Laplace-Spectra as Fingerprints for Shape Matching

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Overview

- Shape Matching
- Laplace Spectrum
- Applications and Results
- Questions

Shape Matching

- How similar are two shapes?
  - Many methods
    - Often require
      - Alignment
      - Specific representation, i.e. 3D polygonal meshes
  - But
    - Alignment is limited
      - Sensitive to data holes / missing parts
      - Rotation is approximate
    - Many different representations

Watermarking

- Driven by the need to identify copyright
  - Particularly industrial applications
- Most methods are limited to meshes
  - NURBS watermarking is not robust
- Most methods mark the representation
  - Washed out by re-parameterization

Desired Properties

- Isometry
- Similarity
- Efficiency
- Completeness
- Compression
- Physicality

Spectrum of the Laplace eigenvalues

- ☑ Isometry
- ☑ Similarity
- ☑ Efficiency
- ☐ Completeness
- ☑ Compression
- ☑ Physicality

- Solids
  - Laplacian
- Surfaces/Images
  - Laplace-Beltrami operator
    - The Laplacian on a Riemannian manifold
- Scale invariance optional

Laplace Spectrum

- Ordered spectrum $0 \le \lambda_1 \le \lambda_2 \le \cdots \uparrow +\infty$
  - Solution to the Laplacian Eigenvalue problem $\Delta f = -\lambda f$
  - For closed surfaces $\lambda_1 = 0$
- Invariance
  - The gradient and divergence are only depenant on the Riemannian manifold
  - Scaling a surface by x scales the eigenvalue by $1/x^2$
- Similarity
  - The spectrum depends continuously on the membrane shape and also on the Riemannian metric

Laplace Spectrum

- Completeness
  - Some spectrum are unique
  - Counter examples exist
    - Are artificial, non-convex and have non-smooth boundaries
    - Only pairs have been found
- Physically
  - The spectrum contains many properties
    - Boundary length, area, Euler charateristic,...
    - "Can you hear the shape of a drum?"

Scale Invariance

1. The first non-zero eigenvalue
   - Useful only for identical shapes
2. The slope of the fitting line
   - ~4 π /Area
3. The volume determined by the spectra
4. The volume determined externally

Computation and Accuracy

- Convert to a variational problem
- Discretize using the Galerkin technique and Finite Element Method and solved.
- The FEM method is proven to converge
  - Other discrete methods don't
  - Accuracy increases with dense sampling
- Verified accuracy against theoretical values
  - Rectangle, circle, sphere, cube, ball, cylinder,...
- Verified geometric data
  - volume, boundary length, Euler characteristic

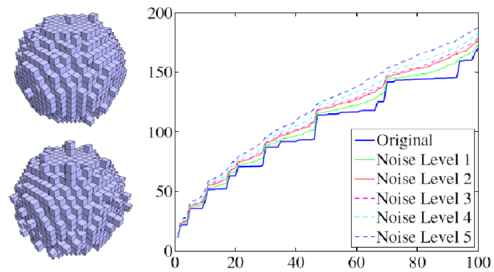
Applications

- Identify shapes for copyright protection
- Database retrieval and matching
- Quality Assurance
- Representation conversion verification

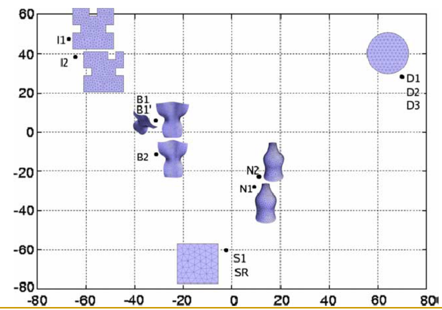
Shape Matching

- Spectrum truncation to 10-100 values
- Euclidean distance
- Database of 1000 random NURBS patches
  - 100% recovery of self + 'noise'
    - 1 order of magnitude safety margin

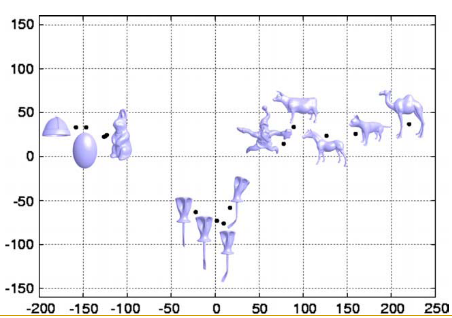
Results



Results



Results



Questions?