# Deriving Multi-Headed Planar Dependency Parses from Link Grammar Parses 

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## Introduction

- This talk is about converting from one annotation style to another.
- The conversion could be hard, where information is fragmented, missing, or ambiguous.
- We use a general technique, Integer Linear Programming to help us do this conversion.


## In Our Case: What We Started With

 the matter may never even be tried in court.

Link Grammar: Parse with undirected edges

## What We Wanted:



Multiheaded parse with directionalized edges

## Why We Wanted That

- We want to develop parsing algorithms for parses that look like this
- We couldn't figure out where to get the data to test them.


## Single-headedness

- Dependency parse treebanks today are either single-headed or not planar.
- Stanford Dependencies are multiheaded but not planar


Some example dependency parse.

## Single-headedness

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Some example dependency parse.
Link Grammar is almost a multiheaded planar corpora! We just need to directionalize the links.

## Why Multi-headedness?

Multi-headedness Can Capture Additional Linguistic Phenomenon

- Control
- Relativization
- Conjunction


## Control



Jill is the subject of two verbs


Jack is the object of one verb and the subject of another

## Relativization



The boy is the object of with as well as the subject of fell.

## Conjunction



Jack and Jill serve as the two arguments to and, but are also subjects of went.

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- Such a corpus could be automatically annotated using Integer Linear Programming
- We explored whether the Link Grammar could be adapted for this purpose.
- The results of this are mixed, but provides a good case study.


## Corpus Building Strategy

- We start with some sentences and parse them with LG Parser
- We take the undirected parses and try to directionalize them.
- We use an ILP to assign consistent directions for each link type.

SENTENCES

I see a brown bear.
I see a blue bird.

I see the red crab.

UNDIRECTED PARSES

## DIRECTED PARSES



## Link Grammars

Grammar-based formalism for projective dependency parsing with undirected links
Original formalism and English Link Grammar created by Davy Temperley, Daniel Sleator, and John Lafferty (1991)

## Link Grammars: How They Work


the


[^0]
## Link Grammars: How They Work



## Link Grammars: How They Work



## Link Grammars: Same Example Parse From Before Again


the matter may never even be tried in court .
Link Parse of a sentence from Penn Tree Bank

## Link Grammars

Compare resulting dependency parse with CoNLL 2007 shared task.


Bottom half is CoNLL. Top half is the directionalized link parse.

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## What is ILP?

## What is Integer Linear Programming?

- An optimization problem where some or all of the variables are integers.
- The objective function and constraints are linear.
- In general, it's NP-Hard! But good solvers exist that work well most of the time.
- Our ILP is encoded as a ZIMPL program and solved using the SCIP Optimization Suite ${ }^{2}$

[^1]
# Integer Linear Programming Model 

Encoded Constraints:

- Acyclicity
- Connectedness
- Consistency of Directionalized Links


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## Integer Linear Programming Model

Encoded Constraints:

- Acyclicity: (No cycles!)
- Connectedness: (Every word is reachable from a root)
- Consistency of Directionalized Links: (Similar links oriented the same way)



## Integer Linear Programming Model

For each sentence, for each edge $i, j$, where $i<j$


Variables:

$$
\begin{aligned}
& x_{i j}, x_{j i} \in \mathbb{Z} \geq 0: \text { orientation of each link } \\
& x_{i j}+x_{j i}=1
\end{aligned}
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An individual link token can either be oriented left or oriented right

## Acyclicity, Connectedness

Acyclicity
Given that node $u$ is the parent of $v$
$n_{v}$ : length of the sentence containing node $v$
$d_{v} \in\left[0, n_{v}\right]$ : depth of the node from the root of the sentence

$$
\begin{equation*}
\left(\forall_{u}\right) d_{v}+\left(1+n_{v}\right) \cdot\left(1-x_{u v}\right) \geq 1+d_{u} \tag{1}
\end{equation*}
$$

Connectedness

$$
\begin{equation*}
\sum_{u} x_{u v} \geq 1 \tag{2}
\end{equation*}
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A word has at least 1 parent

## Consistency of Directionalized Links

Consistency of Directionalized Links
$r_{L}, \ell_{L} \in\{0,1\}$ : whether all links with label $L$ allowed left/right

$$
\begin{equation*}
x_{i j} \leq r_{L} \quad x_{j i} \leq \ell_{L} \tag{3}
\end{equation*}
$$

Objective Function:

$$
\begin{equation*}
\min \left(\sum_{L} r_{L}+\ell_{L}\right) \tag{4}
\end{equation*}
$$

## Consistency of Directionalized Links with Slack

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$r_{L}, \ell_{L} \in\{0,1\}$ : whether all links with label $L$ allowed left/right

$$
x_{i j} \leq r_{L}+s_{i j} \quad x_{j i} \leq \ell_{L}+s_{i j}
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Objective Function:

$$
\begin{equation*}
\min \left(\sum_{L} r_{L}+\ell_{L}\right) \cdot \frac{N_{L}}{4}+\sum_{i j} s_{i j} \tag{4}
\end{equation*}
$$

$s_{i j} \in \mathbb{R} \geq 0$ : slack variable
$N_{L}$ : Number of link tokens with label $L$

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$s_{i j} \in \mathbb{R} \geq 0$ : slack variable
$N_{L}$ : Number of link tokens with label $L$
Slack allows a few links with label $L$ in disallowed directions

## Data Sets

Data Sets taken from:
CoNLL 2007 Shared Task (English)
ACL 2013 Shared Task of Machine Translation (Russian)

|  | Input Sentences | Output Connected Parses |
| :--- | :--- | :--- |
| English | 18,577 | 10,960 |
| Russian | 18,577 | 4,913 |

## Stability of Results

- We were worried that the recovered direction mapping might be unstable and sensitive to the input corpus.
- We compared the results of increasing runs of sentences.



## On the English Data Set:

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Link Data has 8\% additional edges over the CoNLL. (average about 2 multiheaded words per sentence)

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CoNLL Matches
$52 \%$ of links match CoNLL arcs
$57 \%$ of CoNLL arcs match links
Directionality
6.19\% of link types allowed both directions
2.07\% of link tokens required disallowed direction via slack

## ILP Results: Top 25 Most Occurring Labels

| Label | Rightward | Multiheaded | CoNLL Match | CoNLL Dir Match |
| :--- | :--- | :--- | :--- | :--- |
| A | $0 \%(0 / 8501)$ | $0 \%(0 / 8501)$ | $84 \%(7148 / 8501)$ | $98 \%(7002 / 7148)$ |
| AN | $0 \%(0 / 9401)$ | $0 \%(0 / 9401)$ | $83 \%(7825 / 9401)$ | $98 \%(7639 / 7825)$ |
| B | $100 \%(1514 / 1515)$ | $61 \%(919 / 1515)$ | $53 \%(806 / 1515)$ | $84 \%(678 / 806)$ |
| C | $100 \%(3272 / 3272)$ | $0 \%(0 / 3272)$ | $3 \%(85 / 3272)$ | $53 \%(45 / 85)$ |
| CO | $0 \%(0 / 2478)$ | $1 \%(32 / 2478)$ | $5 \%(114 / 2478)$ | $68 \%(78 / 114)$ |
| CV | $100 \%(3237 / 3237)$ | $100 \%(3237 / 3237)$ | $56 \%(1827 / 3237)$ | $28 \%(512 / 1827)$ |
| D | $0 \%(56 / 19535)$ | $0 \%(71 / 19535)$ | $85 \%(16656 / 19535)$ | $100 \%(16608 / 16656)$ |
| E | $0 \%(0 / 1897)$ | $0 \%(2 / 1897)$ | $67 \%(1279 / 1897)$ | $99 \%(1263 / 1279)$ |
| G | $0 \%(0 / 6061)$ | $0 \%(0 / 6061)$ | $70 \%(4258 / 6061)$ | $96 \%(4070 / 4258)$ |
| I | $100 \%(5405 / 5424)$ | $60 \%(3247 / 5424)$ | $95 \%(5168 / 5424)$ | $47 \%(2408 / 5168)$ |
| IV | $100 \%(1626 / 1627)$ | $100 \%(1626 / 1627)$ | $85 \%(1389 / 1627)$ | $97 \%(1353 / 1389)$ |
| J | $98 \%(16400 / 16673)$ | $2 \%(280 / 16673)$ | $87 \%(14522 / 16673)$ | $97 \%(14069 / 14522)$ |
| M | $100 \%(9594 / 9596)$ | $0 \%(16 / 9596)$ | $74 \%(7124 / 9596)$ | $92 \%(6583 / 7124)$ |
| MV | $100 \%(13375 / 13376)$ | $0 \%(61 / 13376)$ | $51 \%(6797 / 13376)$ | $98 \%(6681 / 6797)$ |
| MX | $100 \%(1999 / 1999)$ | $4 \%(83 / 1999)$ | $42 \%(836 / 1999)$ | $91 \%(763 / 836)$ |
| O | $100 \%(11027 / 11028)$ | $0 \%(0 / 11028)$ | $81 \%(8932 / 11028)$ | $96 \%(8535 / 8932)$ |
| P | $100 \%(3755 / 3756)$ | $31 \%(1167 / 3756)$ | $94 \%(3528 / 3756)$ | $100 \%(3523 / 3528)$ |
| S | $97 \%(13138 / 13520)$ | $57 \%(7662 / 13520)$ | $92 \%(12476 / 13520)$ | $5 \%(586 / 12476)$ |
| SJ | $50 \%(2736 / 5468)$ | $0 \%(0 / 5468)$ | $69 \%(3778 / 5468)$ | $93 \%(3502 / 3778)$ |
| TO | $100 \%(1733 / 1734)$ | $0 \%(1 / 1734)$ | $0 \%(5 / 1734)$ | $100 \%(5 / 5)$ |
| VJ | $51 \%(765 / 1500)$ | $1 \%(8 / 1500)$ | $71 \%(1059 / 1500)$ | $89 \%(939 / 1059)$ |
| W | $100 \%(10528 / 10528)$ | $0 \%(5 / 10528)$ | $5 \%(504 / 10528)$ | $46 \%(232 / 504)$ |
| WV | $100 \%(7563 / 7563)$ | $100 \%(7557 / 7563)$ | $57 \%(4345 / 7563)$ | $97 \%(4214 / 4345)$ |
| X | $80 \%(13132 / 16406)$ | $5 \%(806 / 16406)$ | $8 \%(1364 / 16406)$ | $95 \%(1300 / 1364)$ |
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"B" link relative clauses


The dog I had chased was green

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" B " link relative clauses


The dog I had chased was green


I told him I had oranges

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| :--- | :--- | :--- | :--- | :--- |
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"CV" link conjunctions to main verbs of clauses.

## Link Results: Subject-Verb Links are Backwards


the matter may never even be tried in court .


## Link Results: Subject-Verb Links are Backwards



## Link Results: Subject-Verb Links are Backwards

- This is due to a possible inconsistency of the Link Grammar, discovered by our method.



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- But the governing verb usually links to the subject when there is one.
- So this makes the subject a consistent choice to make the head of a clause.

To fix this, we could edit the link grammar, link parses, or the ILP.

## Conclusions

- Link Grammar parses can be oriented into connected DAGs
- A new corpus available for building multi-headed dependency parsers
- ILP can be used to help annotate incomplete or missing data in corpora.

Questions?


[^0]:    ${ }^{1}$ These figures were clipped from the original Link Grammar paper:
    "Parsing English with a Link Grammar" by Sleator and Temperley

[^1]:    ²http://scip.zib.de/

