Ray Casting

Ray Casting Algorithm

For each pixel

1. Compute ray from eye through pixel

2. For each primitive

—Test for ray-object intersection

3. Shade pixel using nearest primitive (or set to background color)

Computing the Rays

Choose eye point, view direction, up direction, fields of view (x and y)

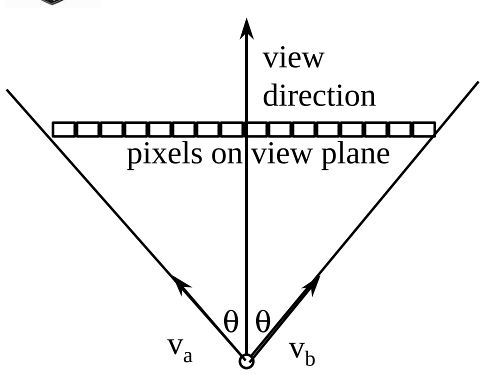
p_t **=** eye **+ t***v **(**v **typically normalized)**

Compute rays to two opposite corners

Compute step sizes, Δx and Δy to go from pixel to pixel

To compute new ray: take step, then normalize

2D ray calculation



Note: take equal-sized steps in viewing plane, not equal angles!

view is normalized view direction

right = (**view**$_y$, **-view**$_x$)
$\mathbf{v_a}$ = **view** - tanθ * **right**
$\mathbf{v_b}$ = **view** + tanθ * **right**
step = ($\mathbf{v_b}$ - $\mathbf{v_a}$) / num_pixels
$\mathbf{v_0}$ = $\mathbf{v_a}$ + **step** / 2
$\mathbf{v_i}$ = $\mathbf{v_{i-1}}$ + **step**

In 3D, we have an additional step size and field-of-view angle as well as an up vector.

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Computing Intersections

Ray is in parametric form (t is parameter)

Represent primitive in implicit form:

$f(x,y,z) = 0$

(any (x,y,z) on surface evaluates to zero)

Substitute (x,y,z) of ray into f(x,y,z) and solve for t

- degree n implicit function will be degree n in t
- quadric surfaces may be solved with quadratic equation -- pick real solution closest to eye

Example Quadric Functions

Sphere: $(x-a)^2 + (y-b)^2 + (z-c)^2 - r^2 = 0$

Circular cylinder (parallel to z-axis):

$(x-a)^2 + (y-b)^2 - r^2 = 0$

Hyperbolic paraboloid:

$y^2/b^2 - x^2/a^2 - z = 0$

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General Quadrics

General quadric has form:

$$Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 2Gy + Hz^2 + 2Iz + J = 0$$

or...

$x^t Q x = 0$, **where** $x^t = [x\ y\ z\ 1]$ **and**

$$Q = \begin{bmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{bmatrix}$$

Quadric Intersections

Quadric: $x^t Q x = 0$

Ray: $x = p + tv$

Substituting ray for x**:**

$$(p + tv)^t Q (p + tv) = 0$$

$$p^t Q p + p^t Q t v + t v^t Q p + t v^t Q t v = 0$$

$$(v^t Q v) t^2 + (p^t Q v + v^t Q p) t + p^t Q p = 0$$

$$(v^t Q v) t^2 + (2 v^t Q p) t + p^t Q p = 0$$

(Q is symmetric)

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Common Ray-tracing Primitives

Sphere, ellipsoid

Cylinders

Plane, triangle

- $Ax + By + Cz + D = 0$

Torus

Bezier/Nurbs patches

- parametric, so use implicit form of ray

  —intersection of two planes

Local Illumination Shading

Compute normal at closest intersection

- $\nabla f = (\partial x, \partial y, \partial z)$ is normal vector field for implicit function, f

For each light

- Use position and normal to compute light contribution
- Accumulate light contributions

Color pixel

- Clamp to avoid overflow

Shadows

Only add contribution from a light if it is visible from the point (and vice versa)

- test for intersections along ray in L direction
- accumulate contribution if no occlusion

(illumination is no longer totally local)

Truncating Primitives

Use another implicit function

- Test which side of the implicit function the intersection is on
- Keep intersection only if it is on the correct side

For example, truncate a cylinder using two plane equations (or perhaps a sphere)

- then cap using the two planes truncated by the cylinder

Constructive Solid Geometry

Perform hierarchical set operations on primitives

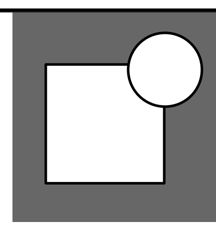
Union: $\cup$

Intersection: $\cap$

Difference: —

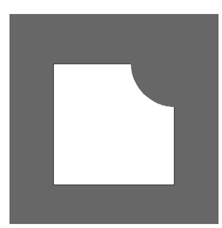
CSG Operators

Square $\cup$ Circle =



Square $\cap$ Circle =

Square — Circle =



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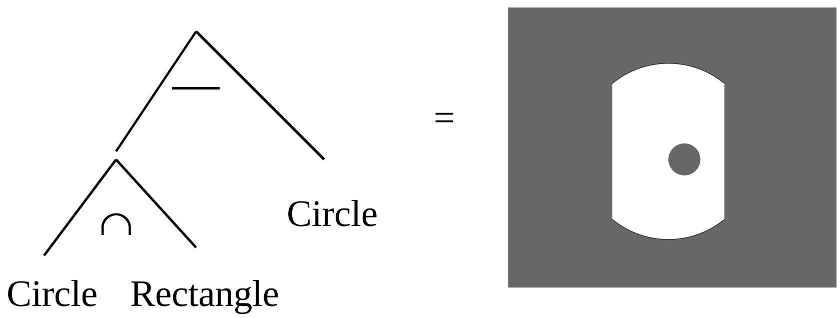
CSG Hierarchy



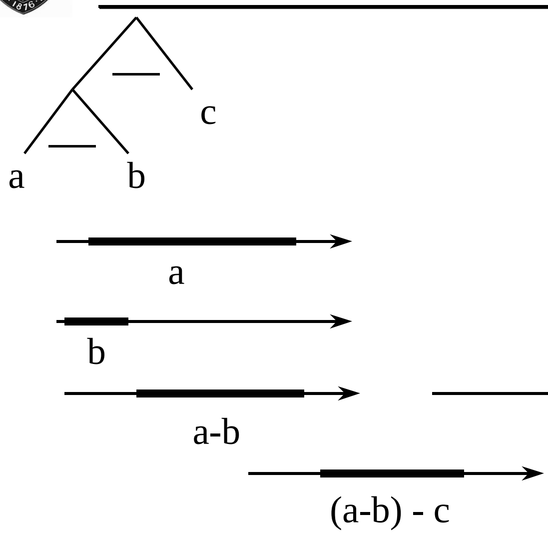
Ray Tracing CSG

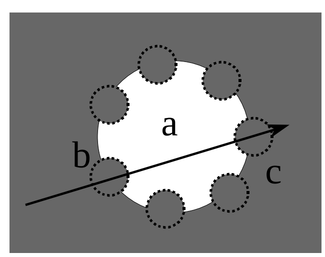
- **Each “object” may be a primitive or a CSG hierarchy**
- **Find** all **ray-primitive intersections for hierarchy**
- **Use CSG operators to determine which** intervals **are solid or vacant**
- **Use start of nearest solid interval as ray-object intersection**

CSG Tracing Algorithm

- Start at root of CSG Hierarchy
- Trace ray through left child - result is ordered list of intersections, forming solid and vacant intervals
- Trace ray through right child
- Merge lists of intersections/intervals by applying CSG operator of current node

CSG Example - golf ball





Some CSG Details

Each interval endpoint associated with intersection of ray with some surface

Normal computed from surface of intersection

Material parameters may come from either primitive