# Fast smoothed shock filtering*

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## Abstract

*Shock filters and related tools, like coherence-enhancing filters, are popular methods for denoising and creating artistic effects. They iteratively apply morphological operators with a constant structuring element. We propose in this article to improve the original shock filtering scheme using smoothed local histograms. Our method exhibits better performance and control of the erosion and dilation operators and serves as an easily-controlled and fast denoising algorithm, in comparison with other shock filters in the literature. We also show application of our method for water-colorization and medical image segmentation.*

## 1. Introduction

Shock filters are morphological image enhancement techniques based on partial differential equations (PDEs), which locally "shock" an image by erosion and dilation to create ruptures between local maxima and minima. As the original shock filter introduced in [6] is unable to remove some basic types of noise, like uniform "salt and pepper" noise, Gaussian noise, *etc*., many authors have proposed improvements to this scheme using various diffusion processes. For example, [1] uses a smoothed Laplacian term while [3] regularizes the shock filter in the complex domain.

Another major property of the shock filter is its enhancement of flow-like patterns, like a fingerprints, a lion's mane, or long hair. This principle was investigated in [10], which proposed the coherence-enhancing shock filter. Iterative applications of this filter provide interesting image abstractions [5].

More generally, numerous filters exist in the literature for denoising and stylization. A wide variety of them may be re-interpreted in terms of local histograms, including the median filter, the bilateral filter [7], the local mode filter [8], *etc.* Kass and Solomon [4] propose an elegant formulation and fast algorithms to evaluate these filters with *smoothed local histograms*. They also show that many algorithms may fail to remove noise in case of high image alterations. Their median, however, succeeds at this task, because they employ isotropic and smoothly-varying weights.

Another contribution of [4] is a generalization of the median filter to the formulation of smoothed morphological filters. They are able to parametrize these operations, smoothly changing from erosion to median filtering to dilation using a single percentage value. A 5% erosion or a 85% dilation are therefore possible, with the smoothed median filter at the 50% mark in between.

In this article, we introduce a *fast smoothed shock filter* using the smoothed local histograms formalization and discretization from [4] to produce parametrized erosions and dilations. Our improvement smooths homogeneous regions while preserving edges, with more flexible and efficient structuring elements. We iterate the filter to increase its strength and to produce interesting water-colorization effects.

## 2. Smoothing the shock filter

The original shock filter [6] processes each pixel $\mathbf{p_i}$ of an image $I$ using the PDE scheme, given at iteration $t$ by $I^t(\mathbf{p_i})$:

$$I^t(\mathbf{p_i}) = -\mathrm{sign}(\Delta I^{t-1}(\mathbf{p_i}))|\nabla I^{t-1}(\mathbf{p_i})|,\ t \geq 0,\quad (1)$$

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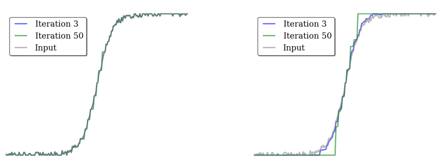
with $I^0(\mathbf{p_i}) = I(\mathbf{p_i})$. $\Delta I^t(\mathbf{p_i})$ is the Laplacian computed at pixel $\mathbf{p_i}$, while $\nabla I$ is the spatial gradient at $\mathbf{p_i}$. At each iteration $t \geq 0$ of this process, the shock filter performs morphological operators depending on the sign of the Laplacian:

$$\begin{cases} \Delta I^{t-1}(\mathbf{p_i}) < 0 \Rightarrow I^t(\mathbf{p_i}) = I^{t-1}(\mathbf{p_i}) \oplus D \,; \\ \Delta I^{t-1}(\mathbf{p_i}) > 0 \Rightarrow I^t(\mathbf{p_i}) = I^{t-1}(\mathbf{p_i}) \ominus D, \end{cases} \quad (2)$$

where $D$ is a disk-shaped structuring element of ra-




(a) Shock (b) Smoothed shock

**Figure 1. For a noisy 1D signal, we produce the classic shock filter (a) and our technique (b) for 3 and 50 iterations.**

dius 1, while $\oplus$ and $\ominus$ are the symbols of classic dilation and erosion operators. Shock filtering therefore produces such morphological processes of radius $t$ near minima and maxima during each iteration. This algorithm has been designed to create ruptures in inflection zones, although even authors admit that it is not able to handle noise efficiently, as illustrated in Figure 1.

The smoothed local histogram of the neighborhood $\mathcal{V}(\mathbf{p_i})$ of a pixel $\mathbf{p_i}$ introduced by [4] is modeled as:

$$\hat{f}_{\mathbf{p_i}}(s_k) = \sum_{\mathbf{p_j} \in \mathcal{V}(\mathbf{p_i})} K(I(\mathbf{p_j}) - s_k) W(|| \, \mathbf{p_i} - \mathbf{p_j} \, ||_2)$$

where $k \in \{1, n_b\}$, $K, W$ are generally Gaussian kernels and $s_k$ is the $k$-th bin of the histogram of size $n_b$. Here, we focus on 1D histograms that could be applied to gray-scale images or to one channel of three-channel images (here, we filter the V channel of images in HSV space). In this case, $n_b$ is an oversample of the processed 1D histogram.

With this formalism, we can define the median filter using the calculation of an integral over the smoothed histogram, which is the same as computing the bins:

$$R_k(\mathbf{p_i}) = 1 - \big(C(I(.) - s_k) * W\big)(\mathbf{p_i}), \quad (3)$$

with $k \in \{1, n_b\}$. $C$ is the integral of $K$, expressed as an integral function (ERF) and $*$ is the convolution operator. To obtain a smoothed median filter, we simply find the $s_k$ value such that $R_k(\mathbf{p_i}) = t$, with $t = \frac{1}{2}$. We can process a smoothed dilation if we choose $\frac{1}{2} < t \leq 1$, and a smoothed erosion with $0 \leq t < \frac{1}{2}$.

Our fast smoothed shock filter makes use of these smoothed morphological operators inside the classic shock scheme. In particular, we replace Equation 2 by the calculation of the bin $s_k$ such that:

$$R_k(\mathbf{p_i}) = \left(\tfrac{1}{2} + \rho \Delta I(\mathbf{p_i})\right), \quad (4)$$

where we have $\Delta I(\mathbf{p_i}) \in [-1; 1]$ and $\rho \in [-\frac{1}{2}; \frac{1}{2}]$. Then, we just have to set $I(\mathbf{p_i})$ with this value of $R_k(\mathbf{p_i})$. This equation means that we generate smoothed erosions of parameter $t = \frac{1}{2} - \rho$ when the Laplacian is positive and smoothed dilations of parameter $t = \frac{1}{2} + \rho$ otherwise. With this formulation, we can process noisy signals using a few iterations of our algorithm. In Figure 1, we can notice that the filtering of the maxima and minima zones are more efficient with our proposal than with the original shock filter scheme (Figure 1-(b)), whereas the inflection zone is smoothed. Moreover, with the increasing number of iterations, it also sharpens the signal. In Figure 2, we describe our

**input** : An image $I$, a number of iterations $n_i$
**output**: $I$ is filtered by smoothed shock
**for** $it = 1$ **to** $n_i$ **do**
  compute $R_k(\mathbf{p_i})$ for all $k \in \{1, n_b\}$, $\mathbf{p_i} \in I$ ;
  **for each** $\mathbf{p_i} \in I$ **do**
    $t \leftarrow \frac{1}{2} + \rho \Delta I(\mathbf{p_i})$ ;
    $v \leftarrow R_1(\mathbf{p_i}) + t(R_{n_b}(\mathbf{p_i}) - R_1(\mathbf{p_i}))$ ;
    **for** $k = 1$ **to** $n_b - 1$ **do**
      **if** $R_k(\mathbf{p_i}) \leq v \wedge R_{k+1} \geq v$ **then**
        $I(\mathbf{p_i}) \leftarrow \frac{s_k + (s_{k+1} - s_k)(v - R_k(\mathbf{p_i}))}{(R_{k+1}(\mathbf{p_i}) - R_k(\mathbf{p_i}))}$ ;
**return** $I$ ;

**Figure 2. Our algorithm.**

whole method, which combines the shock scheme together with smoothed local histograms.

## 3. Experimental results and analysis

We first show the impact of possible parameter choices in our method. In Figure 3, we filter the same image (from the database of http://www.tela-botanica.org/) with $n_i = 20$, $\rho = 0.1$ and several values of standard deviation $\sigma_w$ for the spatial Gaussian kernel $W$ in Equation 3. We can increase the width of the blurring effect of our filter within homogeneous regions using the value of $\sigma_w$, while still preserving edges. Our technique could be applied as a water-colorization effect, with high values of $\sigma_w$.

In our formulation, $\rho$ modulates the effect of the Laplacian in the computation of smoothed morphological operators (see Figure 4, where (a) is under creative commons licence, from Daniel Giffard), with $n_i = 20$. The increase of $\rho$ decreases the smoothing impact of the

(a) Input (b) $\rho = 0.1$ (c) $\rho = 0.2$ (d) $\rho = 0.3$

**Figure 4. Impact of parameter $\rho$ upon smoothed morphological operators.**

(a) Input (b) $\sigma_w = 3$ (c) $\sigma_w = 7$ (d) $\sigma_w = 11$

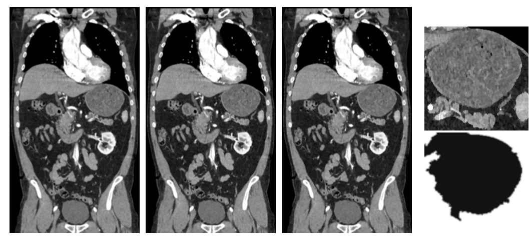
**Figure 3. About increasing the blurring effect with the smoothing kernel $W$.**

filter, giving a classic shock filter if we choose $\rho = 0.5$. We are able to conserve the structural patterns including shingles and bricks even with small values of $\rho$.

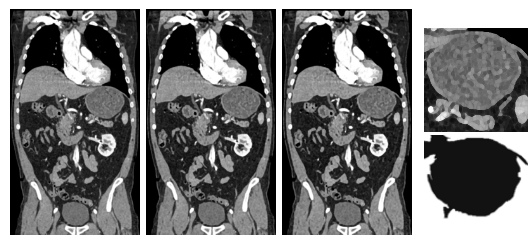
We now consider the following list of some comparable methods to ours:

**Shock-OR:** Original shock filter [6] ;
**Shock-AM:** Regularized shock filter from [1] ;
**Shock-GSZ:** Regularized complex shock filter [3] ;
**Shock-W:** Coherence-enhancing shock filter [10] ;
**FSShock:** Our proposal,

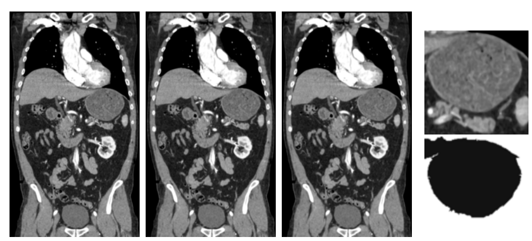
whose output on a 2D slice of a CT image is shown in Figure 5. Regularization schemes such as Shock-AM and Shock-GSZ improve the quality of denoising but resort to a slow iterative process to accomplish these results. The gain between 10 and 30 iterations is slightly perceptible. Our method, denoted FSShock, is able to produce better filtering with a small number of iterations (*e.g.* 10 iterations on the lung image). It produces clean enhancement of the CT slice, in which organs and bones are clearly distinguishable. It can be verified with the segmentations we performed with the algorithm from [2]. Here, we do not present results from Shock-W, since it is more a flow-like pattern enhance-



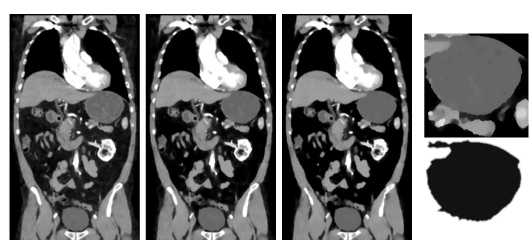
(a) Shock-OR



(b) Shock-AM



(c) Shock-GSZ



(d) FSShock

**Figure 5. Output of the tested methods applied on a CT scan slice, for 10, 20, 30 iterations. A zoomed part at 30 iterations is depicted with its segmentation (right).**

ment technique than a filtering algorithm and is therefore unable to produce an edge-preserving blur.

We evaluate the quality of FSShock using the DenoiseLab benchmark (available at http://www.stanford.edu/~slansel/DenoiseLab/), which is composed of 13 gray-scale 512×512 images, altered with various noises, applied with several standard deviations (from 5 to 25). We consider the SSIM (structural similarity) measure to estimate the perceptual quality of the obtained images [9]. The SSIM between a ground-truth image $U$ and a tested image $V$ leads to a value belonging to $[0; 1]$, $1$ being the perfect equality between $U$ and $V$. We have tuned our SSIM with the most used parameter values in the literature, and we set $\rho = 0.1$, $\sigma_w = 3$, because they give the best SSIM.

Our contribution does not always produce the best SSIM values, but it is the fastest technique to converge to its best SSIM. In particular, if we have a more precise look at the behavior of the algorithms while they iterate, we notice in Figure 6 that our method achieves its best performance within a few iterations. We have depicted our measures for the image Barbara, with an additive white Gaussian noise (AWGN) of standard deviation 25, knowing that the shapes of the curves are similar in all the other cases. We have computed each method on a workstation laptop Dell® XPS™ M1730 with a processor Intel® Core™ Duo 2.4GHz with 3.9Gb RAM. We get the following time results:

**Shock-GSZ:** 0.1 second per iteration, its best SSIM is reached between 1 and 50 iterations, *i.e.* between 0.1 and 5.0 seconds ;

**FSShock:** 0.4 second per iteration, its best SSIM is reached between 1 and 5 iterations, *i.e.* between 0.4 and 2.0 seconds.

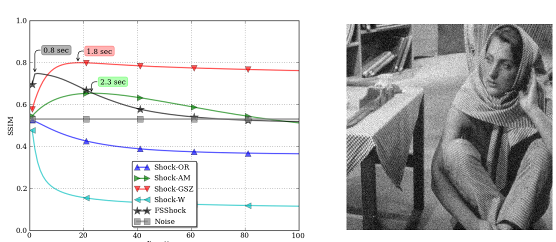



**Figure 6. SSIM during iterative processes for Barbara, AWGN of strength 25.**

We can conclude that our method may be easily controlled, leading to its best performance in a few iterations. Furthermore, it yields a better estimation of execution time, contrary to Shock-GSZ, where a maybe very long iterative process can occur.

## 4. Conclusion and future work

In this article, we have presented a very promising technique for 2D image filtering that can be efficiently used (1) as a stylization tool for water-colorization and (2) as a fast and easily-controlled denoising method.

The adaptation of high-performance filtering techniques on smartpones is a real challenge, because it implies a good optimization of potentially long iterative processes, and a high control of memory. The shock filter and the coherence-enhancing shock filter have been successfully embedded on smartphones (see http://www.shockmypic.com/), and we expect to have interesting results with our contribution on this kind of mobile phones.

An other important issue is to keep on comparing our proposal with the other filtering techniques of the literature, and to study the behavior of SSIM under various types of noises, with variable standard deviations, *etc.*

We also plan to use our fast smoothed shock filter for medical imaging applications. We have shown that it could have high potential for CT image segmentation, and integrating robust statistical noise models would enable filtering of many other modalities.

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