KB-AGENT( percept) returns an action
static: KB, a knowledge base
t, a counter, initially 0, indicating time
TELL(KB, MAKE-PERCEPT-SENTENCE( percept, t))
action \leftarrow ASK(KB, MAKE-ACTION-QUERY(t))
TELL(KB, MAKE-ACTION-SENTENCE(action, t))
t \leftarrow t + 1
return action
```

- The agent must be able to
 - represent states, actions, etc.
 - incorporate new percepts
 - update internal representations of the world
 - deduce hidden properties of the world
 - deduce appropriate actions



example

Wumpus World PEAS Description

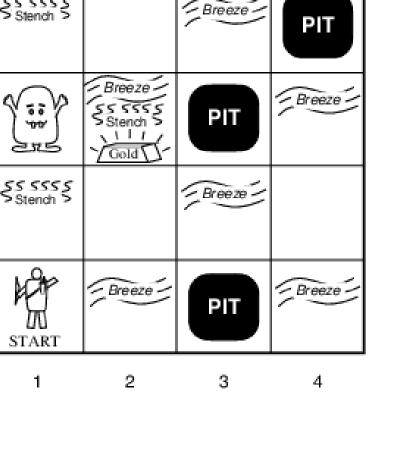
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3.

2

1

- Performance measure
 - **–** gold +1000, death -1000
 - -1 per step, -10 for using the arrow
- Environment
 - squares adjacent to wumpus are smelly
 - squares adjacent to pit are breezy
 - glitter iff gold is in the same square
 - shooting kills wumpus if you are facing it
 - shooting uses up the only arrow
 - grabbing picks up gold if in same square
 - releasing drops the gold in same square
- Actuators Left turn, Right turn, Forward, Grab, Release, Shoot
- Sensors Breeze, Glitter, Smell

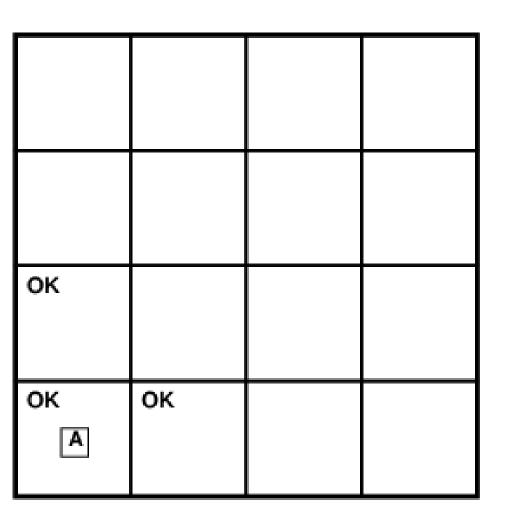




Wumpus World Characterization



- Observable? No—only local perception
- Deterministic? Yes—outcomes exactly specified
- Episodic? No—sequential at the level of actions
- Static? Yes—Wumpus and Pits do not move
- Discrete? Yes
- Single-agent? Yes—Wumpus is essentially a natural feature

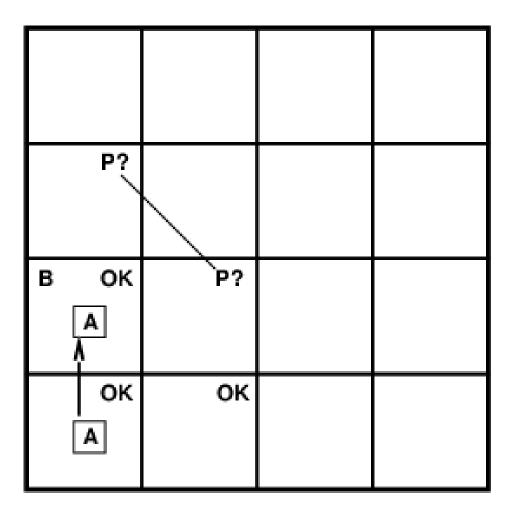


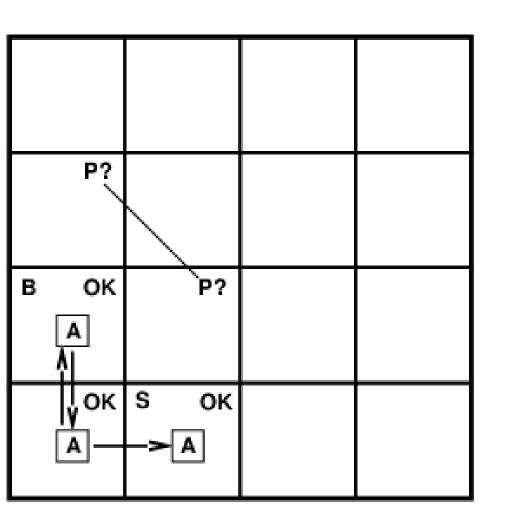




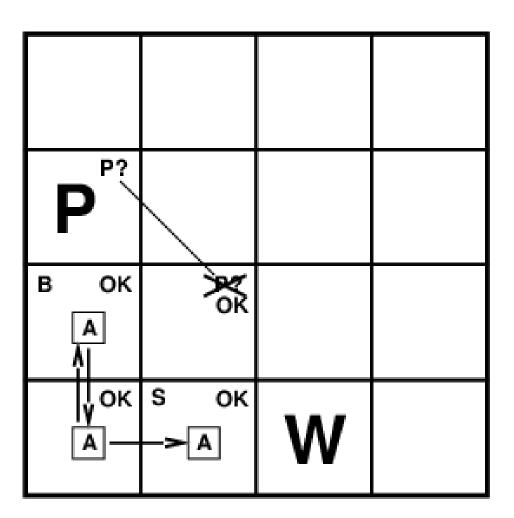
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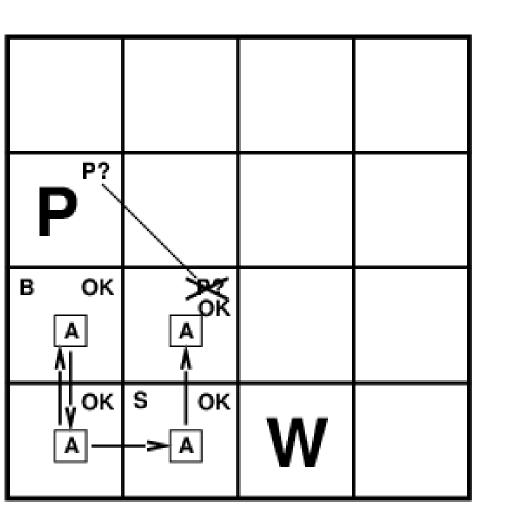






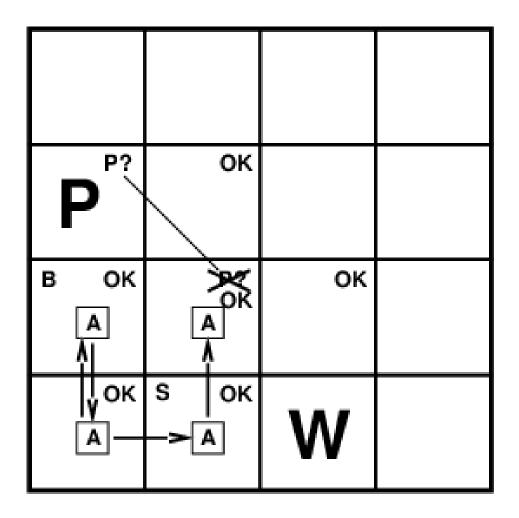




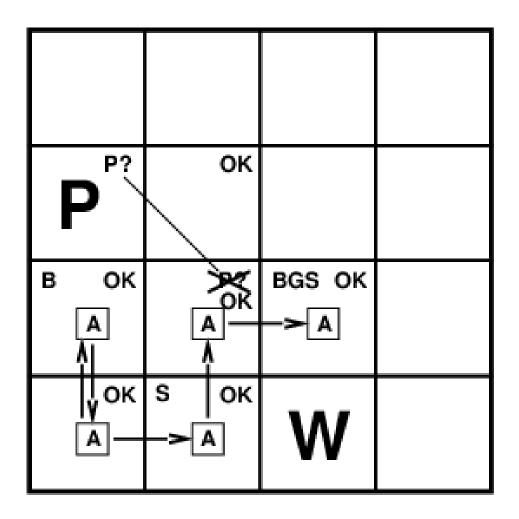






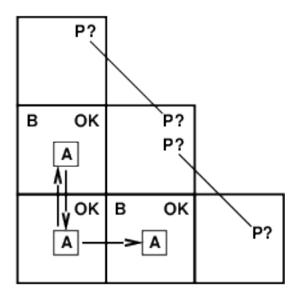






Tight Spot

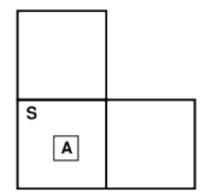




- Breeze in (1,2) and (2,1) \implies no safe actions
- Assuming pits uniformly distributed, (2,2) has pit w/ prob 0.86, vs. 0.31

Tight Spot





• Smell in (1,1)

 \implies cannot move

- Can use a strategy of coercion: shoot straight ahead
 - wumpus was there \implies dead \implies safe
 - wumpus wasn't there \implies safe



logic in general

Logic in General



- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the "meaning" of sentences; i.e., define truth of a sentence in a world
- E.g., the language of arithmetic
 - $x + 2 \ge y$ is a sentence; $x^2 + y > x^2$ is not a sentence
 - $x + 2 \ge y$ is true iff the number x + 2 is no less than the number y
 - $x + 2 \ge y$ is true in a world where x = 7, y = 1 $x + 2 \ge y$ is false in a world where x = 0, y = 6

Entailment



• Entailment means that one thing **follows from** another:

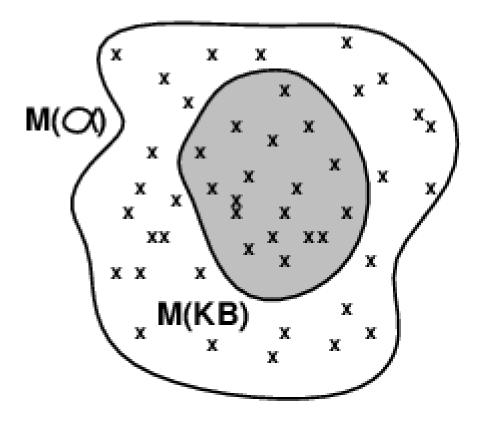
$KB\vDash \alpha$

- Knowledge base *KB* entails sentence α if and only if
 α is true in all worlds where *KB* is true
- E.g., the KB containing "the Ravens won" and "the Jays won" entails "the Ravens won or the Jays won"
- E.g., x + y = 4 entails 4 = x + y
- Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**
- Note: brains process **syntax** (of some sort)

Models

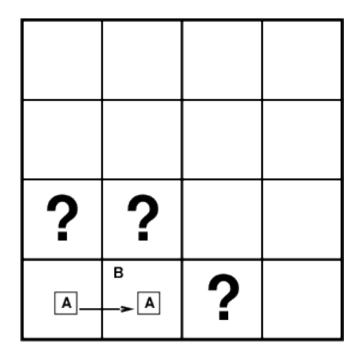


- Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated
- We say *m* is a model of a sentence α if α is true in *m*
- $M(\alpha)$ is the set of all models of α
- \Rightarrow *KB* $\vDash \alpha$ if and only if *M*(*KB*) \subseteq *M*(α)
 - E.g. KB = Ravens won and Jays won α = Ravens won



Entailment in the Wumpus World

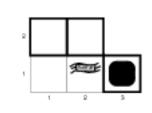


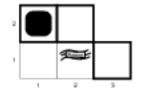


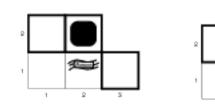
- Situation after detecting nothing in [1,1], moving right, breeze in [2,1]
- Consider possible models for all **?**, assuming only pits
- 3 Boolean choices \implies 8 possible models

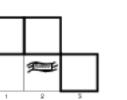
Possible Wumpus Models

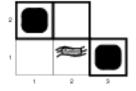


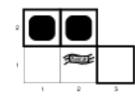


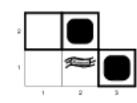


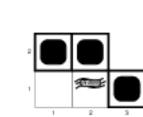






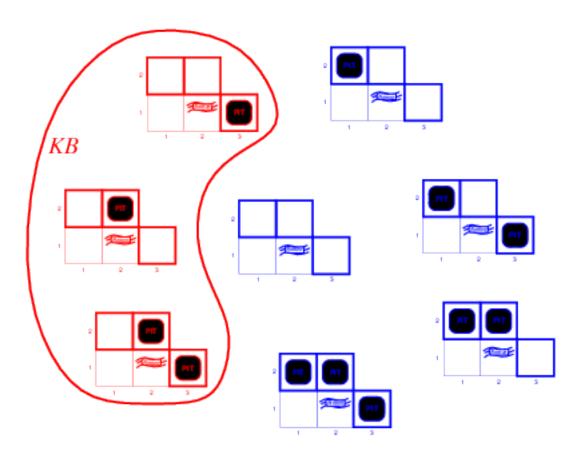






Valid Wumpus Models

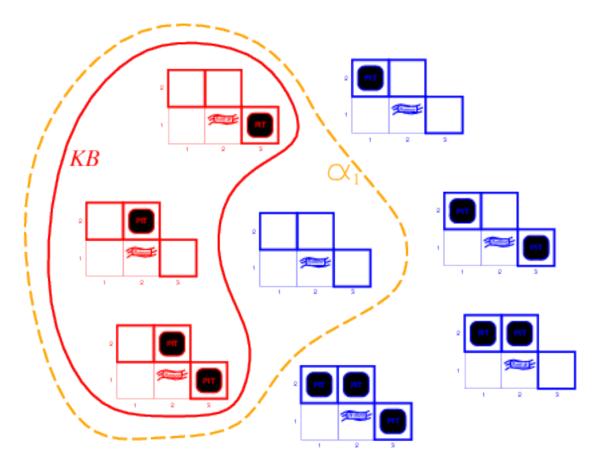




KB = wumpus-world rules + observations

Entailment



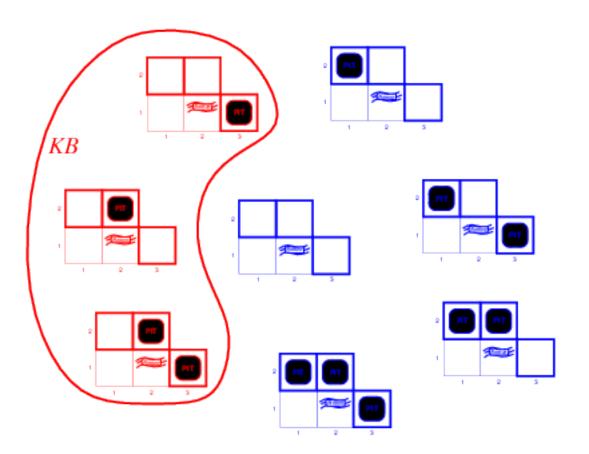


KB = wumpus-world rules + observations

 $\alpha_1 = "[1,2]$ is safe", $KB \vDash \alpha_1$, proved by model checking

Valid Wumpus Models

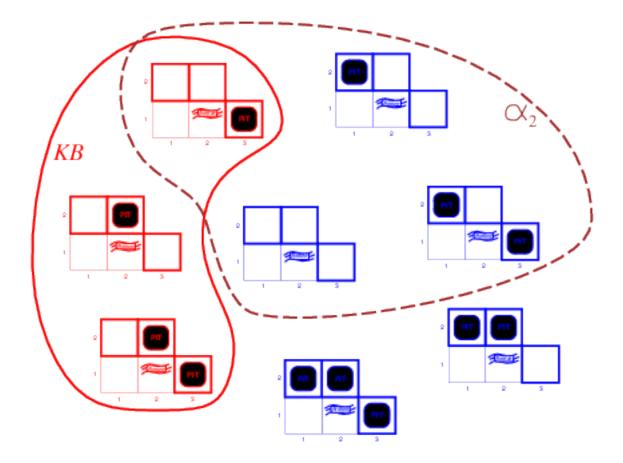




KB = wumpus-world rules + observations

Not Entailed





KB = wumpus-world rules + observations α_2 = "[2,2] is safe", $KB \neq \alpha_2$

Inference



- $KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure *i*
- Consequences of KB are a haystack; α is a needle. Entailment = needle in haystack; inference = finding it
- Soundness: *i* is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \vDash \alpha$
- Completeness: *i* is complete if whenever $KB \vDash \alpha$, it is also true that $KB \succ_i \alpha$
- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
- That is, the procedure will answer any question whose answer follows from what is known by the *KB*.



propositional logic

Propositional Logic: Syntax



- Propositional logic is the simplest logic—illustrates basic ideas
- The proposition symbols P_1 , P_2 etc are sentences
- If *P* is a sentence, $\neg P$ is a sentence (negation)
- If P_1 and P_2 are sentences, $P_1 \wedge P_2$ is a sentence (conjunction)
- If P_1 and P_2 are sentences, $P_1 \vee P_2$ is a sentence (disjunction)
- If P_1 and P_2 are sentences, $P_1 \implies P_2$ is a sentence (implication)
- If P_1 and P_2 are sentences, $P_1 \Leftrightarrow P_2$ is a sentence (biconditional)

Propositional Logic: Semantics



• Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$ true true false

(with these symbols, 8 possible models, can be enumerated automatically)

• Rules for evaluating truth with respect to a model *m*:

$\neg P$	is true iff	P	is false		
$P_1 \wedge P_2$	is true iff	P_1	is true and	P_2	is true
$P_1 \lor P_2$	is true iff	P_1	is true or	P_2	is true
$P_1 \implies P_2$	is true iff	P_1	is false or	P_2	is true
i.e.,	is false iff	P_1	is true and	P_2	is false
$P_1 \Leftrightarrow P_2$	is true iff	$P_1 \implies P_2$	is true and	$P_2 \implies P_1$	is true

• Simple recursive process evaluates an arbitrary sentence, e.g., $\neg P_{1,2} \land (P_{2,2} \lor P_{3,1}) = true \land (false \lor true) = true \land true = true$

Truth Tables for Connectives



P	Q	$\neg P$	$P \wedge Q$	$P \lor Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
true	true	false	true	true	true	true

Wumpus World Sentences



- Let $P_{i,j}$ be true if there is a pit in [i, j]
 - observation $R_1 : \neg P_{1,1}$
- Let $B_{i,j}$ be true if there is a breeze in [i, j].
- "Pits cause breezes in adjacent squares"
 - rule $R_2: B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$
 - rule $R_3: B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$
 - observation R_4 : $\neg B_{1,1}$
 - observation $R_5: B_{2,1}$
- What can we infer about $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, etc.?

Truth Tables for Inference



$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	true	true	true	true	false	false						
false	false	false	false	false	false	true	true	true	false	true	false	false
:	• •	•	•	•	•	•		•		• •	•	•
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
false	true	false	false	false	true	false	true	true	true	true	true	true
false	true	false	false	false	true	true	true	true	true	true	true	true
false	true	false	false	true	false	false	true	false	false	true	true	false
	• •				•					• •	•	
true	false	true	true	false	true	false						

- Enumerate rows (different assignments to symbols $P_{i,j}$)
- Check if rules are satisfied (*R_i*)
- Valid model (*KB*) if all rules satisfied

Inference by Enumeration



• Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB, \alpha) returns true or false
  inputs: KB, the knowledge base, a sentence in propositional logic
          \alpha, the query, a sentence in propositional logic
  symbols \leftarrow a list of the proposition symbols in KB and \alpha
  return TT-CHECK-ALL(KB, \alpha, symbols, [])
function TT-CHECK-ALL(KB, \alpha, symbols, model) returns true or false
  if EMPTY?(symbols) then
     if PL-TRUE?(KB, model) then return PL-TRUE?(\alpha, model)
     else return true
  else do
     P ← FIRST(symbols); rest ← REST(symbols)
     return TT-CHECK-ALL(KB, \alpha, rest, EXTEND(P, true, model)) and
             TT-CHECK-ALL(KB, \alpha, rest, EXTEND(P, false, model))
```

• $O(2^n)$ for *n* symbols; problem is **co-NP-complete**

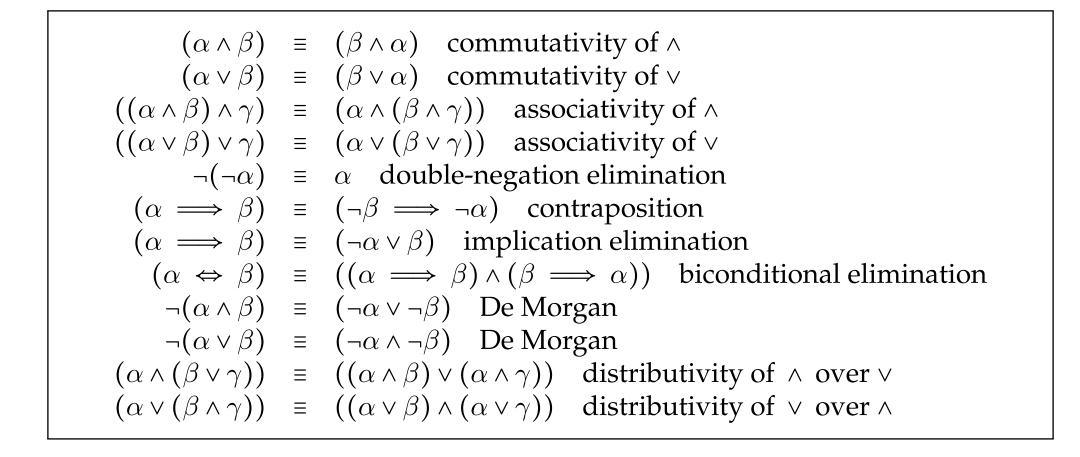


equivalence, validity, satisfiability

Logical Equivalence



• Two sentences are logically equivalent iff true in same models: $\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$



Validity and Satisfiability



- A sentence is valid if it is true in **all** models, e.g., *True*, $A \lor \neg A$, $A \Longrightarrow A$, $(A \land (A \Longrightarrow B)) \Longrightarrow B$
- Validity is connected to inference via the Deduction Theorem: $KB \vDash \alpha$ if and only if $(KB \implies \alpha)$ is valid
- A sentence is satisfiable if it is true in some model e.g., A ∨ B, C
- A sentence is unsatisfiable if it is true in **no** models e.g., $A \land \neg A$
- Satisfiability is connected to inference via the following: *KB* ⊨ α if and only if (*KB* ∧ ¬α) is unsatisfiable i.e., prove α by *reductio ad absurdum*



inference

Proof Methods



- Proof methods divide into (roughly) two kinds
- Application of inference rules
 - Legitimate (sound) generation of new sentences from old
 - Proof = a sequence of inference rule applications
 Can use inference rules as operators in a standard search alg.
 - Typically require translation of sentences into a normal form
- Model checking
 - truth table enumeration (always exponential in *n*)
 - improved backtracking
 - heuristic search in model space (sound but incomplete)
 e.g., min-conflicts-like hill-climbing algorithms

Forward and Backward Chaining



- Horn Form (restricted) KB = **conjunction** of **Horn clauses**
- Horn clause =
 - proposition symbol; or
 - (conjunction of symbols) \implies symbol

e.g., $C \land (B \implies A) \land (C \land D \implies B)$

• Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \dots, \alpha_n, \qquad \alpha_1 \wedge \dots \wedge \alpha_n \implies \beta}{\beta}$$

- Can be used with forward chaining or backward chaining
- These algorithms are very natural and run in **linear** time

Example



• Idea: fire any rule whose premises are satisfied in the *KB*, add its conclusion to the *KB*, until query is found

$$P \Longrightarrow Q$$

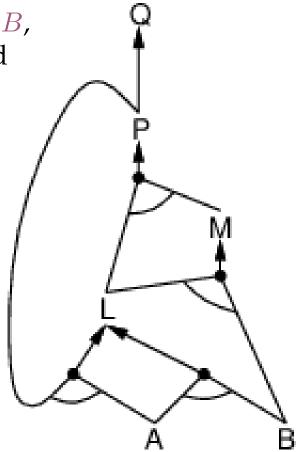
$$L \land M \Longrightarrow P$$

$$B \land L \Longrightarrow M$$

$$A \land P \Longrightarrow L$$

$$A \land B \Longrightarrow L$$

$$A$$





forward chaining

Forward Chaining



- Start with given proposition symbols (atomic sentence) e.g., *A* and *B*
- Iteratively try to infer truth of additional proposition symbols e.g., $A \land B \implies C$, therefor we establish *C* is true
- Continue until
 - no more inference can be carried out, or
 - goal is reached



• Given

$$P \Longrightarrow Q$$

$$L \wedge M \Longrightarrow P$$

$$B \wedge L \Longrightarrow M$$

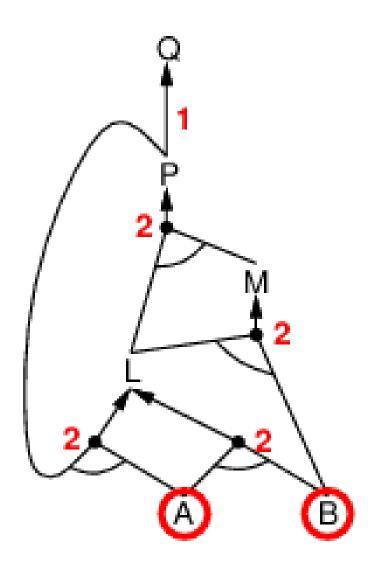
$$A \wedge P \Longrightarrow L$$

$$A \wedge B \Longrightarrow L$$

$$A$$

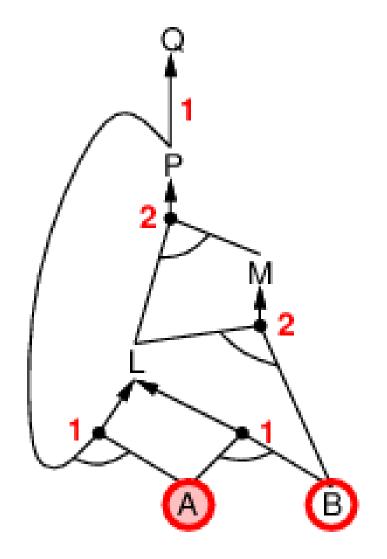
$$B$$

- Agenda: *A*, *B*
- Annotate horn clauses with number of premises



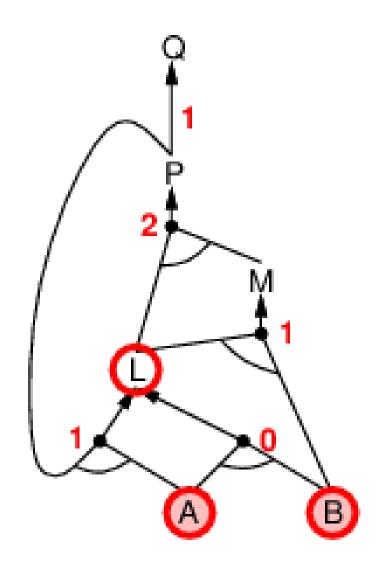


- Process agenda item A
- Decrease count for horn clauses in which *A* is premise



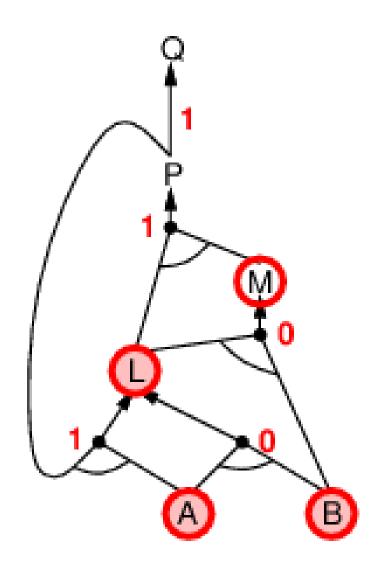


- Process agenda item *B*
- Decrease count for horn clauses in which *B* is premise
- $A \land B \implies L$ has now fulfilled premise
- Add *L* to agenda



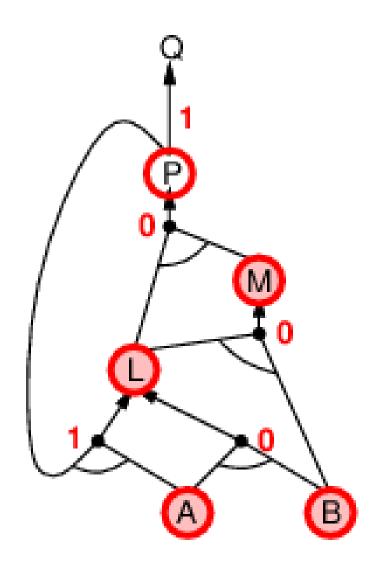


- Process agenda item *L*
- Decrease count for horn clauses in which *L* is premise
- $B \wedge L \implies M$ has now fulfilled premise
- Add *M* to agenda



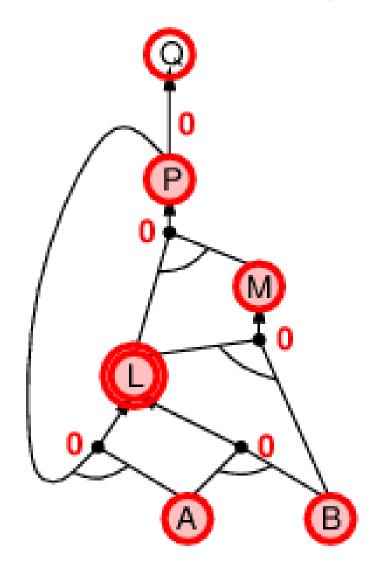


- Process agenda item *M*
- Decrease count for horn clauses in which *M* is premise
- $L \wedge M \implies P$ has now fulfilled premise
- Add *P* to agenda



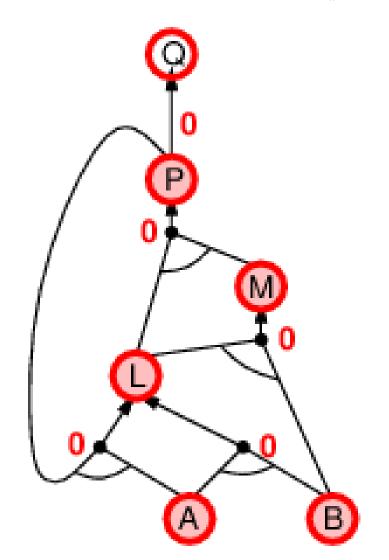


- Process agenda item *P*
- Decrease count for horn clauses in which *P* is premise
- $P \implies Q$ has now fulfilled premise
- Add *Q* to agenda
- $A \land P \implies L$ has now fulfilled premise



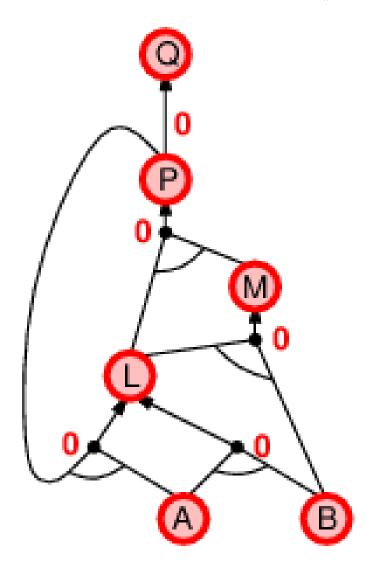


- Process agenda item *P*
- Decrease count for horn clauses in which *P* is premise
- $P \implies Q$ has now fulfilled premise
- Add *Q* to agenda
- $A \land P \implies L$ has now fulfilled premise
- But *L* is already inferred





- Process agenda item *Q*
- *Q* is inferred
- Done



Forward Chaining Algorithm



```
function PL-FC-ENTAILS?(KB, q) returns true or false
  inputs: KB, the knowledge base, a set of propositional Horn clauses
          q, the query, a proposition symbol
  local variables: count, a table, indexed by clause, init. number of premises
                   inferred, a table, indexed by symbol, each entry initially false
                   agenda, a list of symbols, initially the symbols known in KB
  while agenda is not empty do
     p \leftarrow POP(agenda)
     unless inferred[p] do
         inferred[p] ← true
         for each Horn clause c in whose premise p appears do
            decrement count[c]
            if count[c] = 0 then do
               if HEAD[c] = q then return true
                PUSH(HEAD[c], agenda)
  return false
```



backward chaining

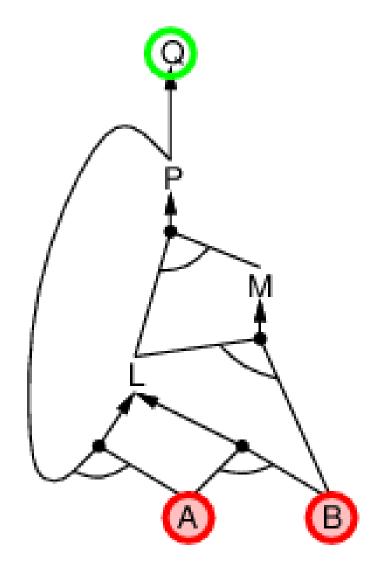
Backward Chaining



- Idea: work backwards from the query Q: to prove Q by BC, check if Q is known already, or prove by BC all premises of some rule concluding q
- Avoid loops: check if new subgoal is already on the goal stack
- Avoid repeated work: check if new subgoal
 - 1. has already been proved true, or
 - 2. has already failed

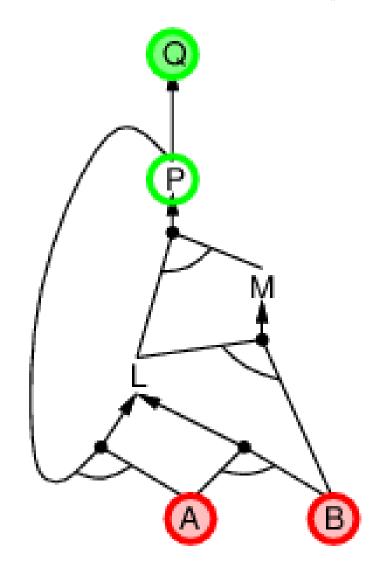


- *A* and *B* are known to be true
- *Q* needs to be proven



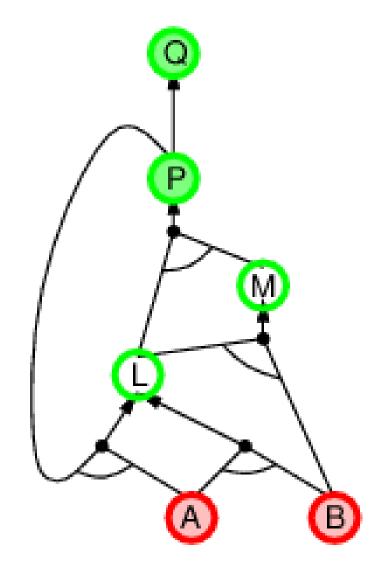


- Current goal: Q
- Q can be inferred by $P \implies Q$
- *P* needs to be proven



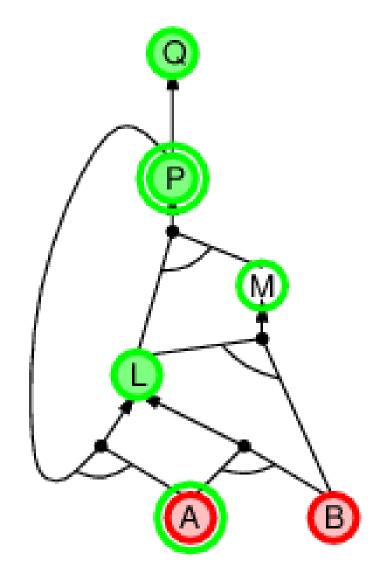


- Current goal: *P*
- P can be inferred by $L \land M \implies P$
- *L* and *M* need to be proven



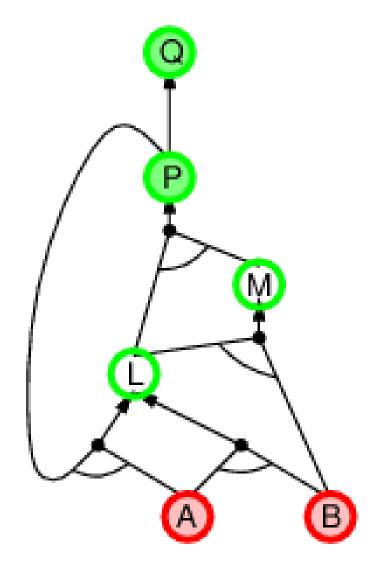


- Current goal: *L*
- *L* can be inferred by $A \land P \implies L$
- *A* is already true
- *P* is already a goal
- \Rightarrow repeated subgoal



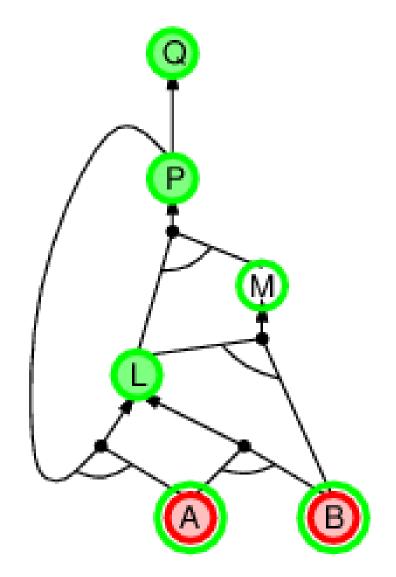


• Current goal: *L*



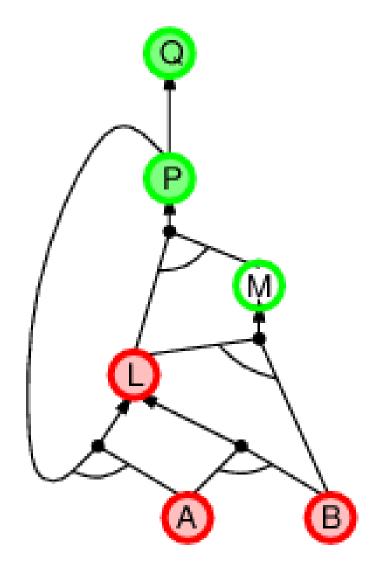


- Current goal: *L*
- *L* can be inferred by $A \land B \implies L$
- Both are true



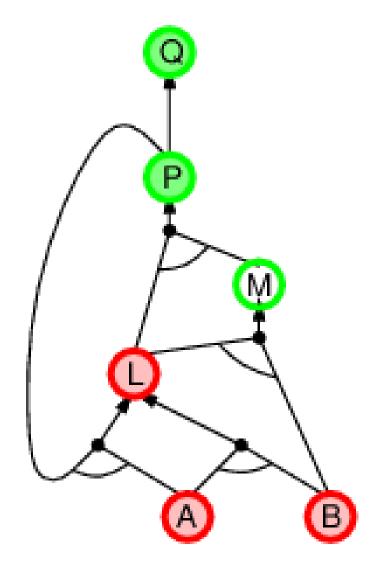


- Current goal: *L*
- *L* can be inferred by $A \land B \implies L$
- Both are true
- \Rightarrow *L* is true



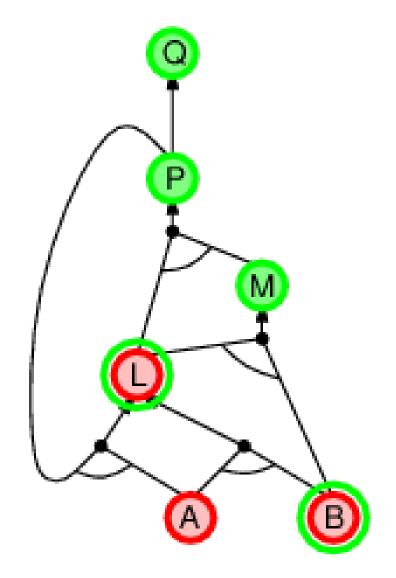


• Current goal: *M*



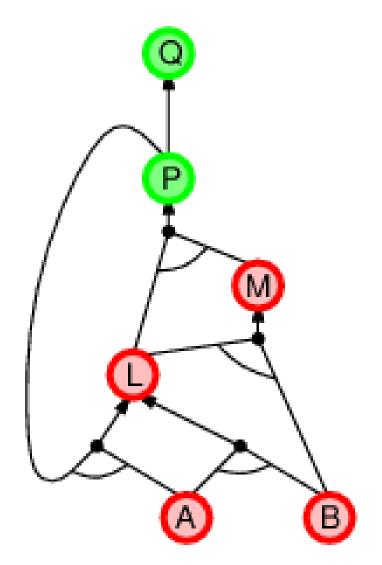


- Current goal: *M*
- *M* can be inferred by $B \wedge L \implies M$



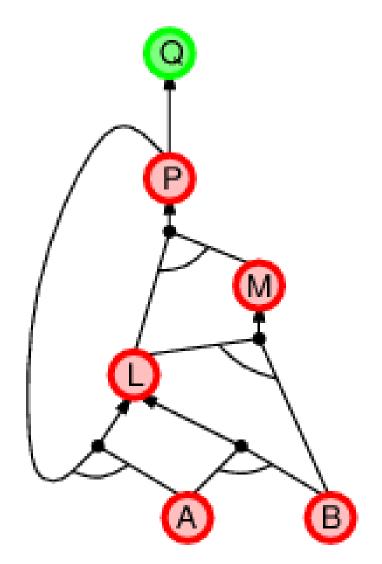


- Current goal: *M*
- *M* can be inferred by $B \wedge L \implies M$
- Both are true
- \Rightarrow *M* is true



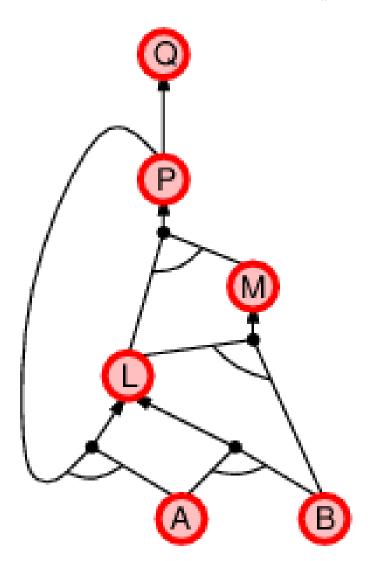


- Current goal: *P*
- P can be inferred by $L \land M \implies P$
- Both are true
- \Rightarrow *P* is true





- Current goal: *Q*
- Q can be inferred by $P \implies Q$
- *P* is true
- $\Rightarrow Q$ is true



Forward vs. Backward Chaining



- FC is data-driven, cf. automatic, unconscious processing, e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving, e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be **much less** than linear in size of KB



resolution

Resolution



- Conjunctive Normal Form (CNF—universal) conjunction of disjunctions of literals clauses E.g., $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$
- Resolution inference rule (for CNF): complete for propositional logic

 $\frac{\ell_1 \vee \cdots \vee \ell_k, \quad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$

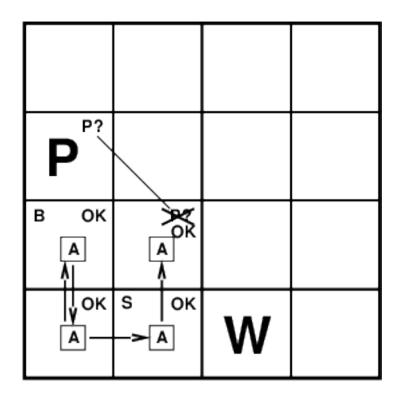
where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

• Resolution is sound and complete for propositional logic

Wampus World





• Rules such as: "If breeze, then a pit adjacent."

 $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

Conversion to CNF



 $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

1. Eliminate \Leftrightarrow , replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \implies \beta) \land (\beta \implies \alpha)$. $(B_{1,1} \implies (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \implies B_{1,1})$

2. Eliminate \Rightarrow , replacing $\alpha \Rightarrow \beta$ with $\neg \alpha \lor \beta$. $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg (P_{1,2} \lor P_{2,1}) \lor B_{1,1})$

3. Move \neg inwards using de Morgan's rules and double-negation: $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})$

4. Apply distributivity law (v over ^) and flatten:

 $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$

Resolution Example



• $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1}))$

reformulated as:

 $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$

- Observation: $\neg B_{1,1}$
- Goal: disprove: $\alpha = \neg P_{1,2}$
- Resolution

$$\frac{\neg P_{1,2} \lor B_{1,1} \quad \neg B_{1,1}}{\neg P_{1,2}}$$

• Resolution

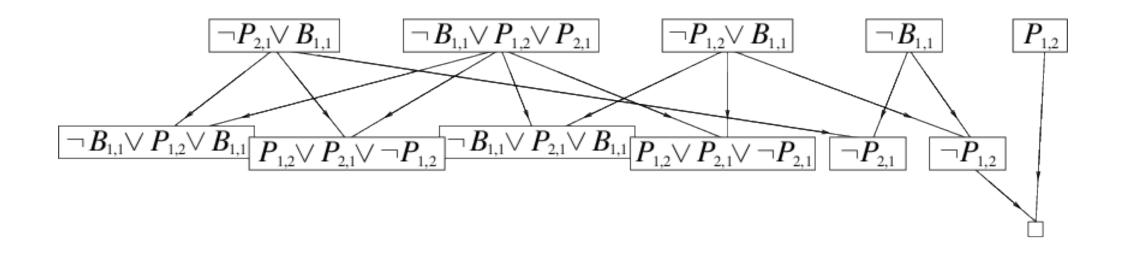
$$\frac{\neg P_{1,2} \quad P_{1,2}}{false}$$

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Resolution Example



• In practice: all resolvable pairs of clauses are combined



Resolution Algorithm



• Proof by contradiction, i.e., show $KB \land \neg \alpha$ unsatisfiable

```
function PL-RESOLUTION(KB, \alpha) returns true or false
  inputs: KB, the knowledge base, a sentence in propositional logic
            \alpha, the query, a sentence in propositional logic
  clauses \leftarrow the set of clauses in the CNF representation of KB \land \neg \alpha
  new \leftarrow \{\}
  loop do
      for each C_i, C_j in clauses do
          resolvents \leftarrow PL-RESOLVE(C_i, C_j)
          if resolvents contains the empty clause then return true
          new \leftarrow new \cup resolvents
      if new clauses then return false
      clauses \leftarrow clauses \cup new
```

Logical Agent



- Logical agent for Wumpus world explores actions
 - observe glitter \rightarrow done
 - unexplored safe spot \rightarrow plan route to it
 - if Wampus in possible spot \rightarrow shoot arrow
 - take a risk to go possibly risky spot
- Propositional logic to infer state of the world
- Heuristic search to decide which action to take

Summary



- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
 - syntax: formal structure of sentences
 - semantics: truth of sentences wrt models
 - entailment: necessary truth of one sentence given another
 - inference: deriving sentences from other sentences
 - soundess: derivations produce only entailed sentences
 - completeness: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, inference to determine state of the world, etc.
- Forward, backward chaining are linear-time, complete for Horn clauses
- Resolution is complete for propositional logic