Homework Assignment 3

600.435 Artificial Intelligence
Spring 2017
Due: April 11th

Logic and Knowledge Representation

Lately we have been focusing on methods for representing knowledge such that an intelligent agent can follow rules and make inferences. In this assignment, you will be using propositional logic, first order logic, and knowledge representation to solve written questions. **Please be sure to show all relevant work.**

Translate the following english sentences into *propositional logic*

1. A and B are both true.
2. If A is true, then B must be true as well.
3. If a student studies for a test, they will do well on it. We can also tell that if a student did well on a test, then they must have studied for it.
4. If a student is completely dry and it is raining outside, it is because they have an umbrella or a hoodie and it is not raining heavily.
5. If a student doesn't hand in the homework late or incomplete, this doesn't necessarily imply that they will not lose points.

Simplify and translate the following *propositional logic* sentence into English

6. $A \lor (A \land B) \iff \neg(A \land B \land C)$

Is the following sentence valid?

7. $A \lor B$

Is the following sentence satisfiable?

8. $A \implies B$

Is the following sentence unsatisfiable?

9. $(A \wedge (B \vee C)) \wedge ((A \wedge B) \vee (A \wedge C))$

Translate the following english sentences into *first order logic*

10. Some students pass English but not Math.
11. Every student is registered in a class and enrolled at a university.
12. If someone is an aunt or uncle, then someone must be their niece or nephew.
13. The old that is strong does not wither.

Translate the following *first order logic* sentence into English

14. $\neg(\forall x, Gold(x) \implies Glitter(x))$

15. Suppose you are given the following axioms:
    1. $0 \leq 3$
    2. $7 \leq 9$
    3. $\forall x, x \leq x$
    4. $\forall x, x \leq x + 0$
    5. $\forall x, x + 0 \leq x$
    6. $\forall x, y, x + y \leq y + x$
    7. $\forall w, x, y, z, w \leq y \wedge x \leq z \implies w + x \leq y + z$
    8. $\forall x, y, z, x \leq y \wedge y \leq z \implies x \leq z$

    A. Give a backward-chaining proof of the sentence $7 \leq 3 + 9$. (Be sure, of course, to use only the axioms given here, not anything else you may know about arithmetic.) Show only the steps that lead to success, not the irrelevant steps.

    B. Give a forward-chaining proof of the sentence $7 \leq 3 + 9$. Again, show only the steps that lead to success.

For the next several questions, use the following two sentences in first order logic.

Assume that x and y range over the set of natural numbers, and that $\leq$ has the conventional mathematical definition.
(A) $\exists y \forall x (x \leq y)$
(B) $\forall x \exists y (x \leq y)$

16. Translate (A) and (B) into English

17. Is (A) true?

18. Is (B) true?

19. Does (A) entail (B)?

20. Does (B) entail (A)?

21. Using resolution, try to prove that (A) follows from (B). You should either complete the proof (if possible), or continue until the proof breaks down and you cannot continue (if impossible). Show the unifying substitution in each step. If the proof fails, explain where, how, and why it fails.

22. Define an ontology in first order logic for tic-tac-toe. The ontology should contain situations, actions, squares, players, marks (X, O, or blank), and the notion of winning, losing, or drawing a game. Also define the notion of a forced win (or draw): a position from which a player can force a win (or draw) with the right sequence of actions. Write axioms for the domain. (Note: The axioms that enumerate the different squares and that characterize the winning positions are rather long. You need not write these out in full, but indicate clearly what they look like.)

23. A popular children's riddle is "Brothers and sisters have I none, but that man's father is my father's son." Using the rules of a family domain (objects are people, predicates are Parent, Sibling, Brother, etc) to show who that man is. You may apply any of the inference methods described in class. Why do you think this riddle is difficult to grasp?

24. Given the following clauses in first order logic, prove by resolution that $\neg istype(Tuna, Mammal)$; that is, prove that a Tuna is not a Mammal.
$istype(Tuna, Fish)$
$\neg equal(Mammal, Fish)$
$istype(p, Type(p))$
$\neg istype(p, k) \vee equal(Type(p), k)$
$\neg equal(x, y) \vee \neg equal(y, z) \vee equal(x, z)$
$\neg equal(x, y) \vee equal(y, x)$

Submission Requirements

You should upload a document containing your answers to the questions to Gradescope. There is no programming portion to this assignment, so a zip file is not necessary. Your answers can be typed or hand-written and scanned, as long as everything is legible. You may **not** work in pairs for this assignment. Again, please be sure to show all relevant work.