

---

# Logical Agents

Philipp Koehn

6 October 2015



# Outline

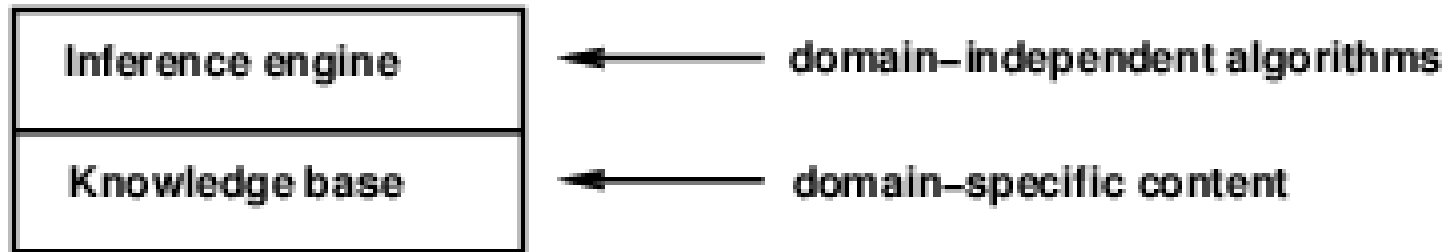


1

- Knowledge-based agents
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - forward chaining
  - backward chaining
  - resolution

# knowledge-based agents

# Knowledge-Based Agent



- Knowledge base = set of sentences in a **formal** language
- Declarative approach to building an agent (or other system):  
    **TELL** it what it needs to know
- Then it can **ASK** itself what to do—answers should follow from the KB
- Agents can be viewed at the knowledge level  
    i.e., **what they know**, regardless of how implemented
- Or at the implementation level  
    i.e., data structures in KB and algorithms that manipulate them

# A Simple Knowledge-Based Agent



```
function KB-AGENT(percept) returns an action  
static: KB, a knowledge base  
         t, a counter, initially 0, indicating time  
TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))  
action ← ASK(KB, MAKE-ACTION-QUERY(t))  
TELL(KB, MAKE-ACTION-SENTENCE(action, t))  
t ← t + 1  
return action
```

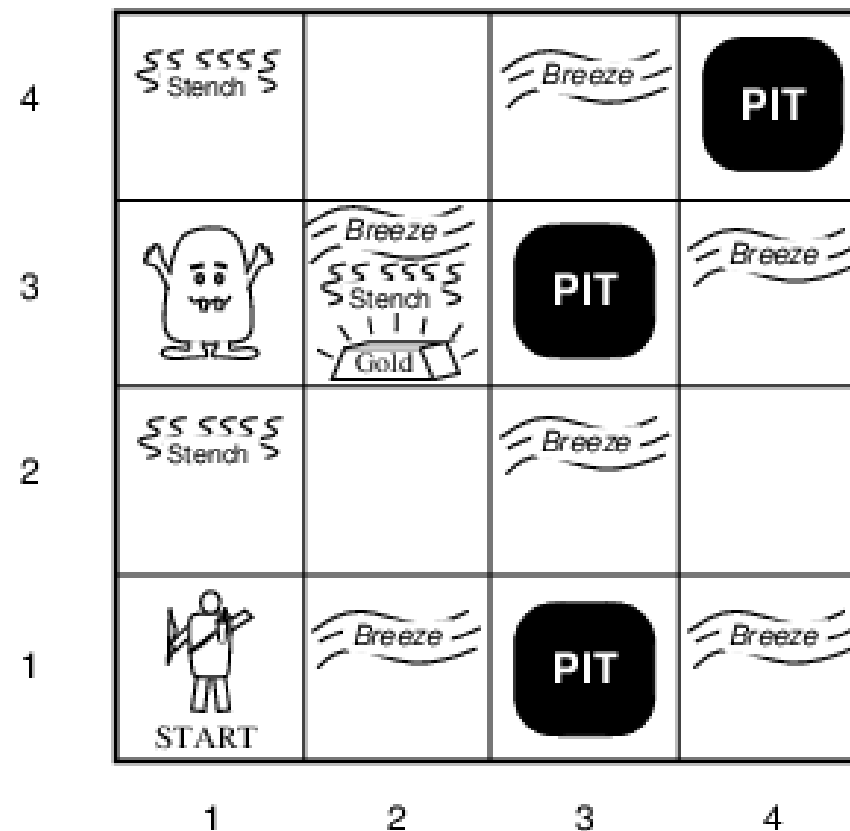
- The agent must be able to
  - represent states, actions, etc.
  - incorporate new percepts
  - update internal representations of the world
  - deduce hidden properties of the world
  - deduce appropriate actions

example

# Wumpus World PEAS Description



- **Performance measure**
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- **Environment**
  - squares adjacent to wumpus are smelly
  - squares adjacent to pit are breezy
  - glitter iff gold is in the same square
  - shooting kills wumpus if you are facing it
  - shooting uses up the only arrow
  - grabbing picks up gold if in same square
  - releasing drops the gold in same square
- **Actuators** Left turn, Right turn, Forward, Grab, Release, Shoot
- **Sensors** Breeze, Glitter, Smell



# Wumpus World Characterization



- **Observable?** ■ No—only local perception
- **Deterministic?** ■ Yes—outcomes exactly specified
- **Episodic?** ■ No—sequential at the level of actions
- **Static?** ■ Yes—Wumpus and Pits do not move
- **Discrete?** ■ Yes
- **Single-agent?** ■ Yes—Wumpus is essentially a natural feature

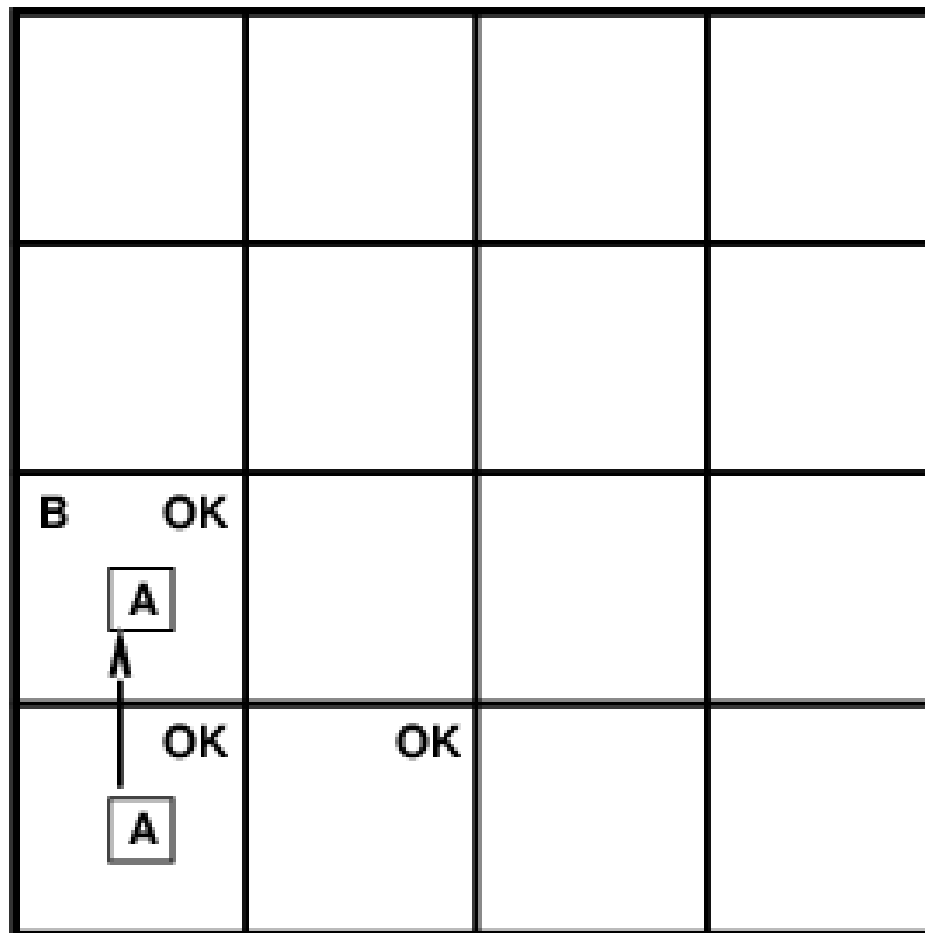


# Exploring a Wumpus World

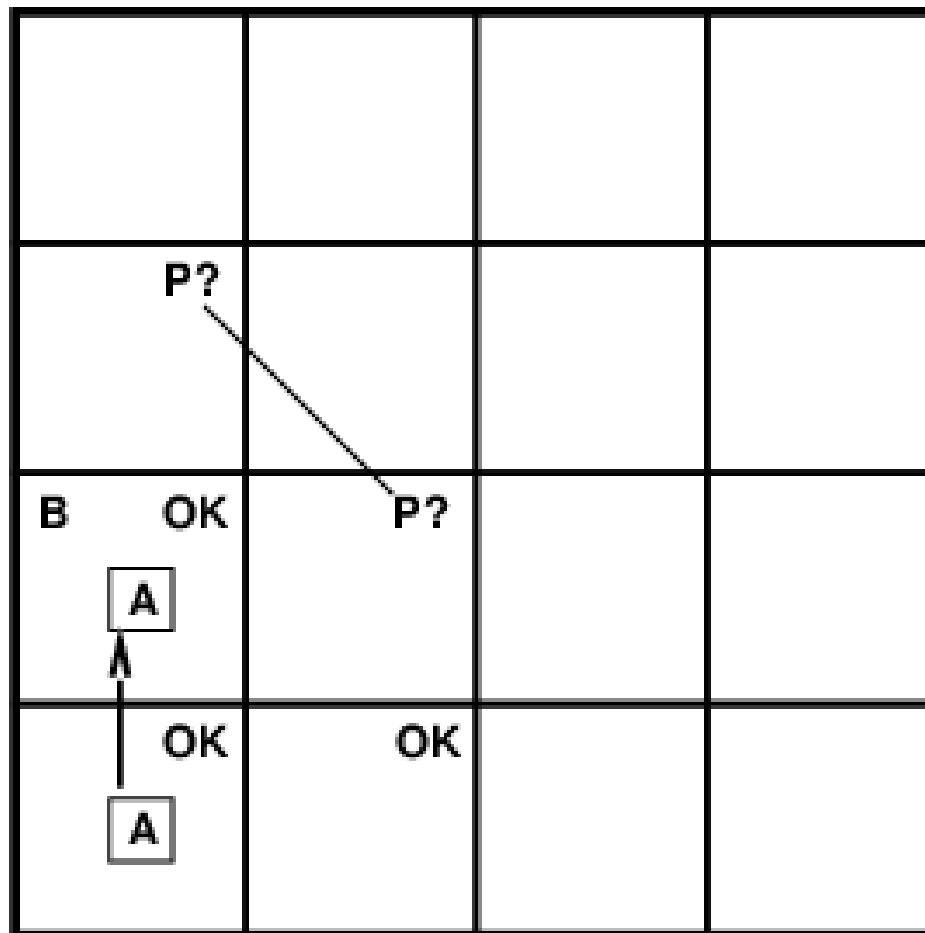


OK			
OK <span style="border: 1px solid black; padding: 2px;">A</span>	OK		

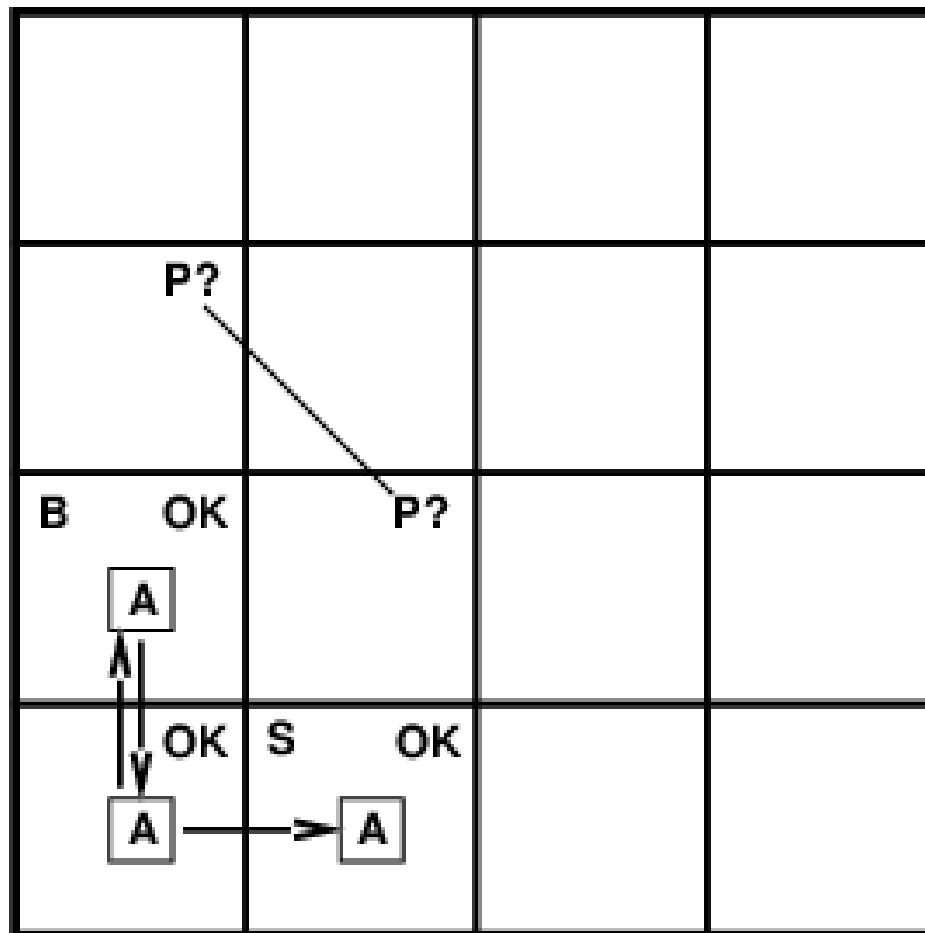
# Exploring a Wumpus World



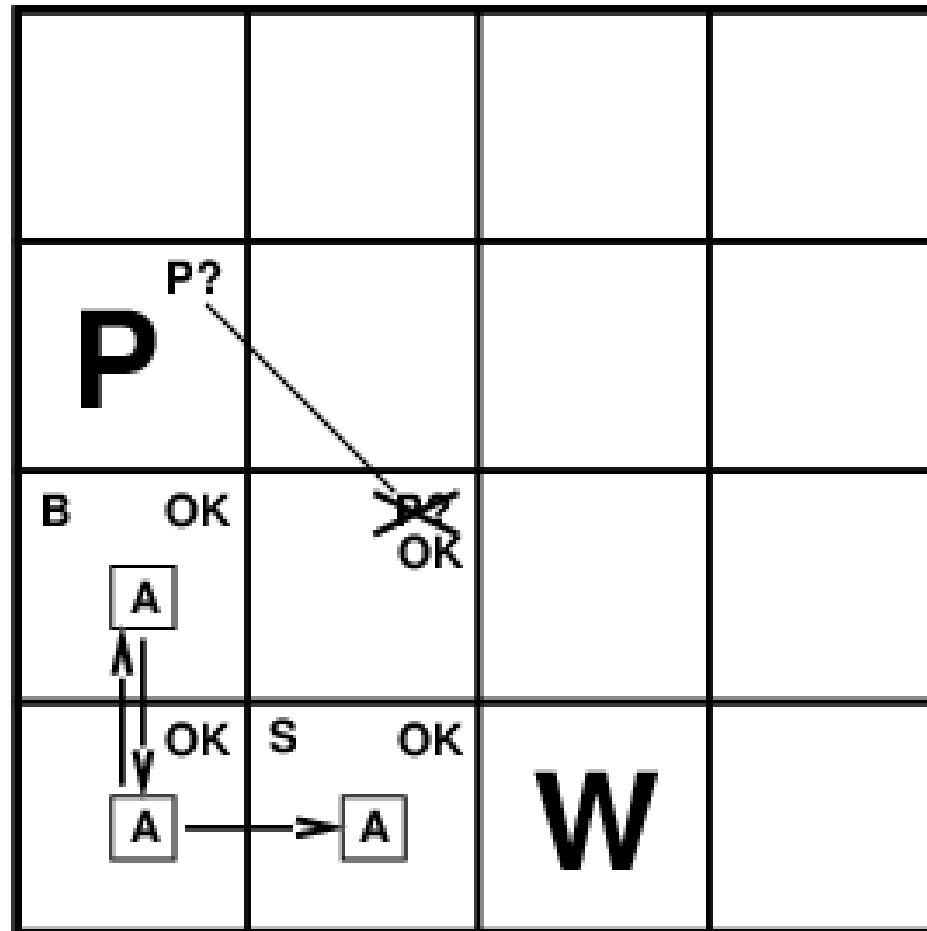
# Exploring a Wumpus World



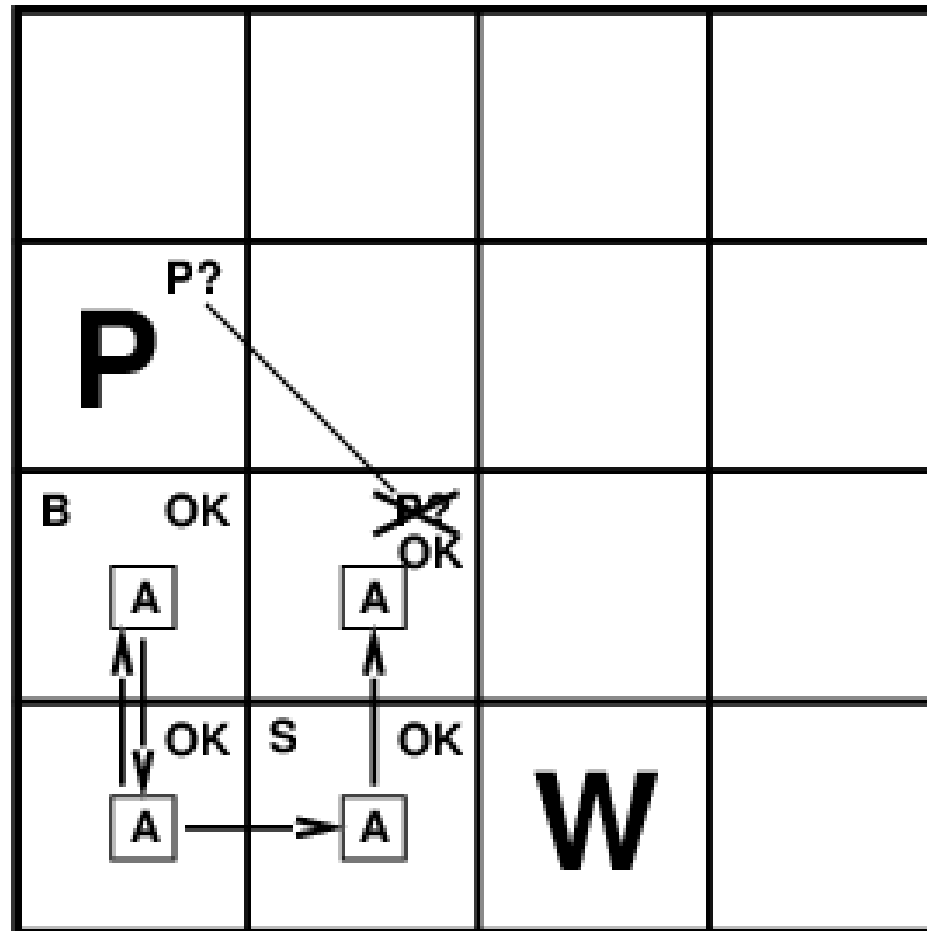
# Exploring a Wumpus World



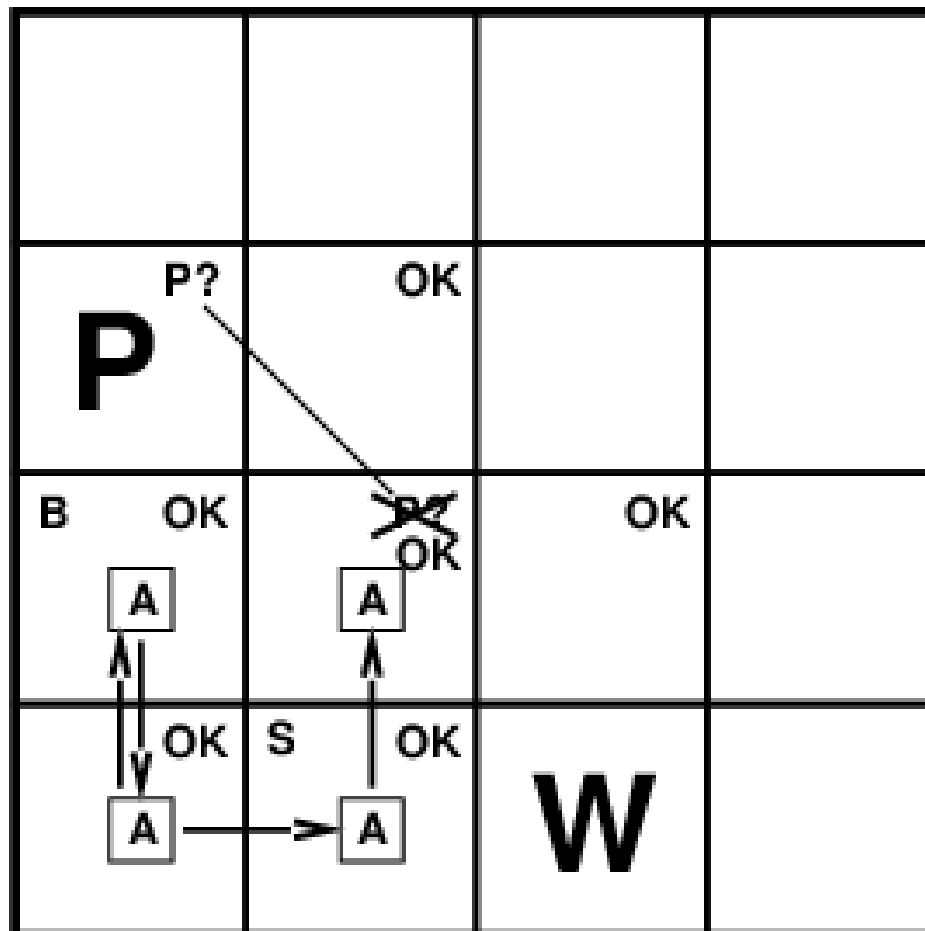
# Exploring a Wumpus World



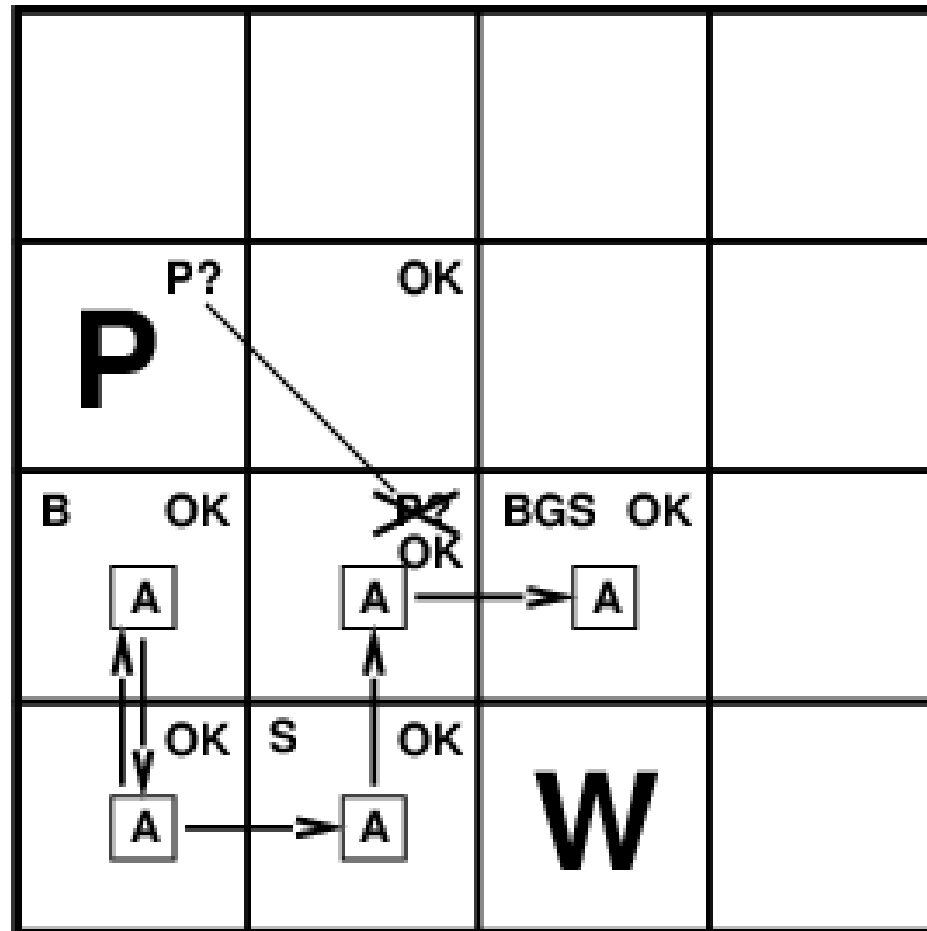
# Exploring a Wumpus World



# Exploring a Wumpus World

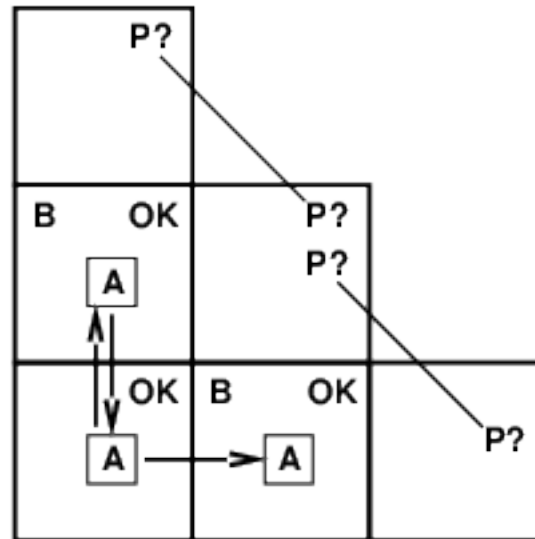


# Exploring a Wumpus World



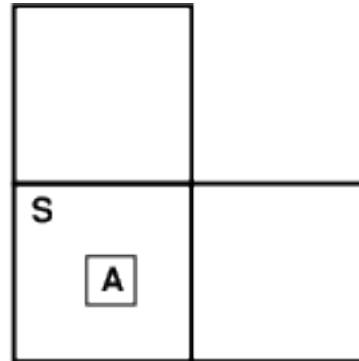


# Tight Spot



- Breeze in (1,2) and (2,1)  
     $\implies$  no safe actions
- Assuming pits uniformly distributed,  
    (2,2) has pit w/ prob 0.86, vs. 0.31

# Tight Spot



- Smell in (1,1)  
     $\implies$  cannot move
- Can use a strategy of **coercion**: shoot straight ahead
  - wumpus was there  $\implies$  dead  $\implies$  safe
  - wumpus wasn't there  $\implies$  safe

# logic in general

- **Logics** are formal languages for representing information such that conclusions can be drawn
- **Syntax** defines the sentences in the language
- **Semantics** define the “meaning” of sentences; i.e., define **truth** of a sentence in a world
- E.g., the language of arithmetic
  - $x + 2 \geq y$  is a sentence;  $x^2 + y >$  is not a sentence
  - $x + 2 \geq y$  is true iff the number  $x + 2$  is no less than the number  $y$
  - $x + 2 \geq y$  is true in a world where  $x = 7, y = 1$   
 $x + 2 \geq y$  is false in a world where  $x = 0, y = 6$

# Entailment

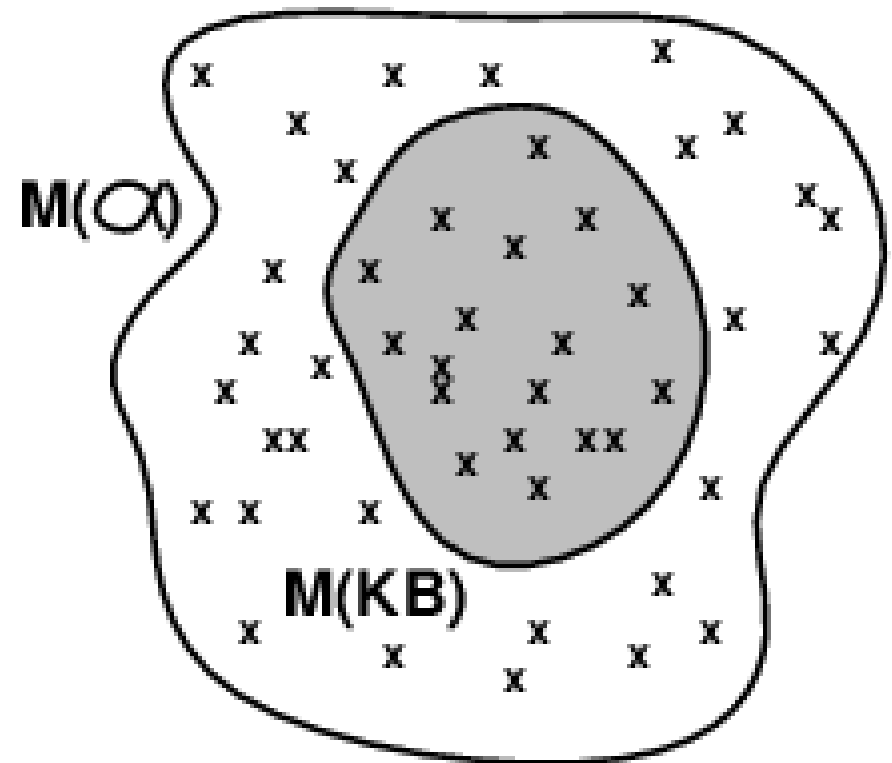


- **Entailment** means that one thing **follows from** another:

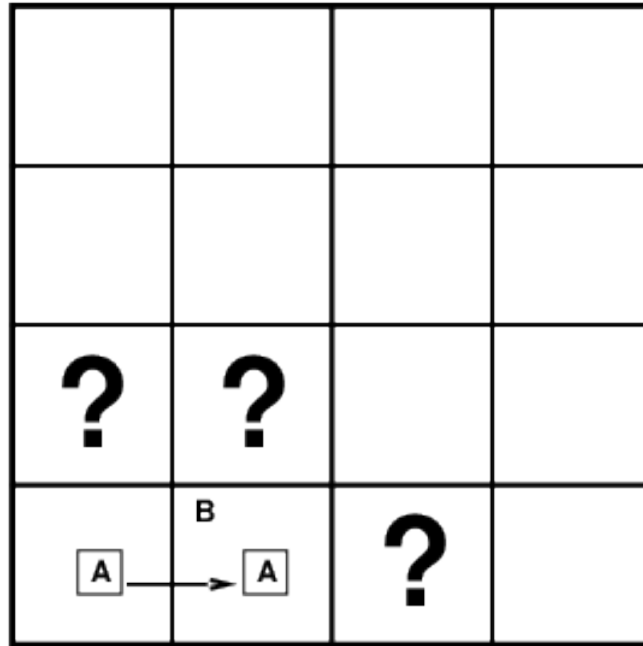
$$KB \models \alpha$$

- Knowledge base  $KB$  entails sentence  $\alpha$   
if and only if  
 $\alpha$  is true in all worlds where  $KB$  is true
- E.g., the KB containing “the Giants won” and “the Reds won”  
entails “Either the Giants won or the Reds won”
- E.g.,  $x + y = 4$  entails  $4 = x + y$
- Entailment is a relationship between sentences (i.e., **syntax**)  
that is based on **semantics**
- Note: brains process **syntax** (of some sort)

- Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated
  - We say  $m$  is a **model** of a sentence  $\alpha$  if  $\alpha$  is true in  $m$
  - $M(\alpha)$  is the set of all models of  $\alpha$
- $\Rightarrow KB \models \alpha$  if and only if  $M(KB) \subseteq M(\alpha)$
- E.g.  $KB =$  Giants won and Reds won  
 $\alpha =$  Giants won

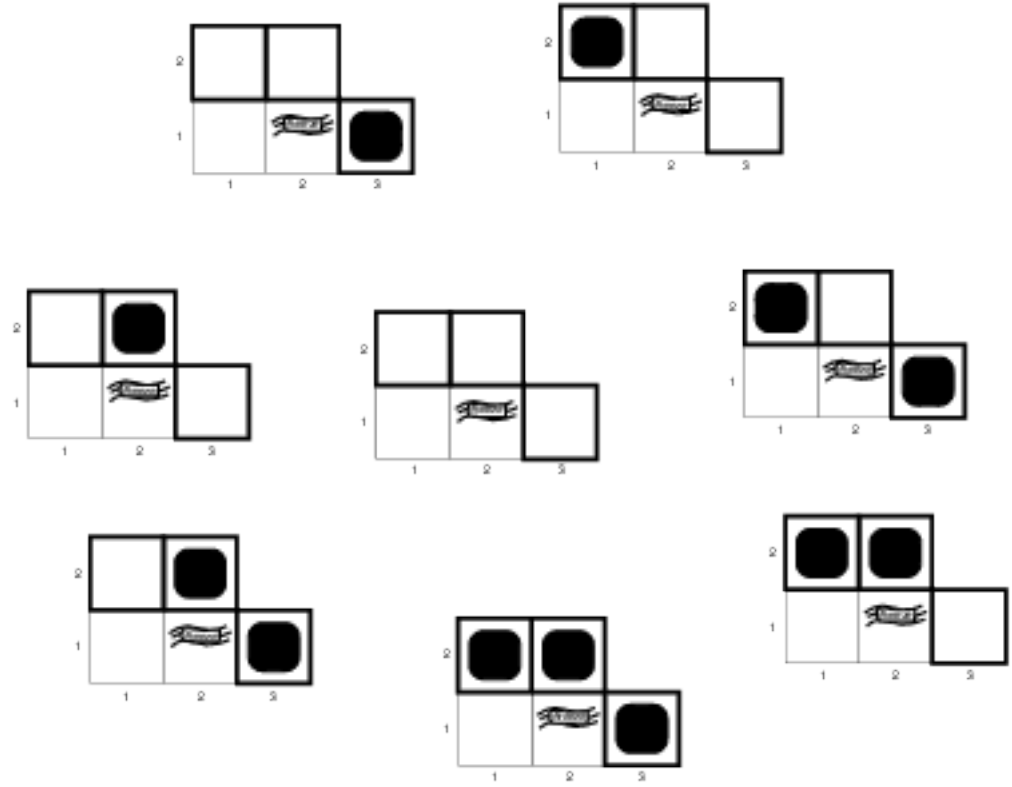


# Entailment in the Wumpus World



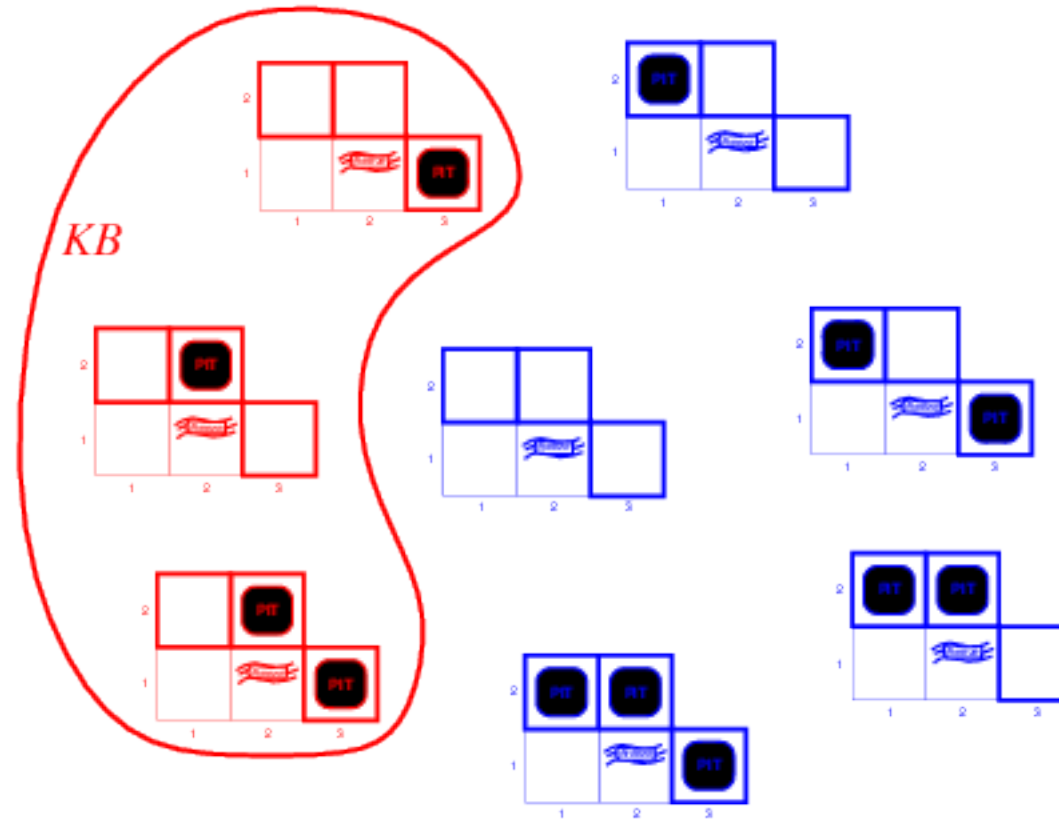
- Situation after detecting nothing in [1,1], moving right, breeze in [2,1]
- Consider possible models for ?s, assuming only pits
- 3 Boolean choices  $\implies$  8 possible models

# Wumpus Models



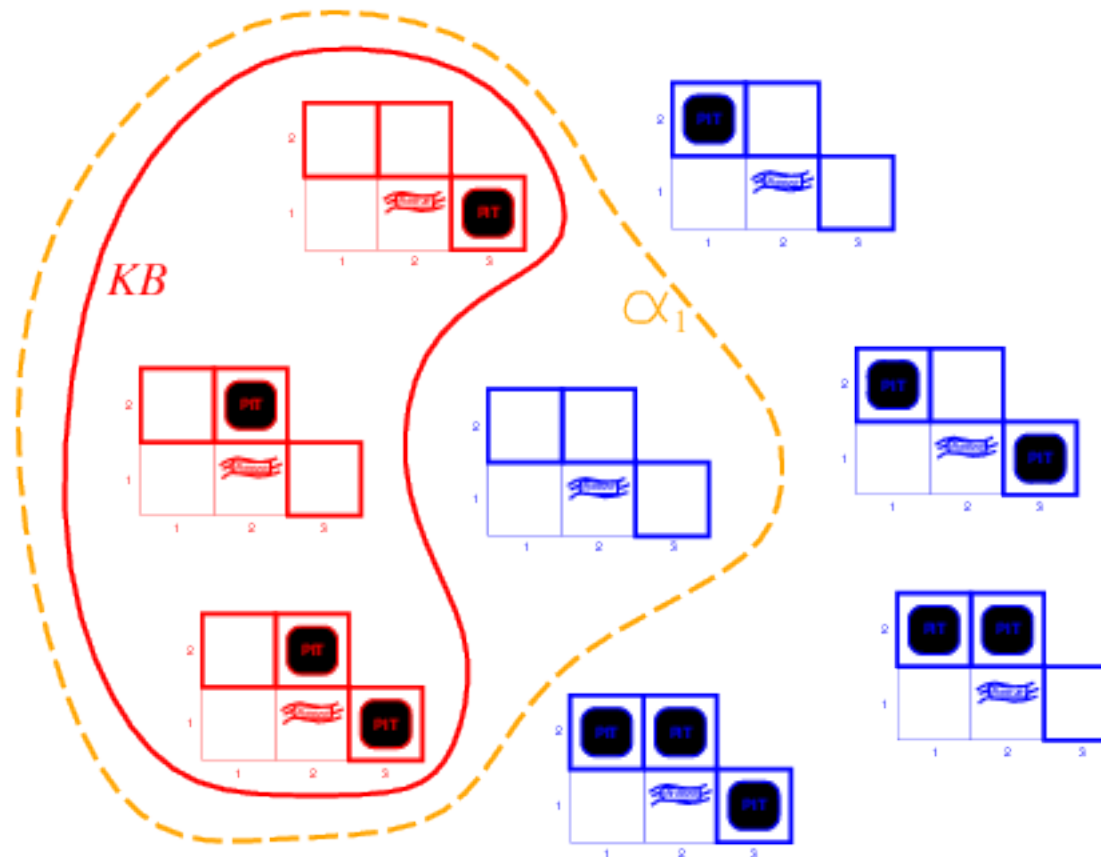


# Wumpus Models



$KB$  = wumpus-world rules + observations

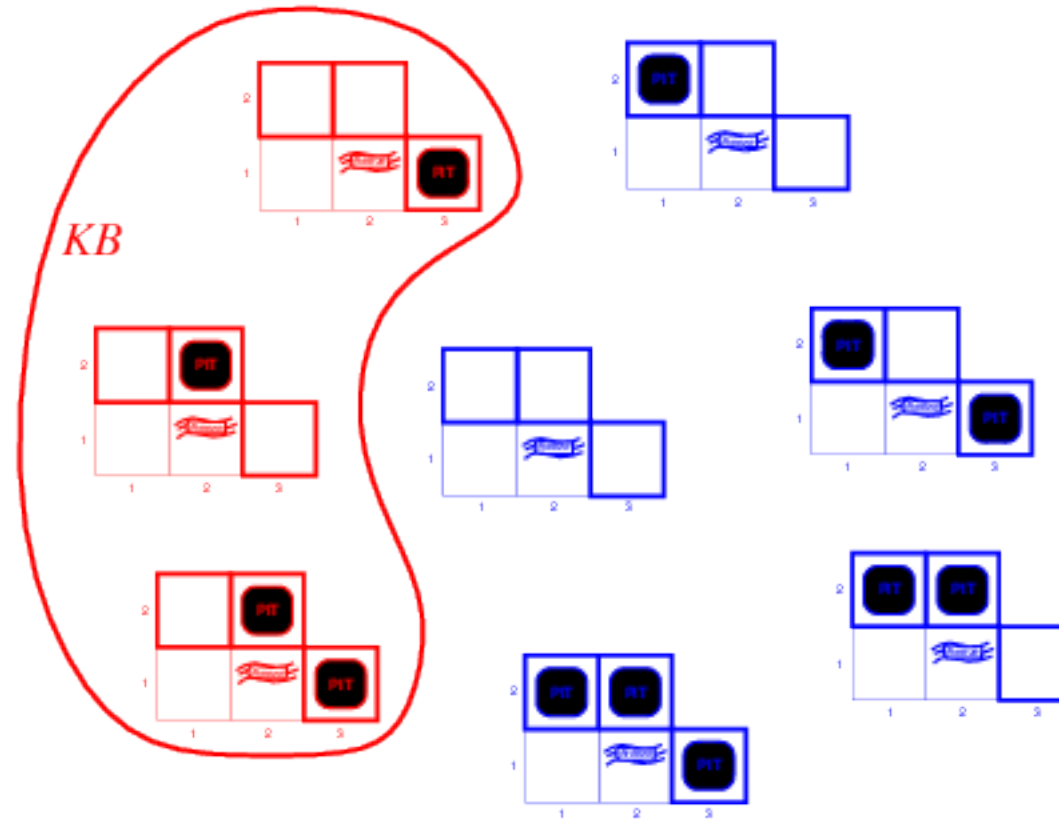
# Wumpus Models



$KB$  = wumpus-world rules + observations

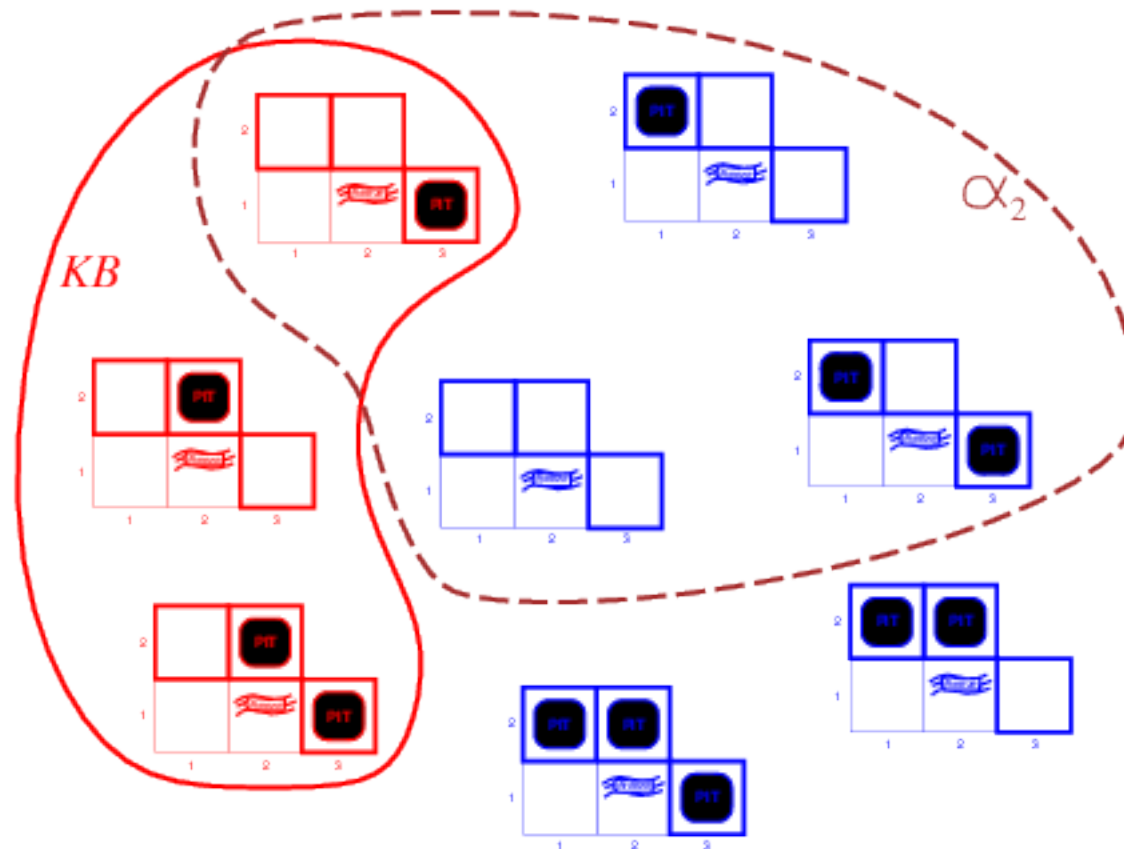
$\alpha_1$  = “[1,2] is safe”,  $KB \models \alpha_1$ , proved by model checking

# Wumpus Models



$KB$  = wumpus-world rules + observations

# Wumpus Models



$KB$  = wumpus-world rules + observations

$\alpha_2$  = "[2,2] is safe",  $KB \not\models \alpha_2$

- $KB \vdash_i \alpha$  = sentence  $\alpha$  can be derived from  $KB$  by procedure  $i$
- Consequences of  $KB$  are a haystack;  $\alpha$  is a needle.  
Entailment = needle in haystack; inference = finding it
- **Soundness:**  $i$  is sound if  
whenever  $KB \vdash_i \alpha$ , it is also true that  $KB \models \alpha$
- **Completeness:**  $i$  is complete if  
whenever  $KB \models \alpha$ , it is also true that  $KB \vdash_i \alpha$
- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
- That is, the procedure will answer any question whose answer follows from what is known by the  $KB$ .

# propositional logic

# Propositional Logic: Syntax



- Propositional logic is the simplest logic—illustrates basic ideas
- The proposition symbols  $P_1, P_2$  etc are sentences
- If  $S$  is a sentence,  $\neg S$  is a sentence (negation)
- If  $S_1$  and  $S_2$  are sentences,  $S_1 \wedge S_2$  is a sentence (conjunction)
- If  $S_1$  and  $S_2$  are sentences,  $S_1 \vee S_2$  is a sentence (disjunction)
- If  $S_1$  and  $S_2$  are sentences,  $S_1 \implies S_2$  is a sentence (implication)
- If  $S_1$  and  $S_2$  are sentences,  $S_1 \iff S_2$  is a sentence (biconditional)

# Propositional Logic: Semantics

- Each model specifies true/false for each proposition symbol

E.g.  $P_{1,2}$   $P_{2,2}$   $P_{3,1}$   
*true true false*

(with these symbols, 8 possible models, can be enumerated automatically)

- Rules for evaluating truth with respect to a model  $m$ :

$\neg S$	is true iff	$S$	is false		
$S_1 \wedge S_2$	is true iff	$S_1$	is true <b>and</b>	$S_2$	is true
$S_1 \vee S_2$	is true iff	$S_1$	is true <b>or</b>	$S_2$	is true
$S_1 \implies S_2$	is true iff	$S_1$	is false <b>or</b>	$S_2$	is true
i.e.,	is false iff	$S_1$	is true <b>and</b>	$S_2$	is false
$S_1 \iff S_2$	is true iff	$S_1 \implies S_2$	is true <b>and</b>	$S_2 \implies S_1$	is true

- Simple recursive process evaluates an arbitrary sentence, e.g.,  
 $\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \textit{true} \wedge (\textit{false} \vee \textit{true}) = \textit{true} \wedge \textit{true} = \textit{true}$



# Truth Tables for Connectives

$P$	$Q$	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>

# Wumpus World Sentences

- Let  $P_{i,j}$  be true if there is a pit in  $[i, j]$ 
  - observation  $R_1 : \neg P_{1,1}$  ■
- Let  $B_{i,j}$  be true if there is a breeze in  $[i, j]$ .
- “Pits cause breezes in adjacent squares” ■
  - rule  $R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
  - rule  $R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$  ■
  - observation  $R_4 : \neg B_{1,1}$
  - observation  $R_5 : B_{2,1}$  ■
- What can we infer about  $P_{1,2}, P_{2,1}, P_{2,2}$ , etc.?

# Truth Tables for Inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$KB$
<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>false</i>

- Enumerate rows (different assignments to symbols),  
if **KB** is true in row, check that  $\alpha$  is too

# Inference by Enumeration

- Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB,  $\alpha$ ) returns true or false  
inputs: KB, the knowledge base, a sentence in propositional logic  
          $\alpha$ , the query, a sentence in propositional logic  
symbols  $\leftarrow$  a list of the proposition symbols in KB and  $\alpha$   
return TT-CHECK-ALL(KB,  $\alpha$ , symbols, [ ])
```

---

```
function TT-CHECK-ALL(KB,  $\alpha$ , symbols, model) returns true or false  
if EMPTY?(symbols) then  
    if PL-TRUE?(KB, model) then return PL-TRUE?( $\alpha$ , model)  
    else return true  
else do  
    P  $\leftarrow$  FIRST(symbols); rest  $\leftarrow$  REST(symbols)  
    return TT-CHECK-ALL(KB,  $\alpha$ , rest, EXTEND(P, true, model)) and  
          TT-CHECK-ALL(KB,  $\alpha$ , rest, EXTEND(P, false, model))
```

- $O(2^n)$  for  $n$  symbols; problem is **co-NP-complete**

equivalence, validity, satisfiability

# Logical Equivalence

- Two sentences are **logically equivalent** iff true in same models:  
 $\alpha \equiv \beta$  if and only if  $\alpha \models \beta$  and  $\beta \models \alpha$

$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$	$(\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$	$(\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha)$	$\equiv$	$\alpha$	double-negation elimination
$(\alpha \implies \beta)$	$\equiv$	$(\neg\beta \implies \neg\alpha)$	contraposition
$(\alpha \implies \beta)$	$\equiv$	$(\neg\alpha \vee \beta)$	implication elimination
$(\alpha \iff \beta)$	$\equiv$	$((\alpha \implies \beta) \wedge (\beta \implies \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv$	$(\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta)$	$\equiv$	$(\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

# Validity and Satisfiability

- A sentence is **valid** if it is true in **all** models,  
e.g., *True*,  $A \vee \neg A$ ,  $A \implies A$ ,  $(A \wedge (A \implies B)) \implies B$
- Validity is connected to inference via the **Deduction Theorem**:  
 $KB \models \alpha$  if and only if  $(KB \implies \alpha)$  is valid
- A sentence is **satisfiable** if it is true in **some** model  
e.g.,  $A \vee B$ ,  $C$
- A sentence is **unsatisfiable** if it is true in **no** models  
e.g.,  $A \wedge \neg A$
- Satisfiability is connected to inference via the following:  
 $KB \models \alpha$  if and only if  $(KB \wedge \neg \alpha)$  is unsatisfiable  
i.e., prove  $\alpha$  by *reductio ad absurdum*

# inference



- Proof methods divide into (roughly) two kinds
- Application of inference rules
  - Legitimate (sound) generation of new sentences from old
  - **Proof** = a sequence of inference rule applications
    - Can use inference rules as operators in a standard search alg.
  - Typically require translation of sentences into a **normal form**
- Model checking
  - truth table enumeration (always exponential in  $n$ )
  - improved backtracking, e.g., Davis–Putnam–Logemann–Loveland
  - heuristic search in model space (sound but incomplete)
    - e.g., min-conflicts-like hill-climbing algorithms

# Forward and Backward Chaining



- Horn Form (restricted)  
KB = **conjunction** of **Horn clauses**

- Horn clause =
  - proposition symbol; or
  - (conjunction of symbols)  $\implies$  symbol

e.g.,  $C \wedge (B \implies A) \wedge (C \wedge D \implies B)$

- **Modus Ponens** (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \dots, \alpha_n, \quad \alpha_1 \wedge \dots \wedge \alpha_n \implies \beta}{\beta}$$

- Can be used with **forward chaining** or **backward chaining**
- These algorithms are very natural and run in **linear** time

# Example

- Idea: fire any rule whose premises are satisfied in the *KB*, add its conclusion to the *KB*, until query is found

$$P \implies Q$$

$$L \wedge M \implies P$$

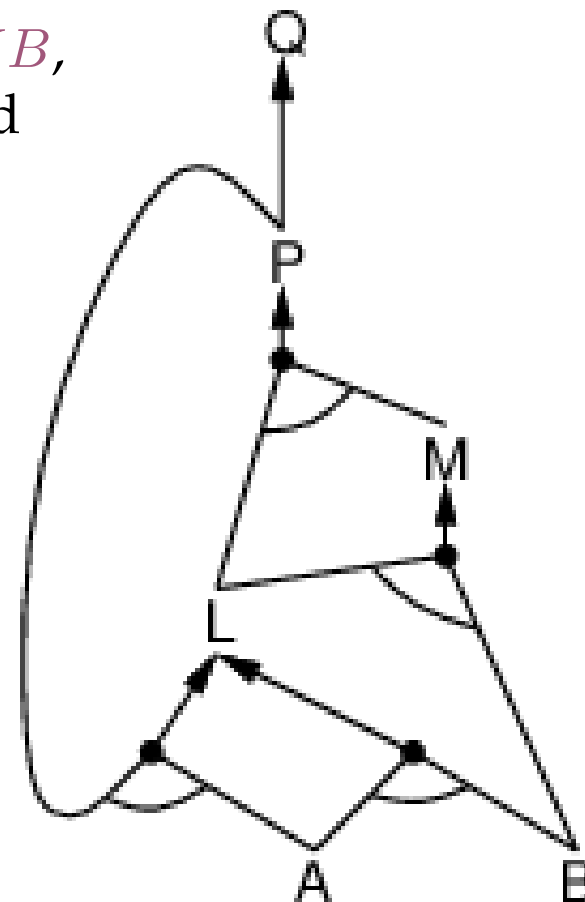
$$B \wedge L \implies M$$

$$A \wedge P \implies L$$

$$A \wedge B \implies L$$

*A*

*B* ■



# forward chaining

# Forward Chaining Algorithm

**function** PL-FC-ENTAILS?(*KB*, *q*) **returns** *true* or *false*

**inputs:** *KB*, the knowledge base, a set of propositional Horn clauses  
*q*, the query, a proposition symbol

**local variables:** *count*, a table, indexed by clause, init. number of premises  
*inferred*, a table, indexed by symbol, each entry initially *false*  
*agenda*, a list of symbols, initially the symbols known in *KB*

**while** *agenda* is not empty **do**

*p* ← POP(*agenda*)

**unless** *inferred*[*p*] **do**

*inferred*[*p*] ← *true*

**for each** Horn clause *c* in whose premise *p* appears **do**

        decrement *count*[*c*]

**if** *count*[*c*] = 0 **then do**

**if** HEAD[*c*] = *q* **then return** *true*

        PUSH(HEAD[*c*], *agenda*)

**return** *false*

# Forward Chaining Example

- Given

$$P \implies Q$$

$$L \wedge M \implies P$$

$$B \wedge L \implies M$$

$$A \wedge P \implies L$$

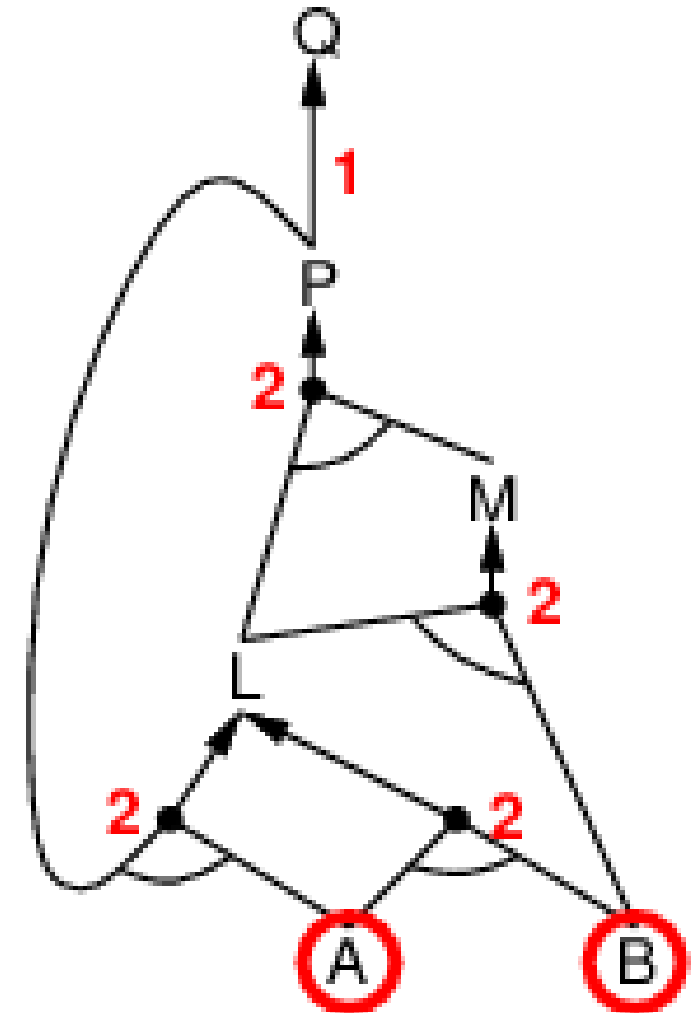
$$A \wedge B \implies L$$

$A$

$B$

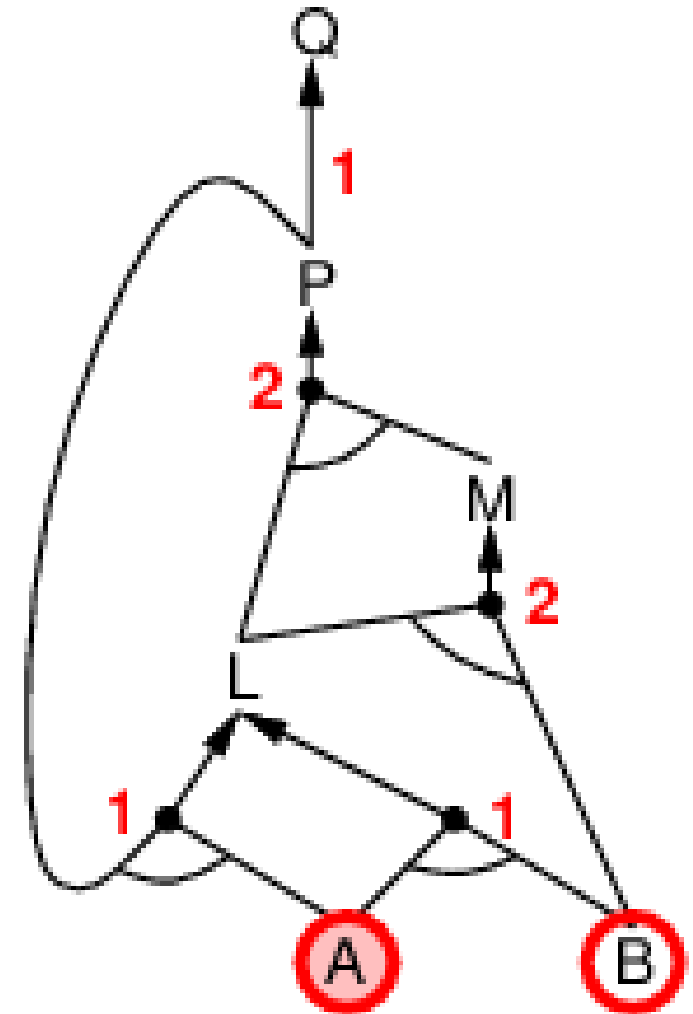
- Agenda:  $A, B$

- Annotate horn clauses with number of premises



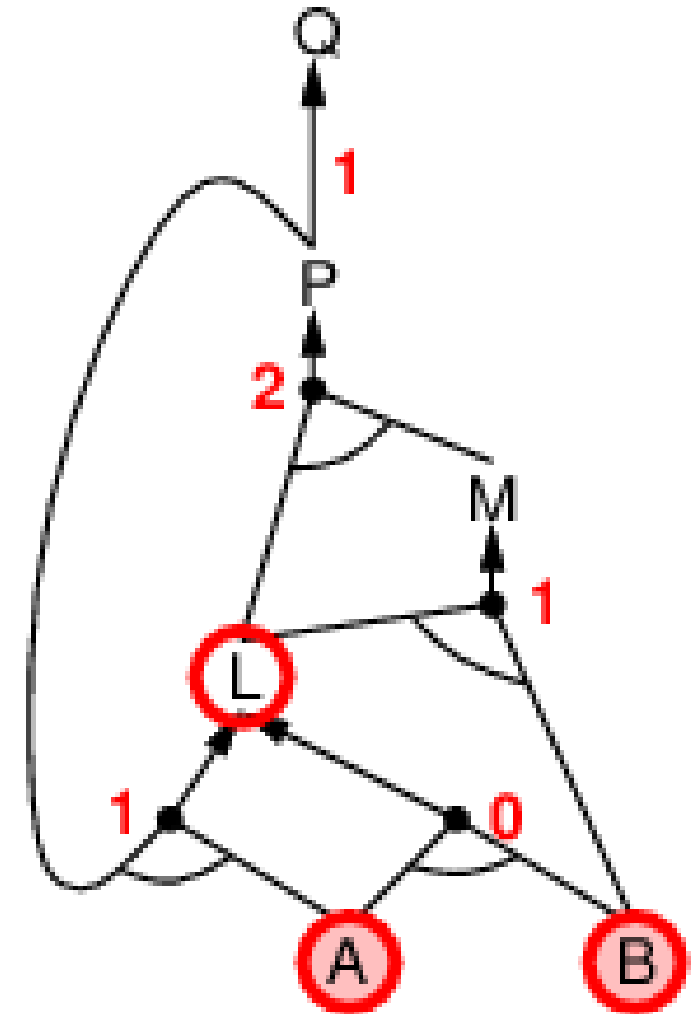
# Forward Chaining Example

- Process agenda item  $A$
- Decrease count for horn clauses in which  $A$  is premise



# Forward Chaining Example

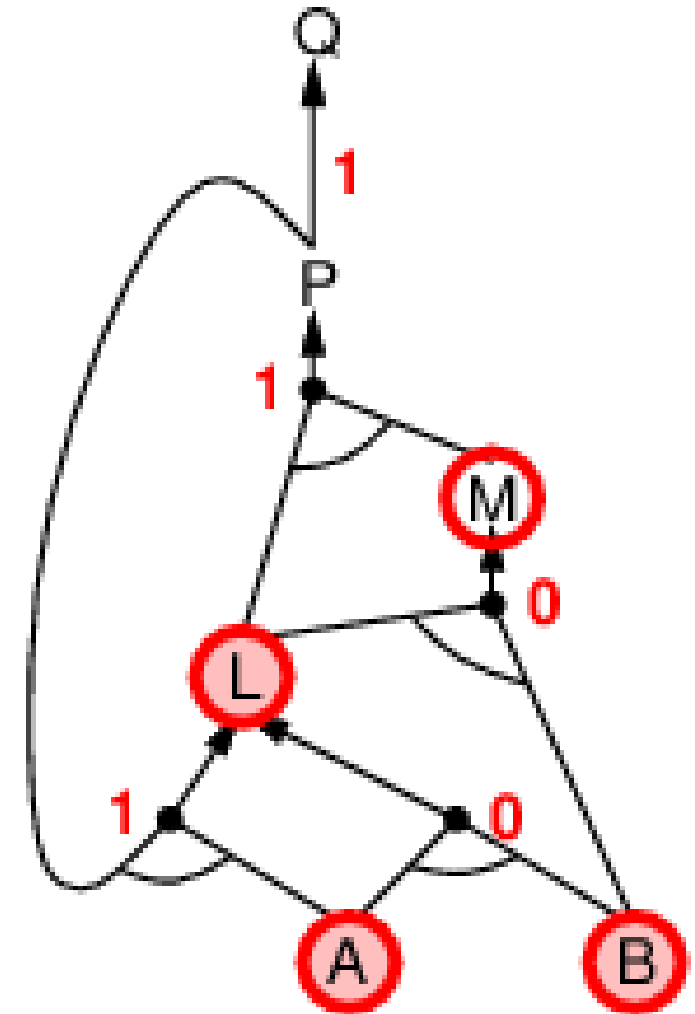
- Process agenda item  $B$
- Decrease count for horn clauses in which  $B$  is premise
- $A \wedge B \implies L$  has now fulfilled premise
- Add  $L$  to agenda





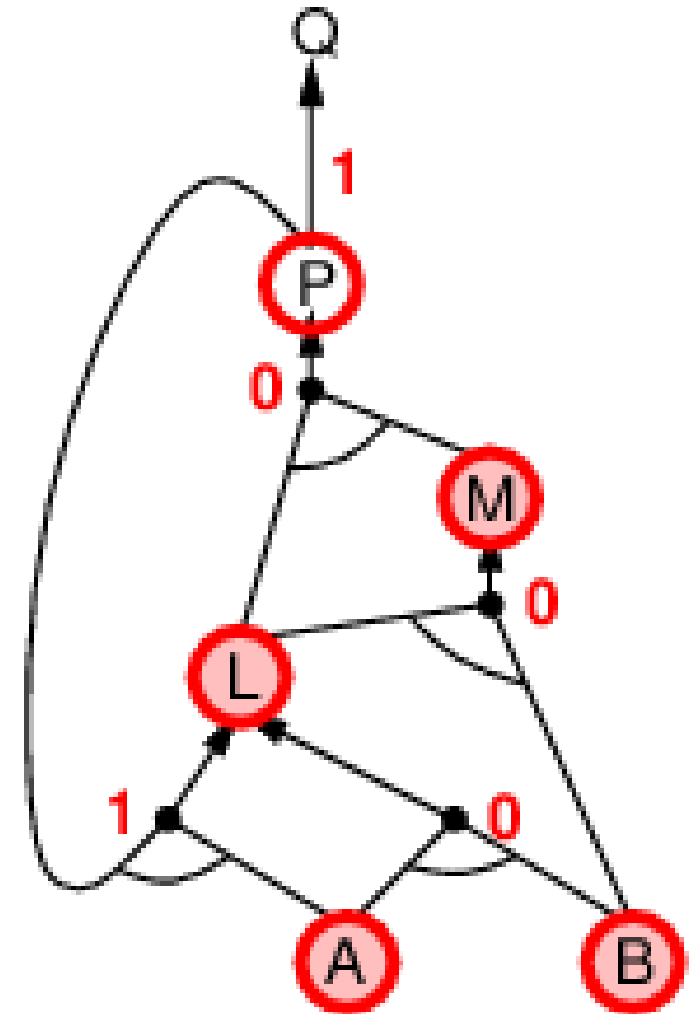
# Forward Chaining Example

- Process agenda item  $L$
- Decrease count for horn clauses in which  $L$  is premise
- $B \wedge L \implies M$  has now fulfilled premise
- Add  $M$  to agenda



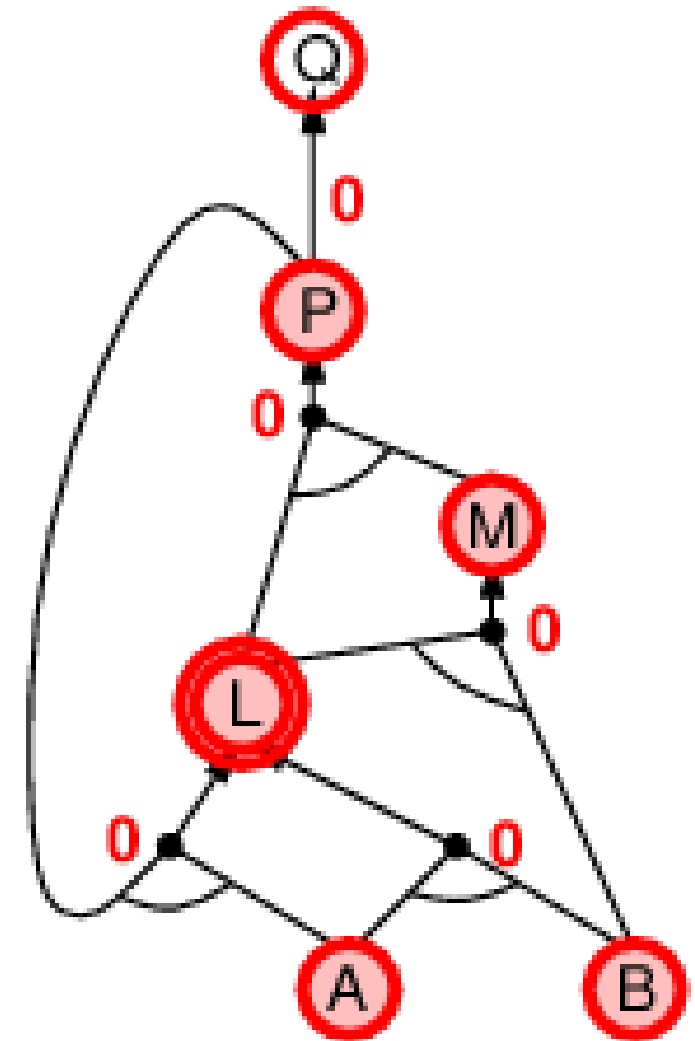
# Forward Chaining Example

- Process agenda item  $M$
- Decrease count for horn clauses in which  $M$  is premise
- $L \wedge M \implies P$  has now fulfilled premise
- Add  $P$  to agenda



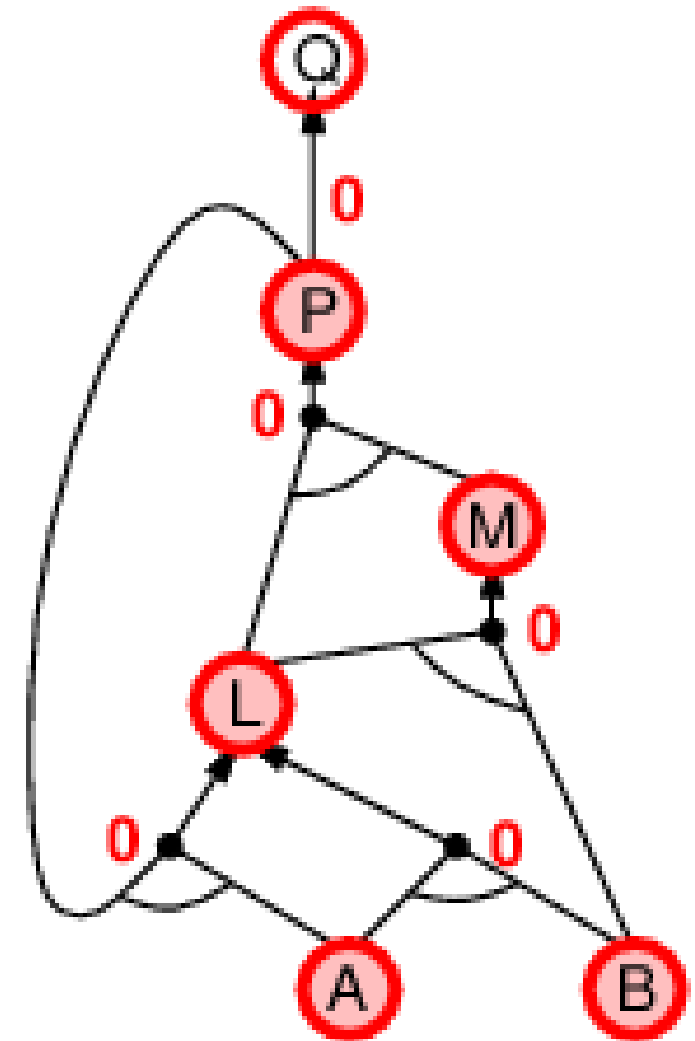
# Forward Chaining Example

- Process agenda item  $P$
- Decrease count for horn clauses in which  $P$  is premise
- $P \implies Q$  has now fulfilled premise
- Add  $Q$  to agenda
- $A \wedge P \implies L$  has now fulfilled premise



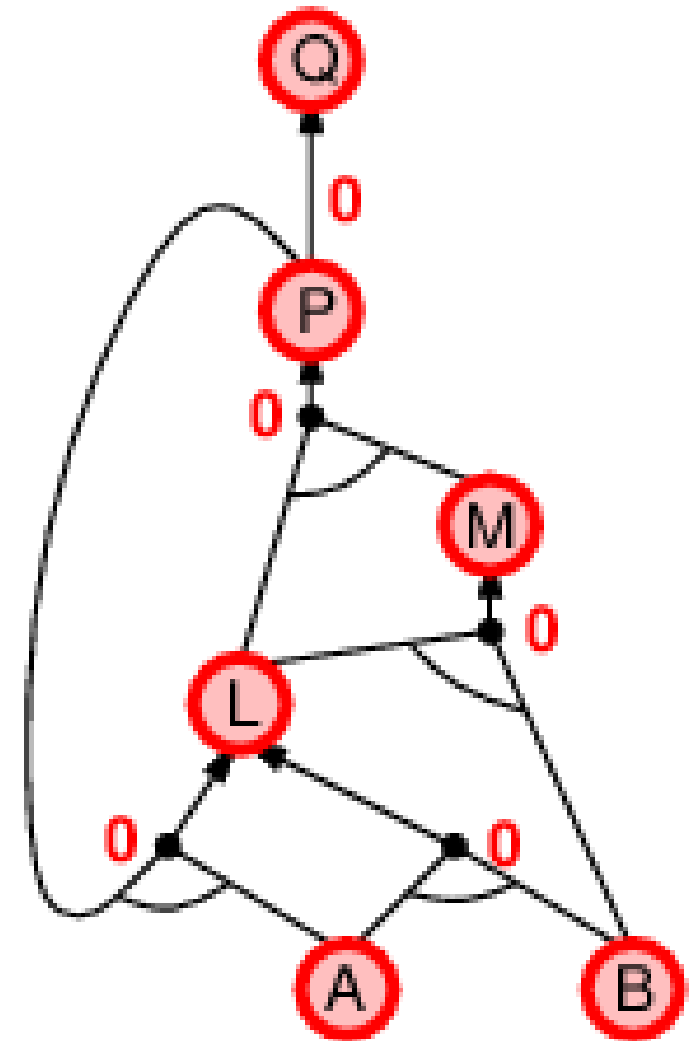
# Forward Chaining Example

- Process agenda item  $P$
- Decrease count for horn clauses in which  $P$  is premise
- $P \implies Q$  has now fulfilled premise
- Add  $Q$  to agenda
- $A \wedge P \implies L$  has now fulfilled premise
- But  $L$  is already inferred



# Forward Chaining Example

- Process agenda item  $Q$
- $Q$  is inferred
- Done



# Proof of Completeness

- FC derives every atomic sentence that is entailed by  $KB$ 
  1. FC reaches a **fixed point** where no new atomic sentences are derived
  2. consider the final state as a model  $m$ , assigning true/false to symbols
  3. every clause in the original  $KB$  is true in  $m$ 

**Proof:** Suppose a clause  $a_1 \wedge \dots \wedge a_k \Rightarrow b$  is false in  $m$   
Then  $a_1 \wedge \dots \wedge a_k$  is true in  $m$  and  $b$  is false in  $m$   
Therefore the algorithm has not reached a fixed point!
  4. hence  $m$  is a model of  $KB$
  5. if  $KB \models q$ ,  $q$  is true in **every** model of  $KB$ , including  $m$
- **General idea:** construct any model of  $KB$  by sound inference, check  $\alpha$

# backward chaining

# Backward Chaining

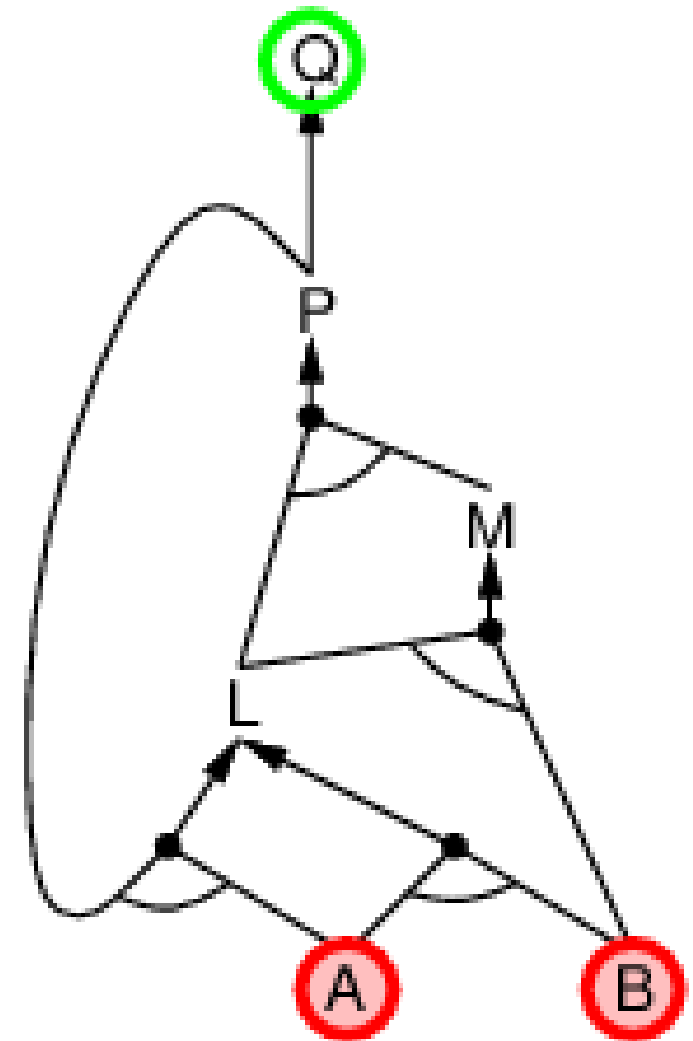


- Idea: work backwards from the query  $q$ :
  - to prove  $q$  by BC,
  - check if  $q$  is known already, or
  - prove by BC all premises of some rule concluding  $q$
- Avoid loops: check if new subgoal is already on the goal stack
- Avoid repeated work: check if new subgoal
  1. has already been proved true, or
  2. has already failed



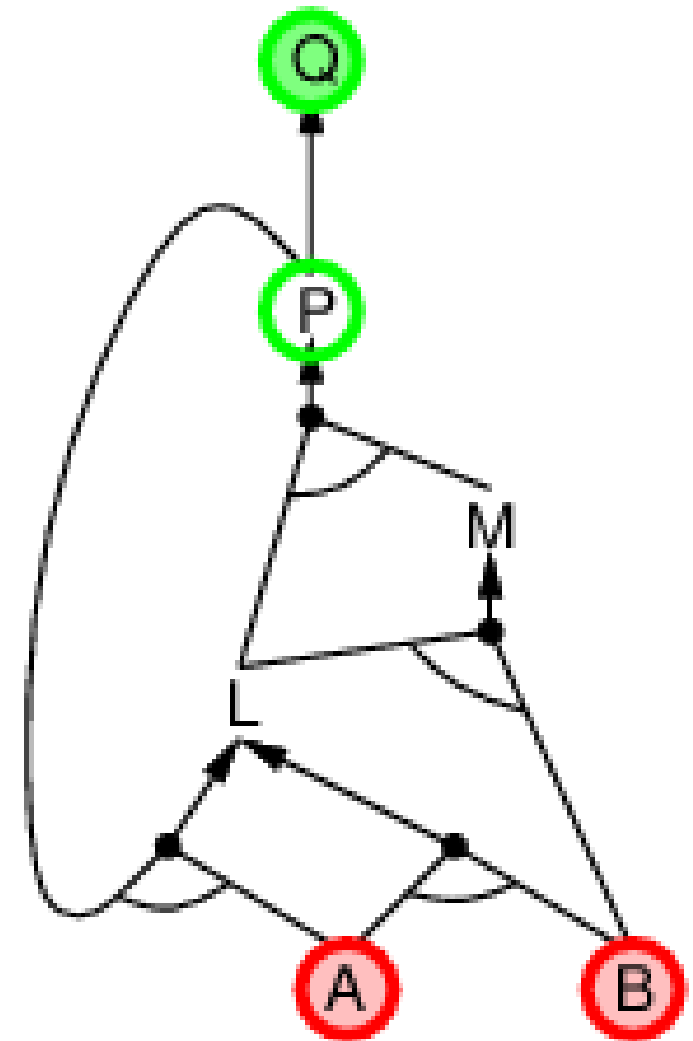
# Backward Chaining Example

- $A$  and  $B$  are known to be true
- $Q$  needs to be proven



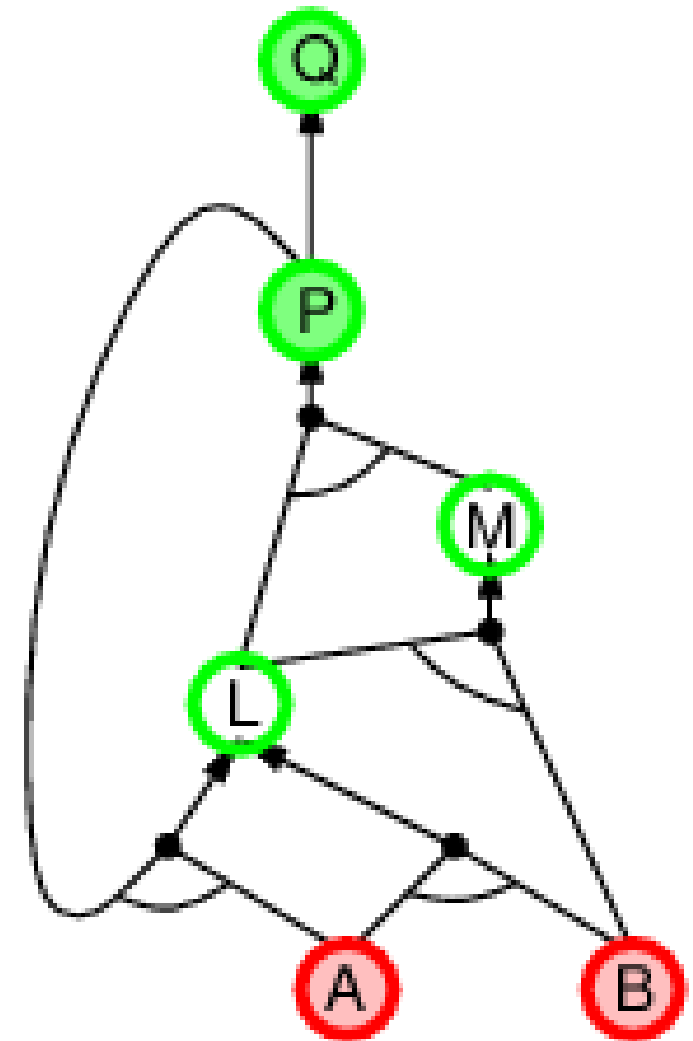
# Backward Chaining Example

- Current goal:  $Q$
- $Q$  can be inferred by  $P \implies Q$
- $P$  needs to be proven



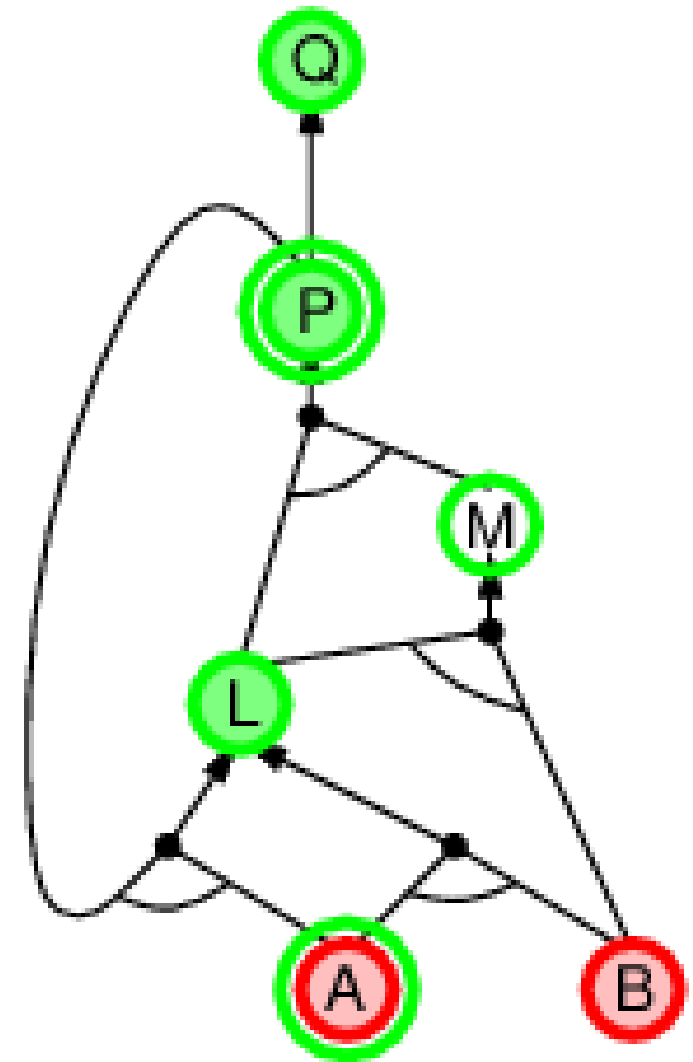
# Backward Chaining Example

- Current goal:  $P$
- $P$  can be inferred by  $L \wedge M \implies P$
- $L$  and  $M$  need to be proven



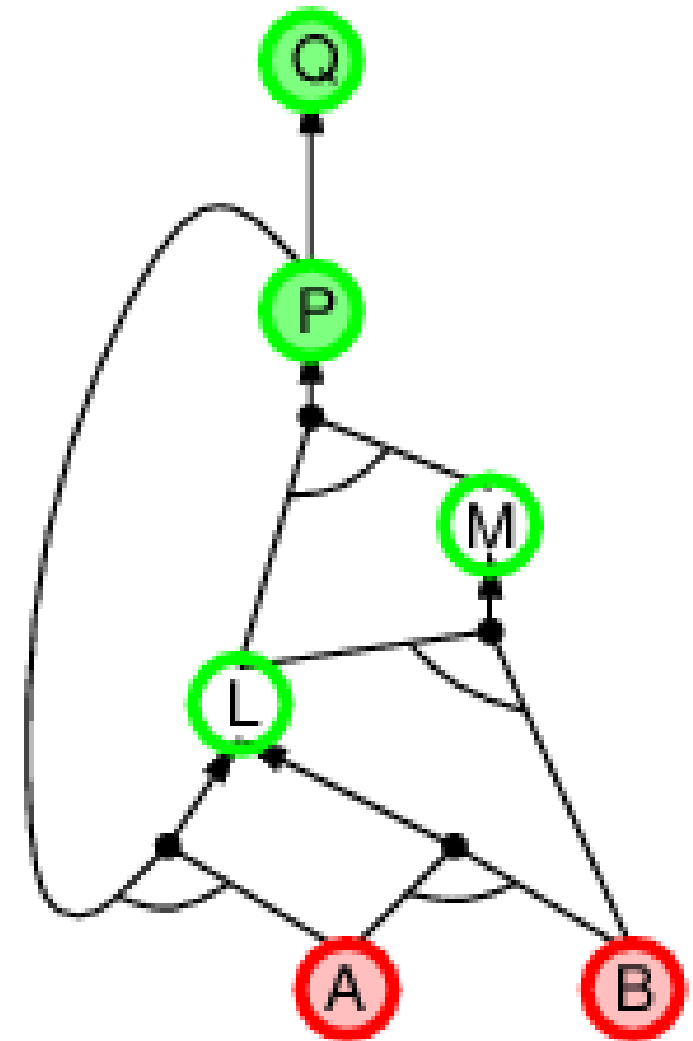
# Backward Chaining Example

- Current goal:  $L$
- $L$  can be inferred by  $A \wedge P \implies L$
- $P$  is already a goal
- $A$  is already true



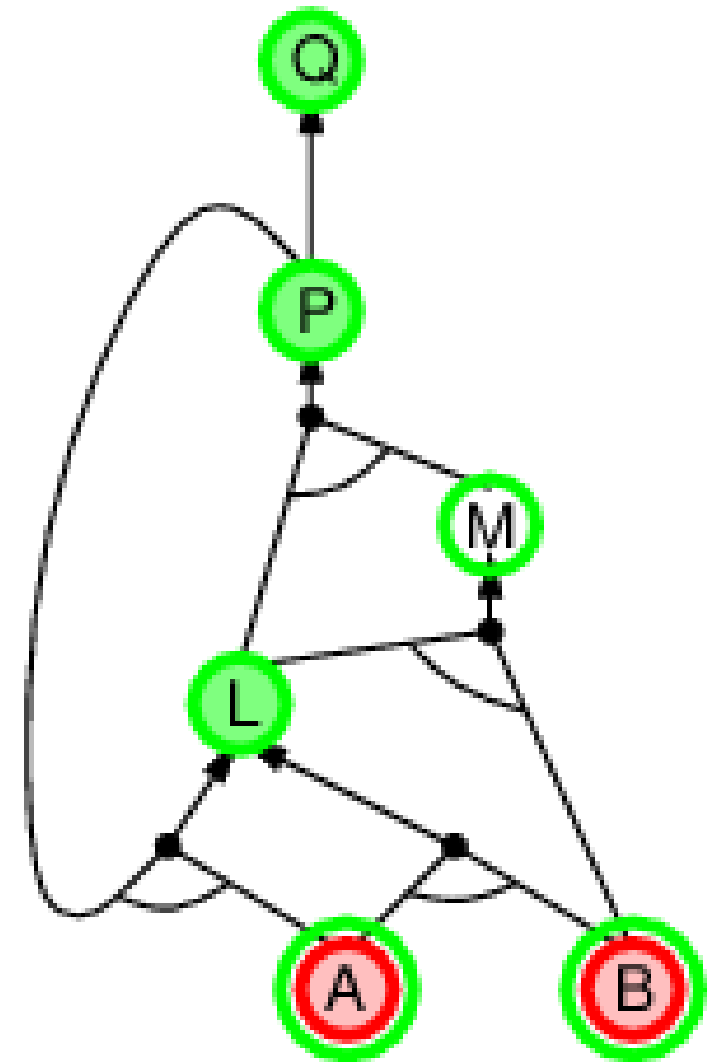
# Backward Chaining Example

- Current goal:  $L$



# Backward Chaining Example

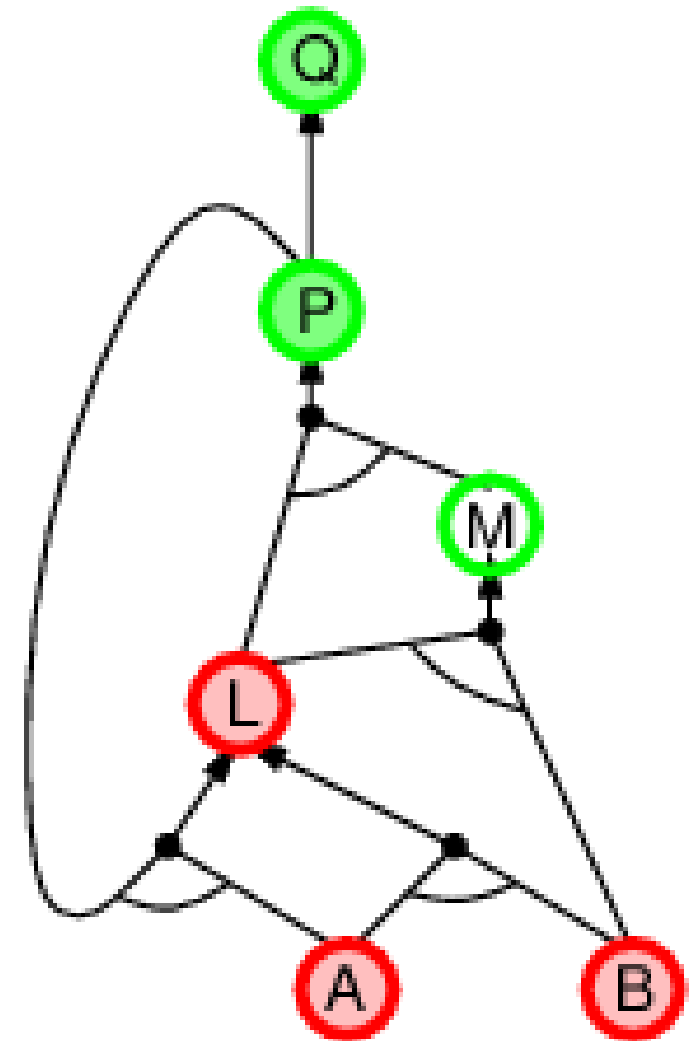
- Current goal:  $L$
- $L$  can be inferred by  $A \wedge B \implies L$
- Both are true



# Backward Chaining Example

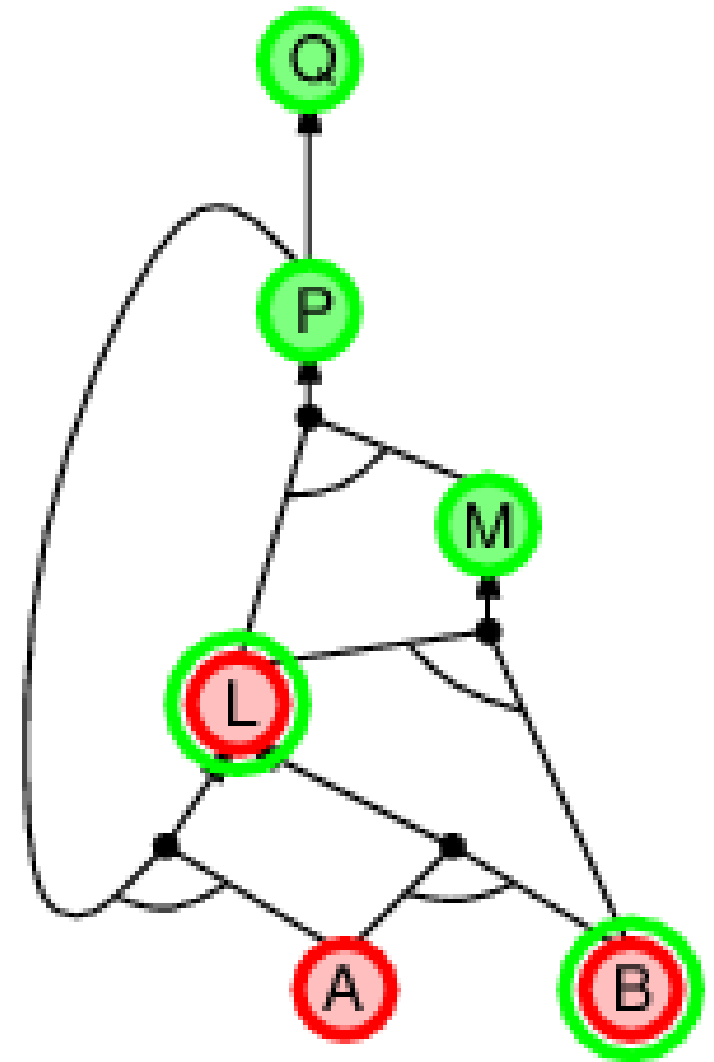
- Current goal:  $L$
- $L$  can be inferred by  $A \wedge B \implies L$
- Both are true

$\implies L$  is true



# Backward Chaining Example

- Current goal:  $M$
- $M$  can be inferred by  $B \wedge L \implies M$

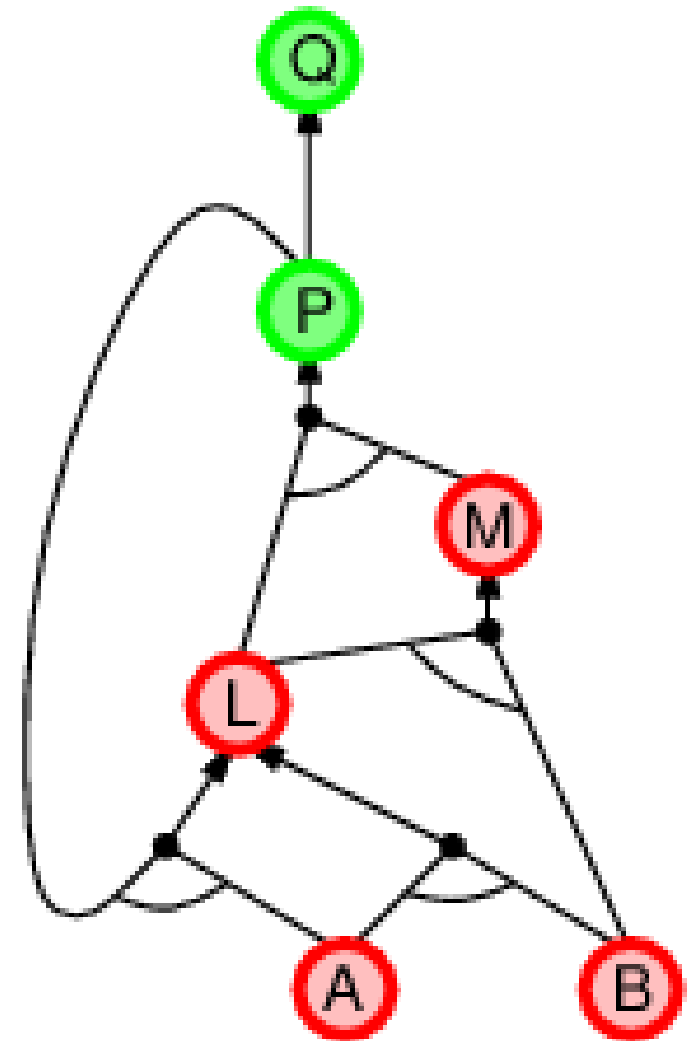




# Backward Chaining Example

- Current goal:  $M$
- $M$  can be inferred by  $B \wedge L \implies M$
- Both are true

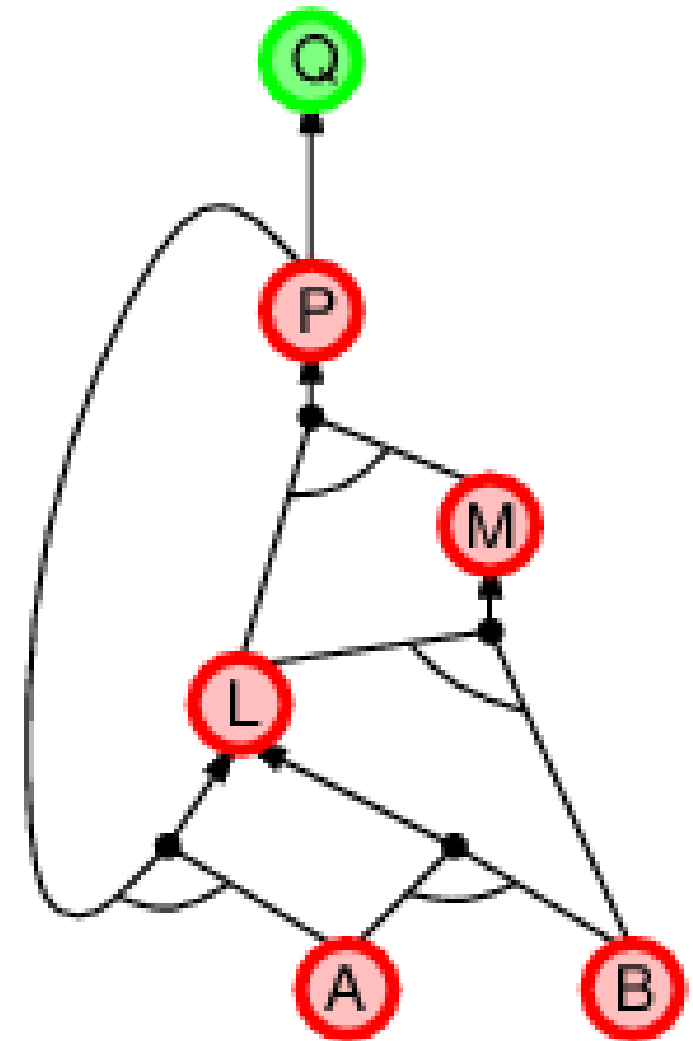
$\implies M$  is true



# Backward Chaining Example

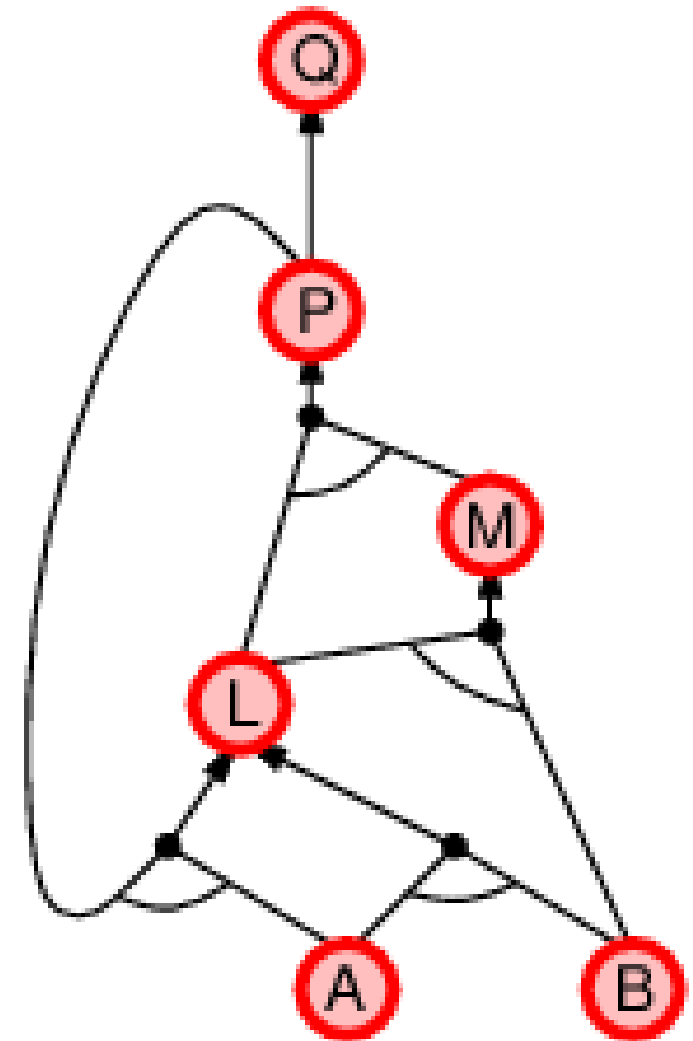
- Current goal:  $P$
- $P$  can be inferred by  $L \wedge M \implies P$
- Both are true

$\implies P$  is true



# Backward Chaining Example

- Current goal:  $Q$
  - $Q$  can be inferred by  $P \implies Q$
  - $P$  is true
- $\implies Q$  is true



# Forward vs. Backward Chaining



- FC is **data-driven**, cf. automatic, unconscious processing, e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is **goal-driven**, appropriate for problem-solving, e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be **much less** than linear in size of KB

# resolution

# Resolution

- Conjunctive Normal Form (CNF—universal)

**conjunction** of **disjunctions** of **literals**  
**clauses**

E.g.,  $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

- **Resolution** inference rule (for CNF): complete for propositional logic

$$\frac{l_1 \vee \dots \vee l_k, \quad m_1 \vee \dots \vee m_n}{l_1 \vee \dots \vee l_{i-1} \vee l_{i+1} \vee \dots \vee l_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

where  $l_i$  and  $m_j$  are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \quad \neg P_{2,2}}{P_{1,3}}$$

- Resolution is sound and complete for propositional logic



# Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate  $\Leftrightarrow$ , replacing  $\alpha \Leftrightarrow \beta$  with  $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$ .

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate  $\Rightarrow$ , replacing  $\alpha \Rightarrow \beta$  with  $\neg\alpha \vee \beta$ .

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move  $\neg$  inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law ( $\vee$  over  $\wedge$ ) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$



# Resolution Algorithm

- Proof by contradiction, i.e., show  $KB \wedge \neg\alpha$  unsatisfiable

```
function PL-RESOLUTION( $KB, \alpha$ ) returns true or false
  inputs:  $KB$ , the knowledge base, a sentence in propositional logic
            $\alpha$ , the query, a sentence in propositional logic
  clauses  $\leftarrow$  the set of clauses in the CNF representation of  $KB \wedge \neg\alpha$ 
  new  $\leftarrow$  { }
  loop do
    for each  $C_i, C_j$  in clauses do
      resolvents  $\leftarrow$  PL-RESOLVE( $C_i, C_j$ )
      if resolvents contains the empty clause then return true
      new  $\leftarrow$  new  $\cup$  resolvents
  if new  $\subseteq$  clauses then return false
  clauses  $\leftarrow$  clauses  $\cup$  new
```

# Resolution Example

- To disprove:  $\alpha = \neg P_{1,2}$

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}))$

reformulated as:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

- Observation:  $\neg B_{1,1}$  ■

- Resolution

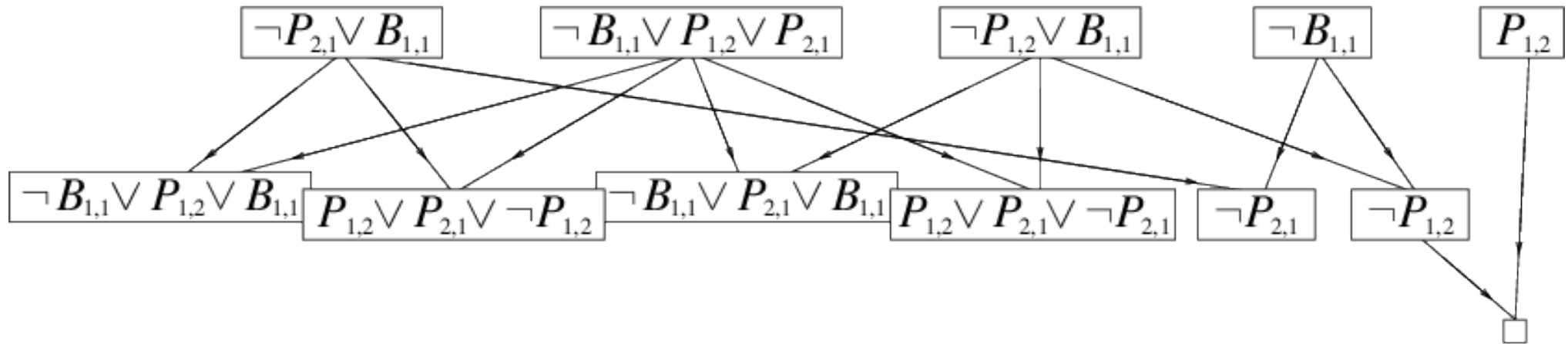
$$\frac{\neg P_{1,2} \vee B_{1,1} \quad \neg B_{1,1}}{\neg P_{1,2}}$$

■

- Resolution

$$\frac{\neg P_{1,2} \quad P_{1,2}}{\text{false}}$$

# Resolution Example



- In practice: all resolvable pairs of clauses are combined

# Logical Agent



- Logical agent for Wumpus world explores actions
  - observe glitter → done
  - unexplored safe spot → plan route to it
  - if Wampus in possible spot → shoot arrow
  - take a risk to go possibly risky spot
- Propositional logic to infer state of the world
- Heuristic search to decide which action to take

# Summary

- Logical agents apply **inference** to a **knowledge base** to derive new information and make decisions
- Basic concepts of logic:
  - **syntax**: formal structure of **sentences**
  - **semantics**: **truth** of sentences wrt **models**
  - **entailment**: necessary truth of one sentence given another
  - **inference**: deriving sentences from other sentences
  - **soundness**: derivations produce only entailed sentences
  - **completeness**: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Forward, backward chaining are linear-time, complete for Horn clauses  
Resolution is complete for propositional logic
- Propositional logic lacks expressive power