Dictionaries

What is a Dictionary?

Container class

- Stores key-element pairs (like priority queue)

Allows "look-up" (find) operation

Allows insertion/removal of elements

May be *unordered* or *ordered*

Dictionary Keys

Must support equality operator

- For ordered dictionary, also support comparitor operator
  - —useful for finding neighboring elements

Sometimes required to be unique

Dictionary Examples

Natural language dictionary

- word is key
- element contains word, definition, pronunciation, etc.

Web pages

- URL is key
- html or other file is element

Any typical database (e.g. student record)

- has one or more search keys
- each key may require own organizational dictionary

Unordered Dictionary ADT

findElement(k): Return element with key k

insertItem(k,e): Insert element e with key k

removeElement(k): Remove element with key k

Special *sentinel*, NO_SUCH_KEY returned when no element with key is present

Log File

Store key-element pairs in unsorted sequence

Always insert using insertLast( )

- $O(1)$ time

findElement( ) by traversing entire list

- $O(n)$ time

Good when inserts are common and finds are rare (e.g. archiving data records)

- number of searches = $O(1) \rightarrow O(n)$ total time
- number of searches = $O(n) \rightarrow O(n^2)$ total time

Hash Table

Provides efficient implementation of unordered dictionary

- Insert, remove, and find all $O(1)$ expected time

Bucket array

- Provides storage for elements

Hash function

- Maps keys to buckets (ranks)
- For each operation, evaluate hash function to find location of item

Bucket Array

Each array element holds 1 or more dictionary elements

Capacity is number of array elements

Load is percent of capacity used

- N is capacity of hash table
- n is size of dictionary
- n/N is load of hash table

Collision is mapping of multiple dictionary elements to the same array element

Simplest Hash Table

Keys are unique integers in range $[0, N\text{-}1]$

Trivial hash function

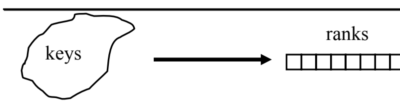
- $h(k) = k$

Uses $O(N)$ space (can be very large)

- okay if $N = O(n)$
- bad if key can be any 32-bit integer
  - —table has 2^{32} entries = 4 gigaentries

find( ), insert( ), and remove( ) all take $O(1)$ time

Hash Function



Maps each key to an array rank

- $h(k)$: K $\rightarrow$ R
- array rank is integer in $[0, N\text{-}1]$

Decomposed into two parts

- *hash code* generation
  - —converts key to an integer
- *compression map*
  - —converts integer hash code to valid rank
- $h(k) = cm(\ hc(\ k\ )\ )$

"Good" hash function

Want to "spread out" values to avoid collisions

Ideally, keys act as random distribution of ranks

- Probability($h(k) = i$) = $1/N$ for all i in $[0, N\text{-}1]$
- Expected keys in bucket i is n/N
  - —this is $O(1)$ if $n = O(N)$

If no collision, operations are $O(1)$

- so *expected* time is $O(1)$ for all operations

Note: worst case time is still $O(n)$

Generating Hash Codes: Java's Object.hashCode()

generates integer for any object

generates same integer for two objects as long as equals( ) method evaluates to true

- different instances with same value are not equal according to Object.equals( )
  - —won't always give expected hashing behavior

exact method is implementation dependent

Generating Hash Codes: Cast to Integer

Works well if key is byte, short, or char type
- can use Float.floatToIntBits() for floats

Disadvantages
- High order bits ignored for longs/doubles
  - —May result in collisions
- Cannot handle more complex keys

Generating Hash Codes: Summing Components

Add up multiple integers to get a single integer
- Ignore overflows
- $hc(x_0, x_1, x_2, ..., x_{k-1}) = \sum_{i=0}^{k-1} x_i$

Examples
- Long or double may be converted to two ints (high order and low order) and summed
- Strings may be broken into multiple characters and summed

Disadvantage
- Ordering of integers is ignored
  - —May result in collisions

Generating Hash Codes: Polynomial Hash Codes

Multiply each component by some constant to a power
- $hc(x_0, x_1, x_2, ..., x_{k-1}) = \sum_{i=0}^{k-1} a^i x_i$

  $= x_0 + a(x_1 + a(x_2 + ...x_{k-1}))...)$
- Makes hash code dependent on order of components

Disadvantages
- k-1 multiplies in hash evaluation
- Choice of *a* makes big difference in “goodness” of hash function

Generating Hash Codes: Cyclic Shift

Cyclic Shift Hash Codes
- Rotates bits of current code by some number of positions before adding each new component
- $hc(x_0, x_1, x_2, ..., x_{k-1}) =$ $\text{rotate}(x_{k-1} + \text{rotate}(x_{k-2} + ...(x_1 + \text{rotate}(x_0))...))$
- no multiplication
  - —only addition and bitwise shifts and ORs

Disadvantages
- Choice of rotation size still makes big difference in “goodness”

Compression Maps

Division Method
- $h(k) = |k| \bmod N$
- N works best if it is a prime number
- Even then, multiples of N map to same position
  - —$h(iN) = 0$, $h(iN+j) = j \bmod N$

MAD (multiply, add, and divide) Method
- $h(k) = |ak+b| \bmod N$
  - —$h(iN) = |aiN + b| \bmod N = b \bmod N$
  - —$h(iN+j) = |aiN + aj + b| \bmod N$ $= |aj + b| \bmod N$
- Not clear that this is much better...

Collision Handling: Chaining

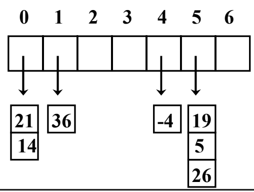
For each bucket, store a sequence of elements that map to the bucket
- effectively a much smaller, auxiliary dictionary

Linearly search sequence to find correct element

Chaining Example

$N = 7, \quad h(k) = |k| \bmod N$

Insert 19 36 5 21 -4 26 14 **(load = 1)**



Collision Handling: Open Addressing

Store only 1 element per bucket
- **No addition space, but requires smaller load**

If multiple elements map to same bucket, use some method to find empty bucket
- **Linear probing**
  - —$h'(k) = (\, h(k) + j\,) \bmod N \quad j = 0, 1, 2, 3, \ldots$
    - »Keep adding 1 to rank to find empty bucket
- **Quadratic probing**
  - —$h'(k) = (\, h(k) + j^2\,) \bmod N \quad j = 0, 1, 2, 3, \ldots$
- **Double hashing**
  - —$h'(k) = (\, h(k) + j*h''(k)\,) \bmod N \quad j = 0, 1, 2, 3, \ldots$

Linear Probing Example

$N = 7, \quad h(k) = |k| \bmod N$

Insert 19 36 5 21 -4 26 14 **(load = 1)**

0	1	2	3	4	5	6
21	36	26	14	-4	19	5

Other Open Addressing Difficulties

Searching
- **For NO_SUCH_KEY, must search until empty bucket found**

Removing
- **Cannot just empty the bucket**
  - —could disconnect colliding keys
- **Easiest method is setting with special DELETED_KEY sentinal**
  - —insert( ) can reuse bucket
  - —find( ) must continue searching beyond bucket

Rehashing

When load of hash table gets too large
- **Allocate new hash table**
- **Generate new hash function**
- **Re-hash old elements into new table**
- **Time cost may be amortized as in dynamic array**
  - —must increase size by $O(n)$ each time

Ordered Dictionary ADT

Unordered Dictionary ADT *plus*:
- **closestKeyBefore(*k*): returns key preceding k**
- **closestElemBefore(*k*): returns element preceding k**
- **closestKeyAfter(*k*): returns key following k**
- **closestElemAfter(*k*): returns element following**

Ordered Dictionaries

Simplest type is "lookup table"

- Store elements in sorted vector
- Insert takes O(n)
- Find takes O(logn)
  - —use *binary search*

Skip Lists

Based on stacked set of linked lists (a hierarchy of lists)

- $\{S_0, S_1, ..., S_h\}$: *h* is *height* of skip list
- S_0 is entire dictionary
- S_i contains a subset of S_{i-1}
  - —Each element of S_i is 50% likely to appear in S_{i+1}

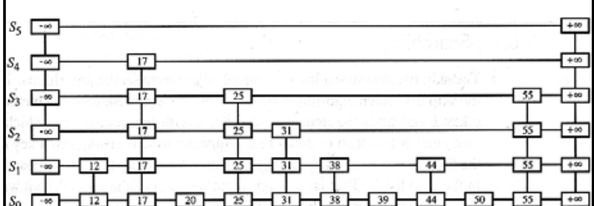
Provides *expected* bounds of *O*(log*n*) for find, insert, and remove

- with "high probability" - uses *randomization*

Example Skip List

Each skip list/row is a *level*
Each column is a *tower*

- links connect elements within level or tower



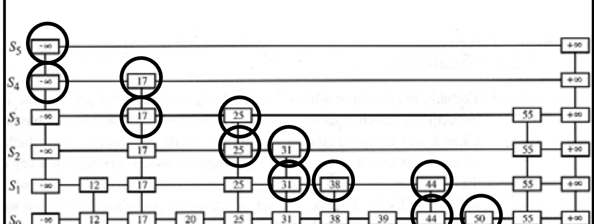
Searching (findElem)

Begin at left end of highest level

1. Scan forward while key ≤ search key
2. If level > 0, drop down to next level. Goto 1.

Search Example

findElement(50)



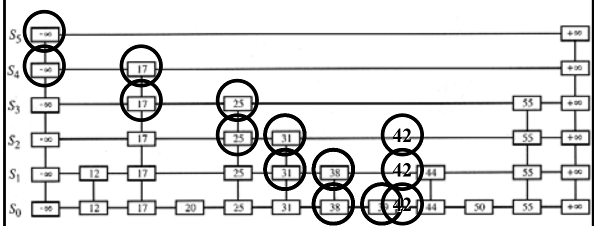
Insertion

1. Find insert position in level 0 using search and do insert
2. Flip coin. If heads, insert in level *i*+1, and repeat 2 until tails

Insert Example

insertItem(42, elem)

- **random( ) returns: H, H, T, . . .**

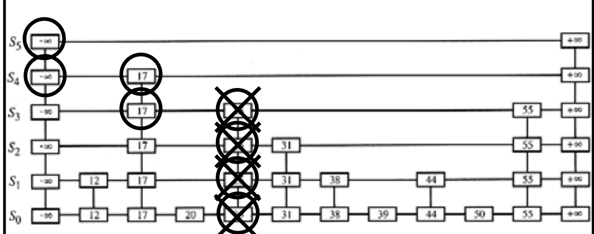


Removal

1. **Find element in level 0 using search**
2. **Remove from level 0 and follow tower to remove from all levels**

Remove Example

removeElement(25)



Basic Analysis - height (drop down)

Probability that given item appears in level i

$1/2^i$: $1/2 * 1/2 * ... * 1/2$

Probability that level i has at least 1 element

$P_i \leq n * 1/2^i = n/2^i$

$P_{\log n} \leq n/2^{\log n} = 1$

$P_{3\log n} \leq n/2^{3\log n} = n/2^{\log n^3} = n/n^3 = 1/n^2$

So height is < 3log$n$ with high probability

- **Expected height is $O(\log n)$**

Basic Analysis - scan forward

Imagine scan in reverse direction

Each element scanned has 1/2 chance of having an element in tower above it

***Expected* number of elements scanned before going up tower is 2 = $O(1)$**

Search Analysis

Number of drop down steps is $O(\log n)$

Number of scan forward steps is $O(\log n)$

Total expected search time is $O(\log n)$

Same applies to insert and remove

Worst case? **O(h + n)**

In Class Example

Work in groups of 2-3

Assume calls to random( ) return:

H T T H   H T T H . . .

Create skip list with these inserts:

10   15   12   5   20   17   25

What is maximum height for any sequence of inserts? Why?

What is expected search time for this random( ) distribution? Why?