An Algebraic Approach to Surface Reconstruction from Gradient Fields

A. Agrawal, R. Chellappa, R. Raskar

Dan Abretske
Johns Hopkins University 2007

Seminar on Advanced Computer Graphics

Motivation

- Goal: The recovery of 3D shape from 2D Data
- Methods:
  - Shape From Shading
    - Exploits Shading Cues
  - Photometric Stereo
    - Exploits Multiple Light Positions

Motivation

- Problems:
  - Estimated Gradient Fields are rarely integrable… But they should be.
  - All SFS Algorithms are forced to make simplifying assumptions due to the fact that these systems are underdetermined.
  - The focus of this paper is on solving the first problem.

Curls, Divs, and Integrability

$$\nabla S(x,y) = (\frac{\partial S}{\partial x}, \frac{\partial S}{\partial y}) = (p,q)$$

$$curl(p,q) = \frac{\partial p}{\partial y} - \frac{\partial q}{\partial x}$$

$$div(p,q) = \frac{\partial p}{\partial x} + \frac{\partial q}{\partial x}$$

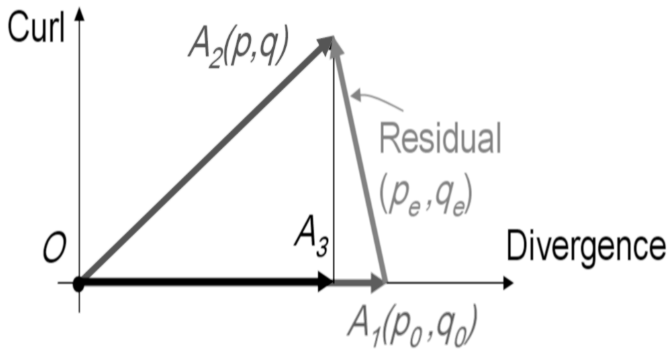
An integrable field has zero curl

Solutions

- Apply any number of different techniques
- More importantly solve the standard

$$\nabla^2 S = div(p,q)$$

- This method only provides a least squares(LS) approximation since it ignores the information contained in the curl.
- Consider this example
  - A_1 is the real field
  - A_2 is the noisy field
  - A_3 is the LS solution



Using the curl

- We can use the information in the curl to solve for a better solution.
- First we discretize the system as:

$$p(y,x) = S(y,x+1) - S(y,x)$$

$$q(y,x) = S(y+1,x) - S(y,x)$$

$$C(y+\frac{1}{2},x+\frac{1}{2}) = p(y+1,x) - p(y,x) + q(y,x) - q(y,x+1)$$

- Note: these equations correspond to loop integrals over 4 pixels, hence the +1/2 terms

Using the curl

- The curl of a non-integralble field is the curl of the residual field.
  1. Non-zero curl implies that at least one of the residual gradients is non zero
  2. A non-zero p residual affects loops to the left and right
  3. A non-zero q residual affects loops to the top and bottom

Finding the residual curl

- The goal is build a system Ax=b using the previous rules

$$x = [p_\varepsilon^1 \cdots p_\varepsilon^{K_p}, q_\varepsilon^1 \cdots q_\varepsilon^{K_q}]^T$$

$$b = [C(y^1, x^1) \cdots C(y^{K_b}, x^{K_b})]^T$$

$$C(y^i, x^i) = [1 \ldots -1 \ldots 1 \ldots -1]x$$

- A is formed by stacking C's as rows

Solving the System

- To solve the previous system rk(A)=K_p+K_q is a necessary condition
- rk(A) is at most (H-1)x(W-1)-1
- In general this means A is rank deficient
- We need a way of increasing the rank of A.

Graphs!

- Define a graph G such that edges are gradients and nodes are pixels.
- Break any edge which is involved in a non-zero curl calculation
- If the graph remains connected the system Ax=b can be solved.
- If the graph breaks into n pieces $rk(A)=(K_p+K_q)-(n-1)$
- The question then is how can the graph be made connected with a minimal number of edges?

Joining Edges

1. A minimal number of edges should be added.

2. Edges added should belong to loops with small curl values

The Algorithm

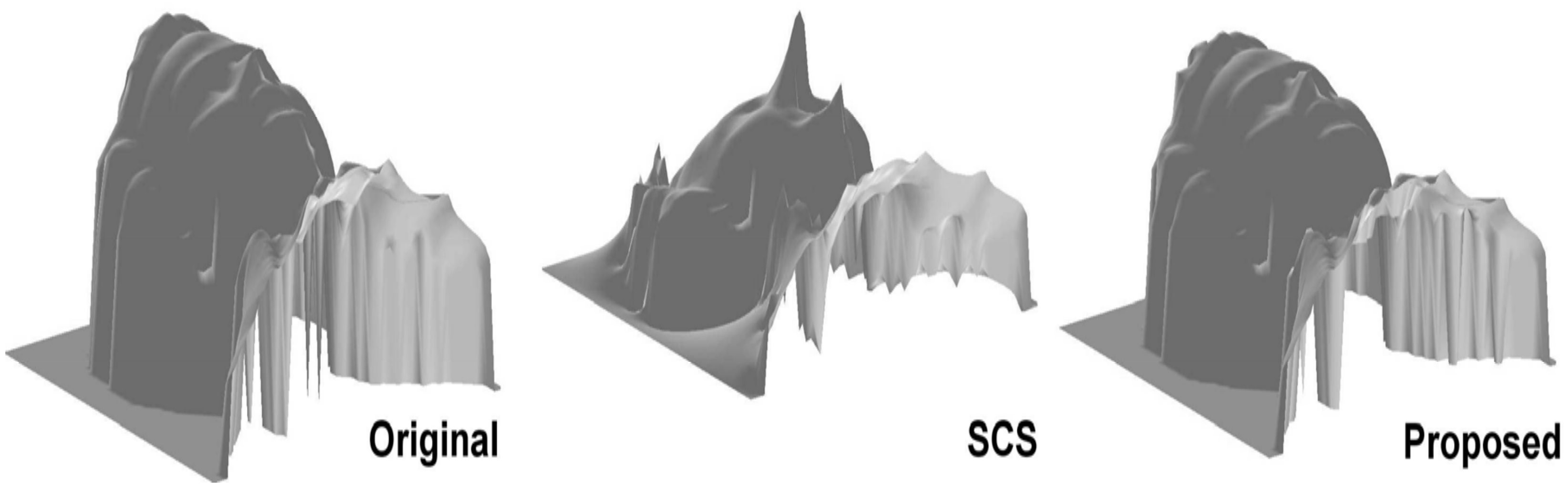
1. Compute the curl for each image loop
2. Place all boundary pixels in B2
3. Place any node with a curl loop greater than Tau in B1 and all other nodes in B2.
4. Break all edges connecting B1 and B2 and all edges within B1

The Algorithm

4. Connect the graph
   I. While B1 is non-empty find the minimal shortest path from a node V2 in B2 to a node V1 in B1.
   II. Move V1 to B2 and add the edge from V2 to V1
5. Solve Ax=b for remaining broken edges

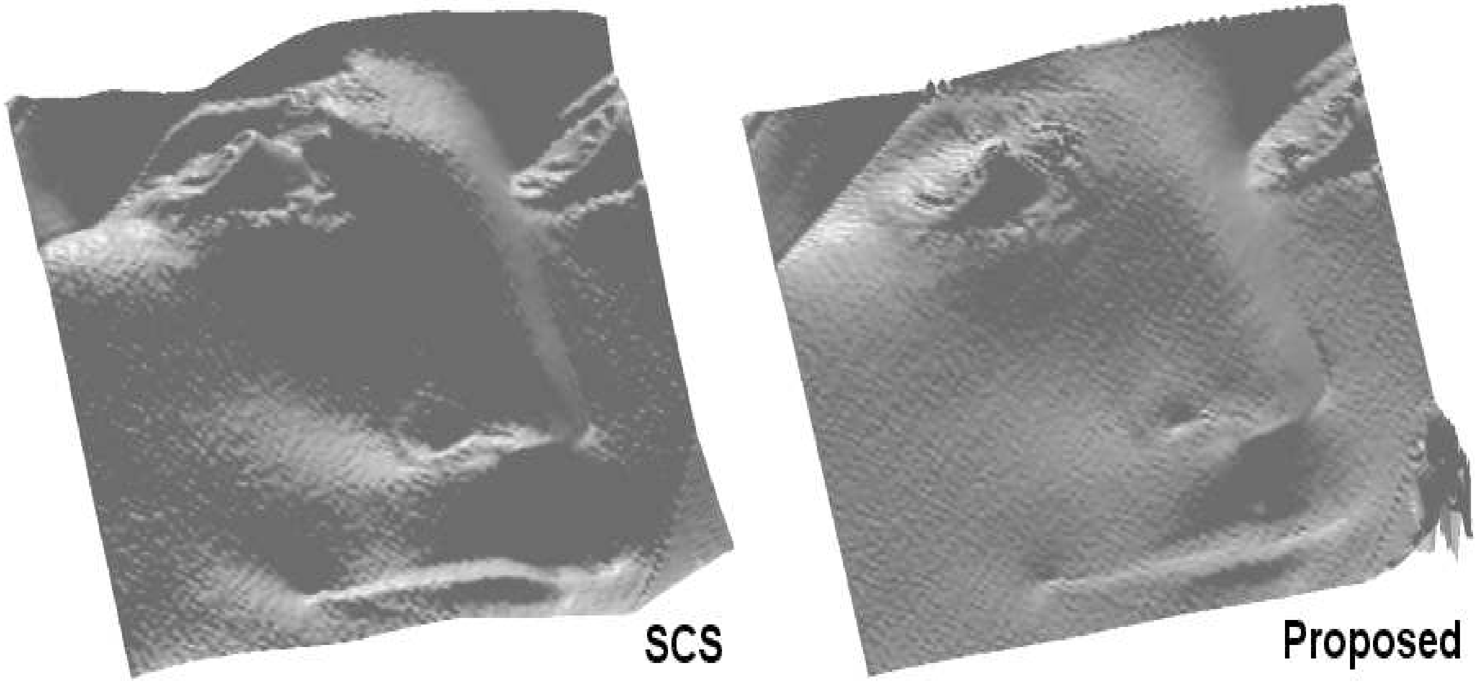
Photometric Stereo Results

- Mozart Data Set



Shape from Shading Results

- The ever popular Lena



Shape from Shading

- A semi ubiquitous vase