Blockchains from Proofs of Space and Time: from Spacemint to Chia

Krzysztof Pietrzak

Guest Lecture, Blockchains and Cryptocurrencies (Spring 2018)

Outline

- Bitcoin and Proofs of Work
- Proofs of Stake
- Proofs of Space
- Proofs of Sequential Work
- Putting it all together (Chia)

Mining Bitcoin (Proofs of Work)

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Ecological: Massive energy & hardware waste.

Economical: Requires high rewards $\Rightarrow$ inflation and/or high transaction fees.

Can we have a more “sustainable” Blockchain?

Alternative Proof Systems: Proof of Stake

PoW based blockchain (Bitcoin): Probability a miner can add a block proportional to its hashing power.

Proof of Stake: Probability proportional to the fraction of coins the miner owns.

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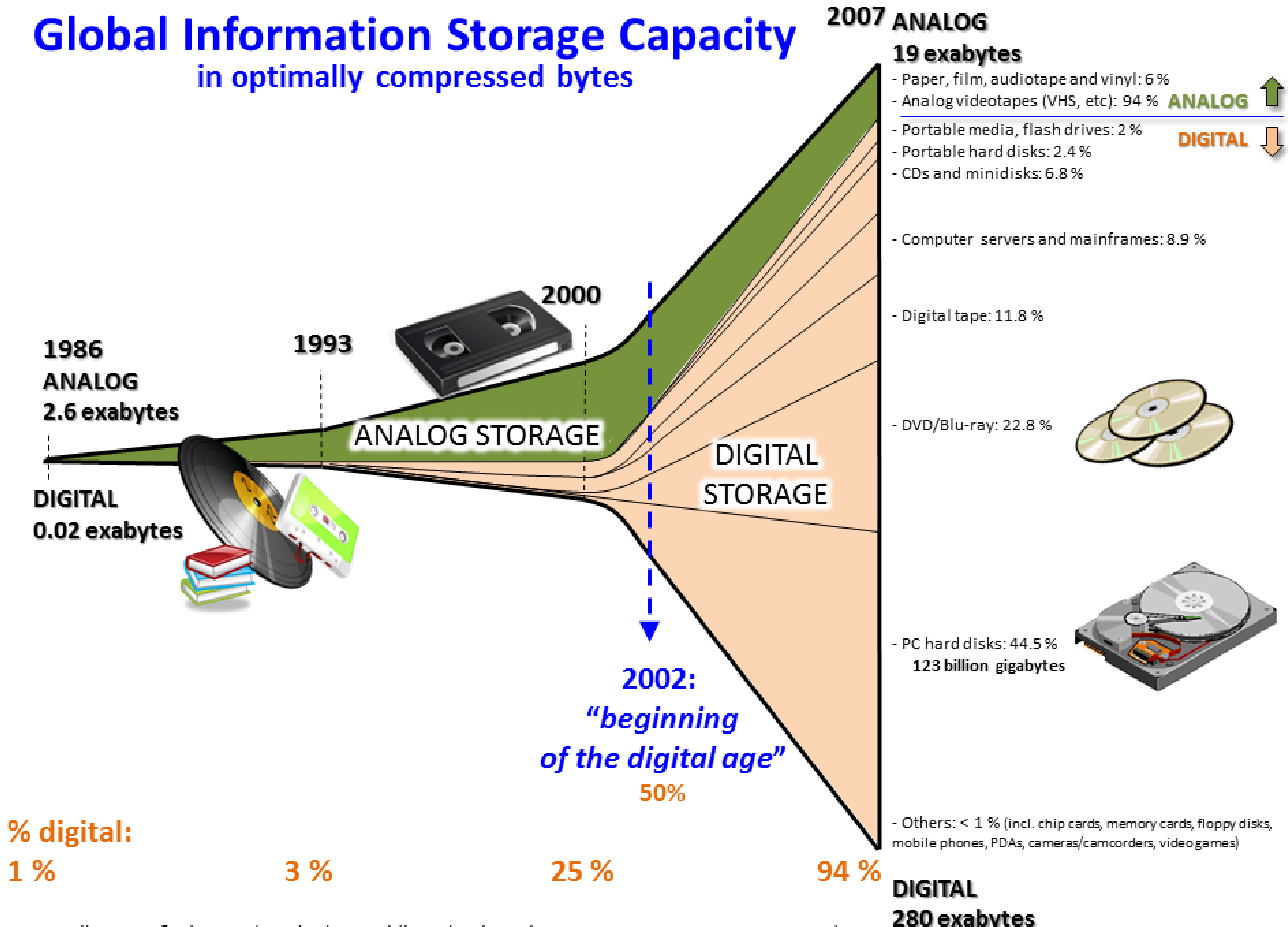
PoW based blockchain (Bitcoin): Probability a miner can add a block proportional to its hashing power.

Proof of Stake: Probability proportional to the fraction of coins the miner owns.

Nxt, Algorand, Snow White, Ouroboros,...

Global Information Storage Capacity
in optimally compressed bytes



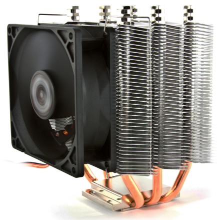
Source: Hilbert, M., & López, P. (2011). The World's Technological Capacity to Store, Communicate, and Compute Information. *Science*, 332(6025), 60 –65. http://www.martinhilbert.net/WorldInfoCapacity.html

First Ingredient

Proofs of Space

Proofs of Space

Dziembowski-Faust-Kolmogorov-Pietrzak 2015



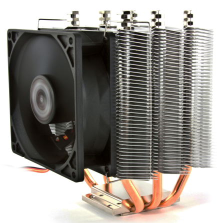
Parameter N

$\mathcal{P}$

$\mathcal{V}$

Proofs of Space
Dziembowski-Faust-Kolmogorov-Pietrzak 2015

N



Parameter N

init

$\mathcal{P}$

$\mathcal{V}$

communication $\tilde{O}(1)$

Proofs of Space

Dziembowski-Faust-Kolmogorov-Pietrzak 2015

Parameter N

N

$\mathcal{P}$

$\mathcal{V}$

communication $\tilde{O}(1)$

Proofs of Space

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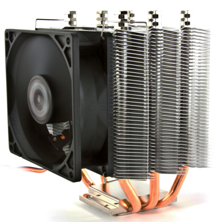


$\tilde{O}(1)$

$\mathcal{P}$

challenge $\longleftarrow$

response $\longrightarrow$

communication $\tilde{O}(1)$

$\mathcal{V}$

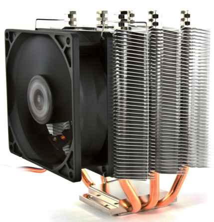
$\tilde{O}(1)$

Proofs of Space

Dziembowski-Faust-Kolmogorov-Pietrzak 2015

Parameter N

N



$\tilde{O}(1)$

$\mathcal{P}$

challenge

response

$\mathcal{V}$

communication $\tilde{O}(1)$

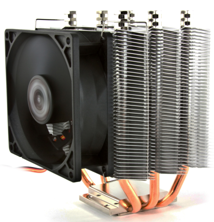
$\tilde{O}(1)$

Proofs of Space

Dziembowski-Faust-Kolmogorov-Pietrzak 2015

Parameter N

S



T

$\mathcal{P}$

challenge

response

$\mathcal{V}$

communication $\tilde{O}(1)$

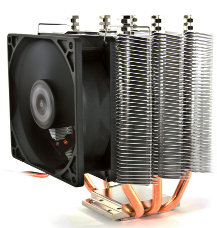
$\tilde{O}(1)$

Proofs of Space

Dziembowski-Faust-Kolmogorov-Pietrzak 2015

Parameter N

S



T

$\mathcal{P}$

challenge

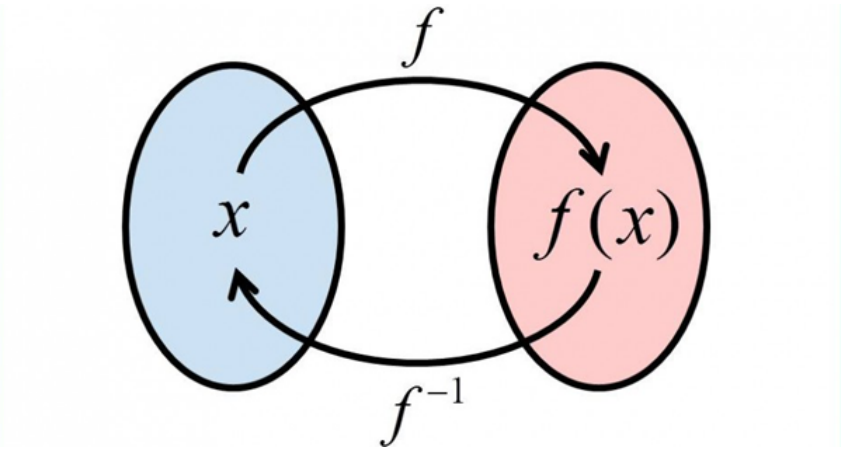
response

$\mathcal{V}$

communication $\tilde{O}(1)$ $\tilde{O}(1)$

Security: either $S \approx N$ space before exec or $T \approx N$ time in exec

Two Types of Proofs of Space



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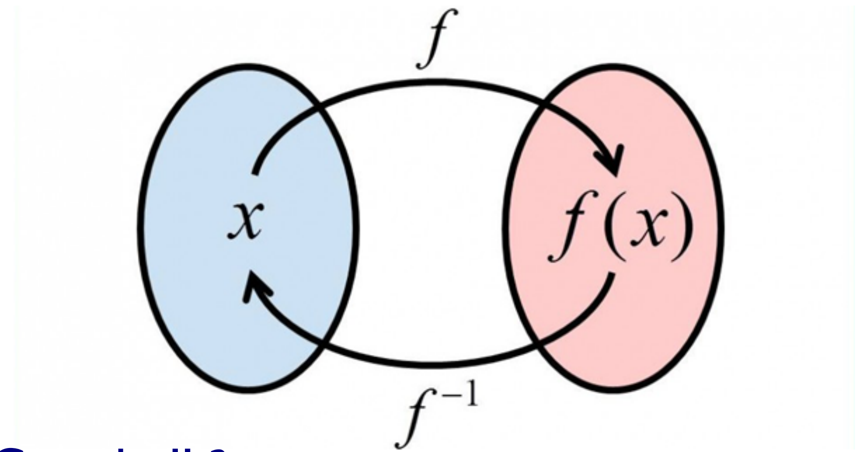

Constructions from "Hard to Pebble Graphs"[a]

- Optimal bounds: either $\Theta(N)$ space or $\Theta(N)$ time
- Non-Interactive Initialization Phase, Complicated

[a] Stefan Dziembowski, Sebastian Faust, Vladimir Kolmogorov, Krzysztof Pietrzak: Proofs of Space. CRYPTO 2015

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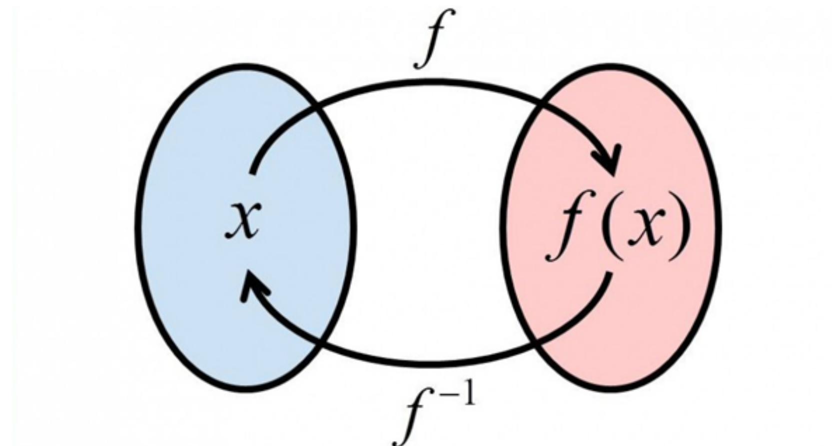

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Inverting Random Functions[a]

- Bounds (only) asymptotically optimal: $T \cdot S^k \geq N^k$ for "small" k, e.g. $S = T = N^{k/(1+k)}$ (proof size exponential in k)
- Non-Interactive Initialization Phase, Simple!

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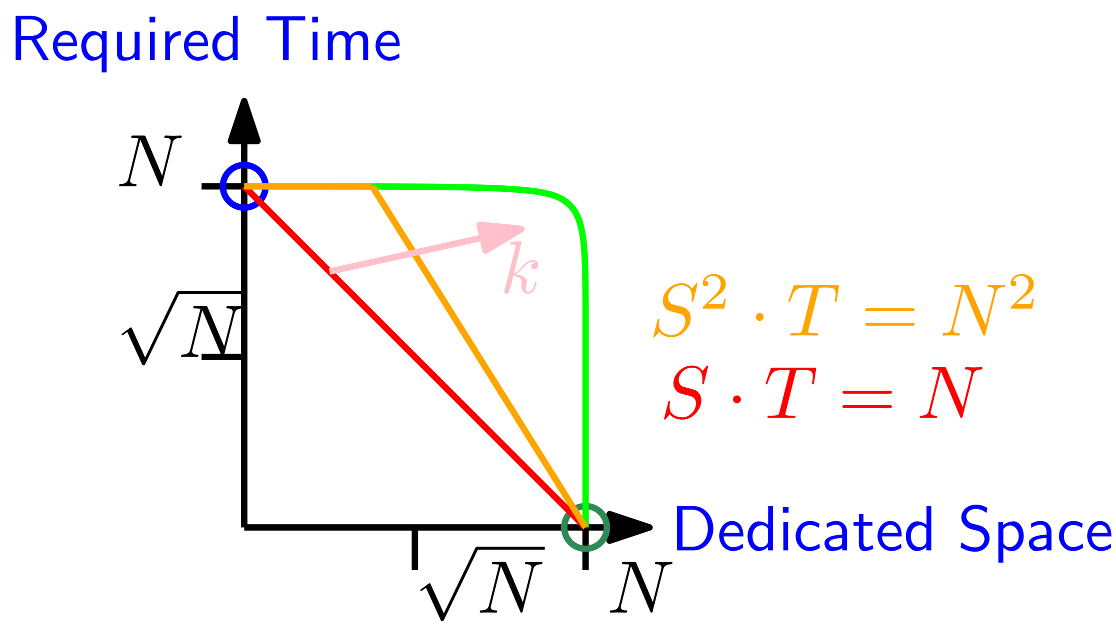

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Spacemint***:**
A Cryptocurrency Based on Proofs of Space

Sunoo Park*, Krzysztof Pietrzak†, Albert Kwon*, Joël Alwen†, Georg Fuchsbauer†, and Peter Gaži†

* MIT
† IST Austria

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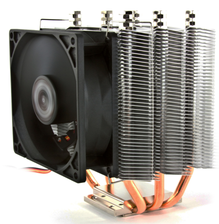
Proofs of Space From Hard to Invert Functions

Beyond Hellman's Time-Memory Trade-Offs with Applications to Proofs of Space

Hamza Abusalah[1], Joël Alwen[1], Bram Cohen[2], Danylo Khilko[3], Krzysztof Pietrzak[1], and Leonid Reyzin[4]

Towards a Simple Construction



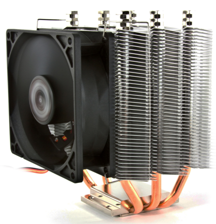
$\mathcal{P}$ $\xleftarrow{\;L\;}$ $\mathcal{V}$

Random Table L

$1, \quad y_1$
$2, \quad y_2$
$\dots$
$\dots$
$N, \quad y_N$

Towards a Simple Construction

N



$$\mathcal{P} \xleftarrow{\;L\;} \mathcal{V}$$

Random Table L

$1,$	y_1
$2,$	y_2
$\dots$	
$\dots$	
$N,$	y_N

$1,$	y_1
$2,$	y_2
$\dots$	
$\dots$	
$N,$	y_N

Towards a Simple Construction

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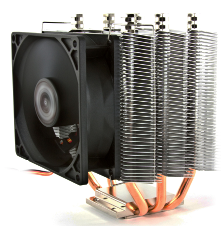

$\mathcal{P}$

$\mathcal{V}$

$1, \quad y_1$
$2, \quad y_2$
$\cdots$ $\cdots$
$N, \quad y_N$

Random Table L

$1, \quad y_1$
$2, \quad y_2$
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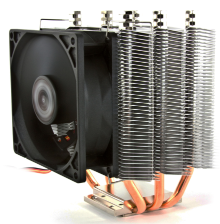

$\mathcal{P}$

$i \leftarrow [N]$

$\mathcal{V}$

Random Table L

$1, \quad y_1$
$2, \quad y_2$
$\dots$
$\dots$
$N, \quad y_N$

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$\dots$
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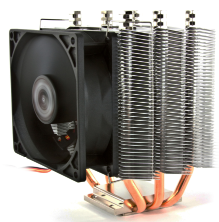


lookup

$1, \quad y_1$
$2, \quad y_2$
$\dots$
$\dots$
$N, \quad y_N$

$\mathcal{P}$

$i \leftarrow [N]$

y_i

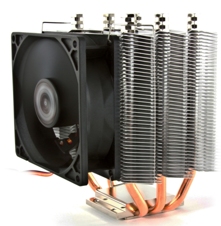
$\mathcal{V}$

Random Table L

$1, \quad y_1$
$2, \quad y_2$
$\dots$
$\dots$
$N, \quad y_N$

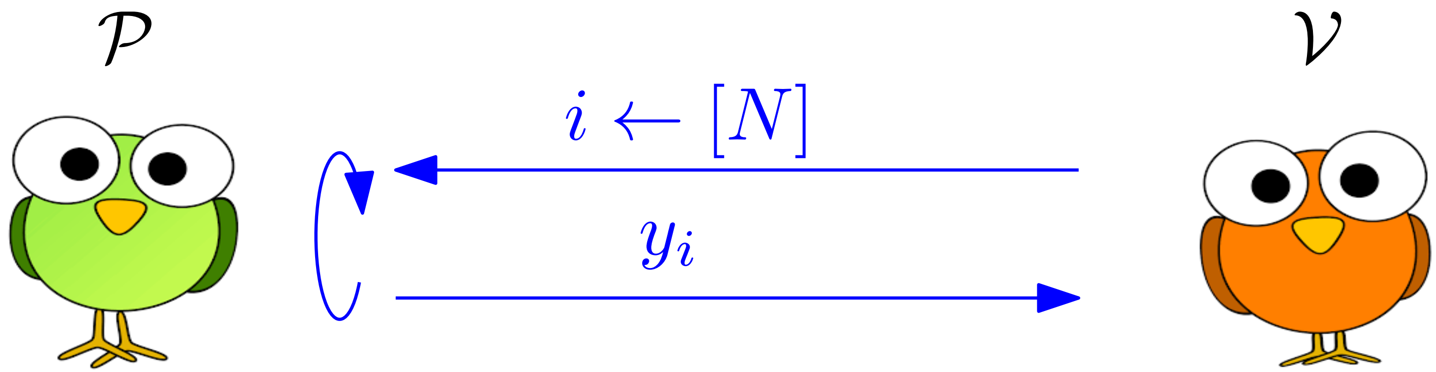
Towards a Simple Construction

N



lookup

$1,$	y_1
$2,$	y_2
$\ldots$	
$\ldots$	
$N,$	y_N



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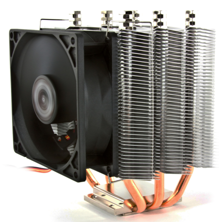

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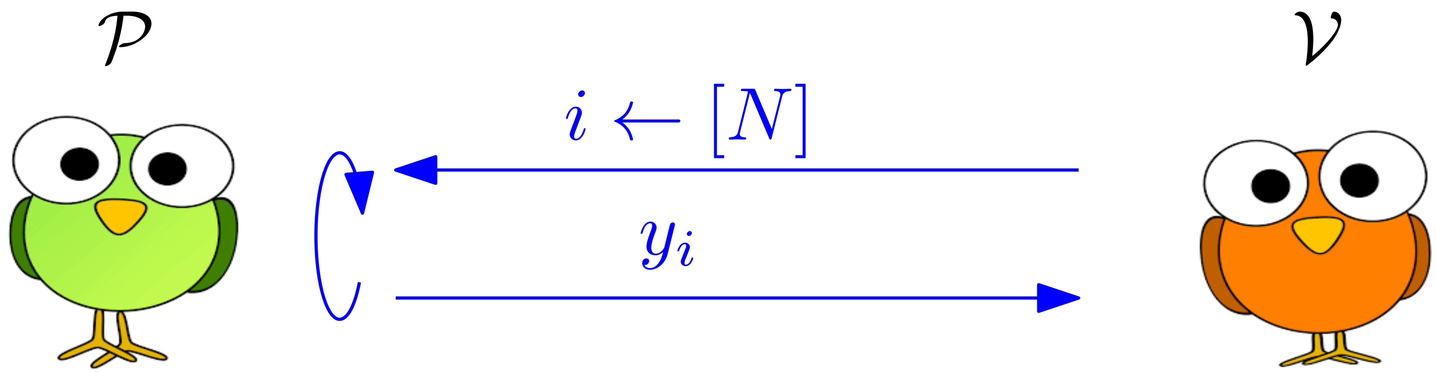


Random Table L

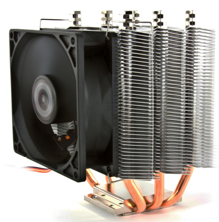
$1, \quad y_1$
$2, \quad y_2$
$\ldots$
$\ldots$
$N, \quad y_N$

Problems:

1. large communication
2. space-inefficient $\mathcal{V}$

Towards a Simple Construction

N



$\mathcal{P}$

$\xleftarrow{\pi}$

$\mathcal{V}$

randomly pick

$\pi : [N] \to [N]$

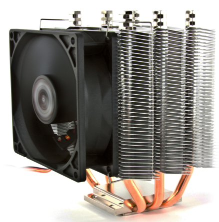
$1, \pi(1)$
$2, \pi(2)$
$\dots$
$\dots$
$N, \pi(N)$

Towards a Simple Construction

N

$\mathcal{P}$

$\mathcal{V}$



randomly pick

$\pi : [N] \to [N]$

$1, \pi(1)$
$2, \pi(2)$
$\cdots$
$\cdots$
$N, \pi(N)$

Towards a Simple Construction

N

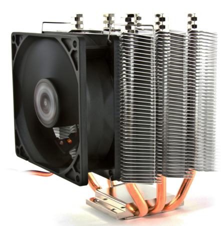
$\mathcal{P}$

$y \leftarrow [N]$

$\mathcal{V}$

randomly pick

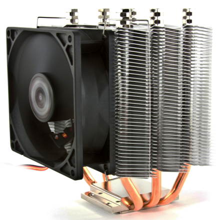
$\pi : [N] \to [N]$



$1,\pi(1)$
$2,\pi(2)$
$\cdots$
$\cdots$
$N,\pi(N)$

Towards a Simple Construction

N



lookup

$1,\pi(1)$
$2,\pi(2)$
$\dots$ $\dots$
$N,\pi(N)$

$\mathcal{P}$

$y \leftarrow [N]$

$\pi^{-1}(y)$

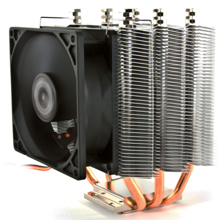
$\mathcal{V}$

randomly pick

$\pi : [N] \to [N]$

Towards a Simple Construction

N



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$\mathcal{P}$

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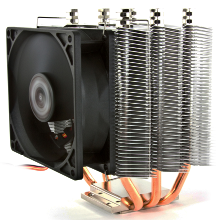
$\mathcal{V}$

randomly pick

$\pi : [N] \to [N]$

Towards a Simple Construction

$\sqrt{N}$



$\sqrt{N}$

$\mathcal{P}$

$y \leftarrow [N]$

$\pi^{-1}(y)$

$\mathcal{V}$

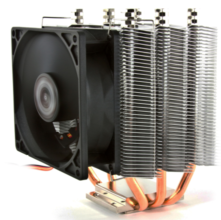
randomly pick

Problem: Hellman 1980

$\pi : [N] \rightarrow [N]$

Towards a Simple Construction

$\sqrt{N}$



$\sqrt{N}$

$\mathcal{P}$

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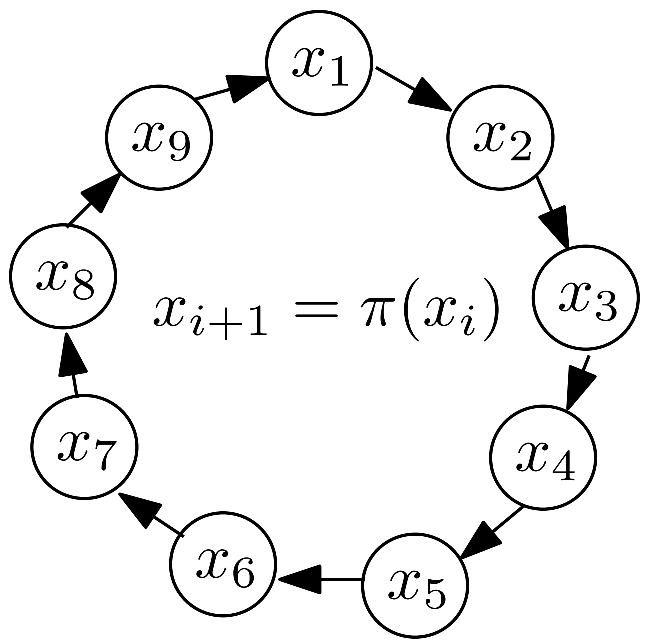

Towards a Simple Construction

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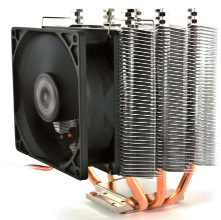

$\sqrt{N}$

$\mathcal{P}$

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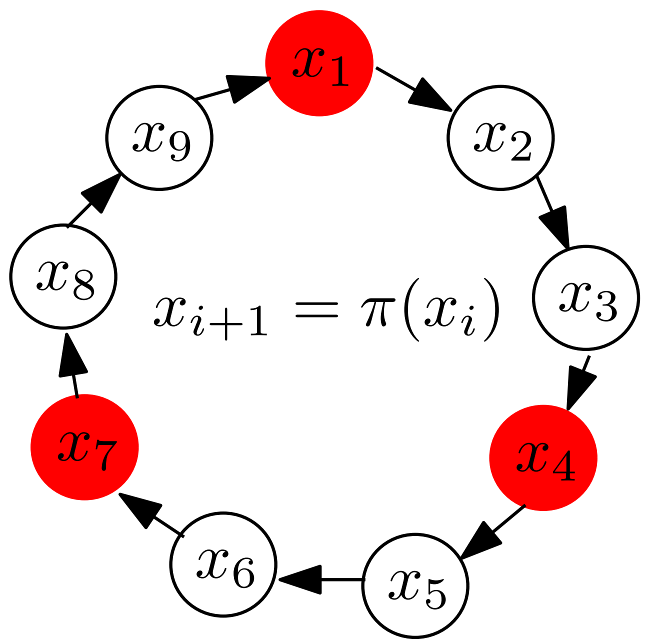
$\pi^{-1}(y)$

$\mathcal{V}$

randomly pick

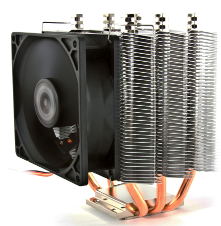
$\pi : [N] \to [N]$

Problem: Hellman 1980



Towards a Simple Construction

$\color{red}{\sqrt{N}}$



$\color{blue}{\sqrt{N}}$

$\mathcal{P}$

$\mathcal{V}$

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randomly pick

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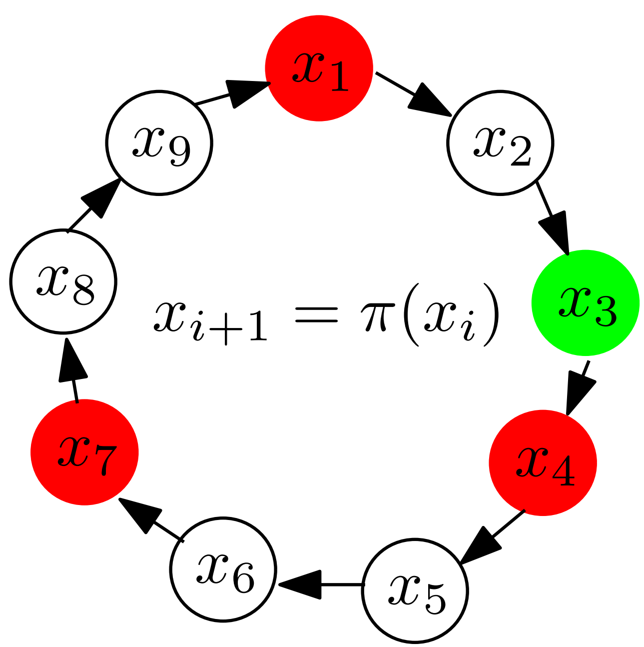
Problem: Hellman 1980



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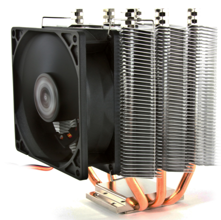

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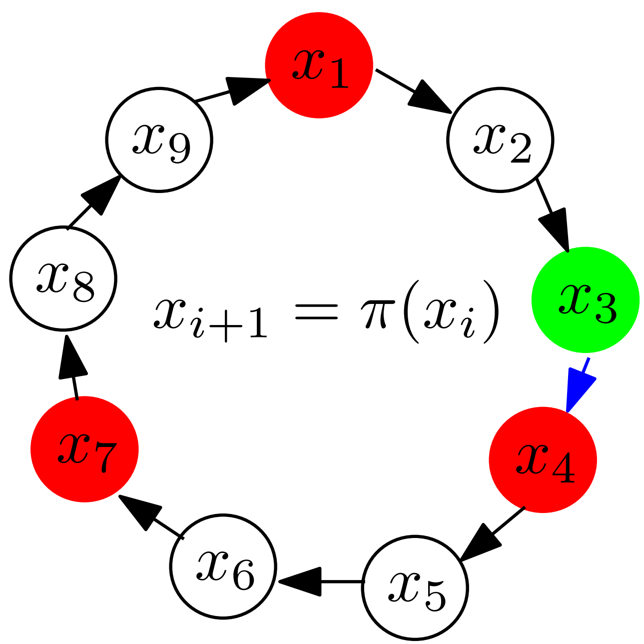

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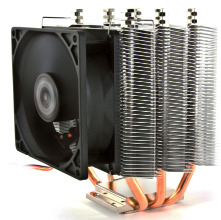

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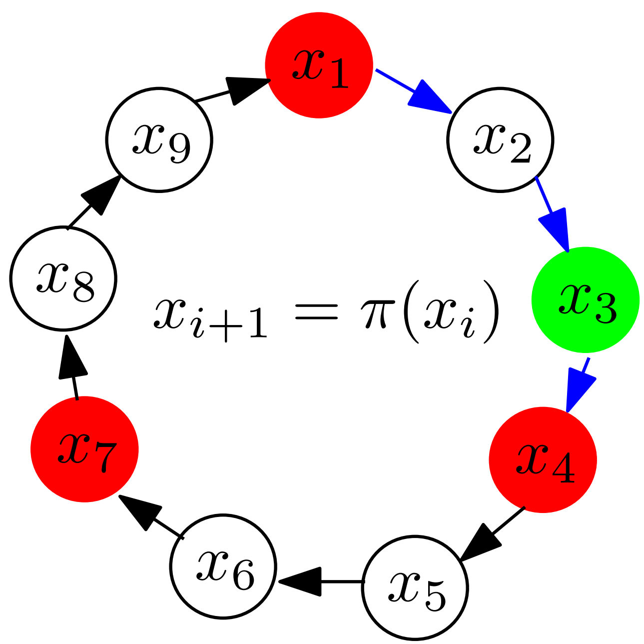

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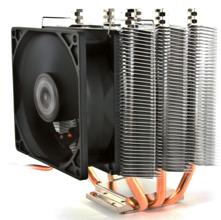

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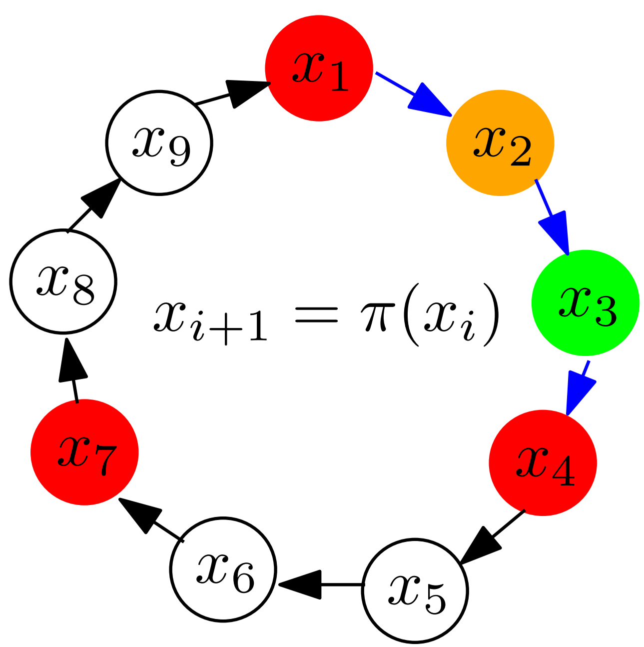

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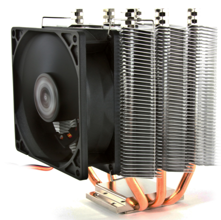
$\sqrt{N}$



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$\mathcal{P}$

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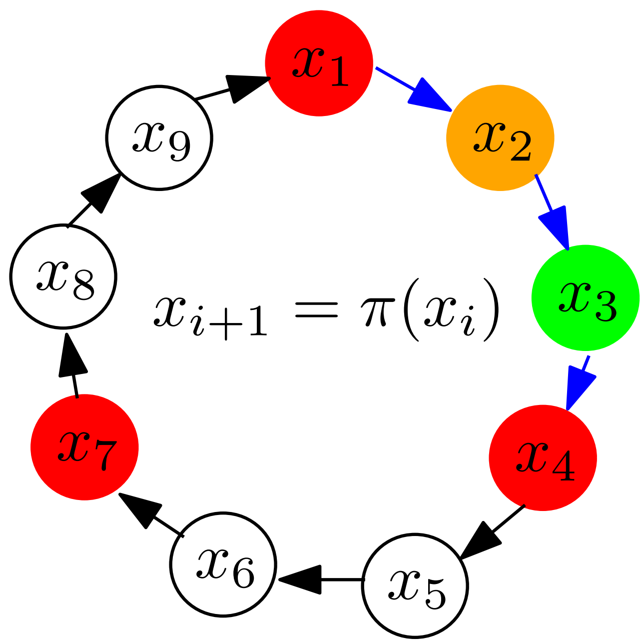
$\pi : [N] \to [N]$

Problem: Hellman 1980

π on $[N] : ST \leq N$



Towards a Simple Construction

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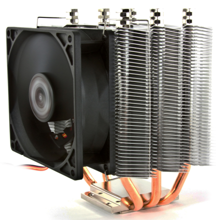

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$\mathcal{P}$

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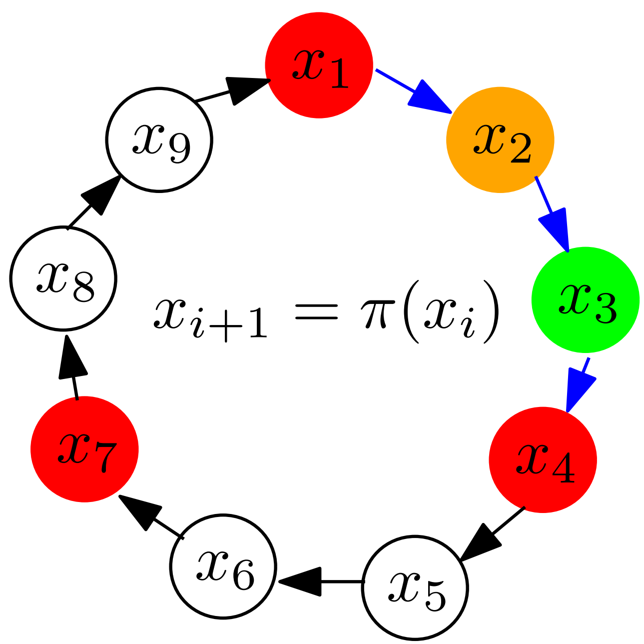
Problem: Hellman 1980

π on $[N]$: $ST \leq N$



Yao 1990: π random: $ST \geq N$

Inverting Functions

$f : [N] \to [N]$

$\mathcal{A}$ inverts f on ϵ fraction with
S bits advice
T oracle queries

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general functions			
our functions			

*: Yao-1990, Gennaro-Trevisan'00, Wee05, De-Trevisan-Tulsiani-2010

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Two Observations

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2) Usefulness for PoS: sufficient that the function table is computable in linear time

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$g : [N] \times [N] \to [N]$

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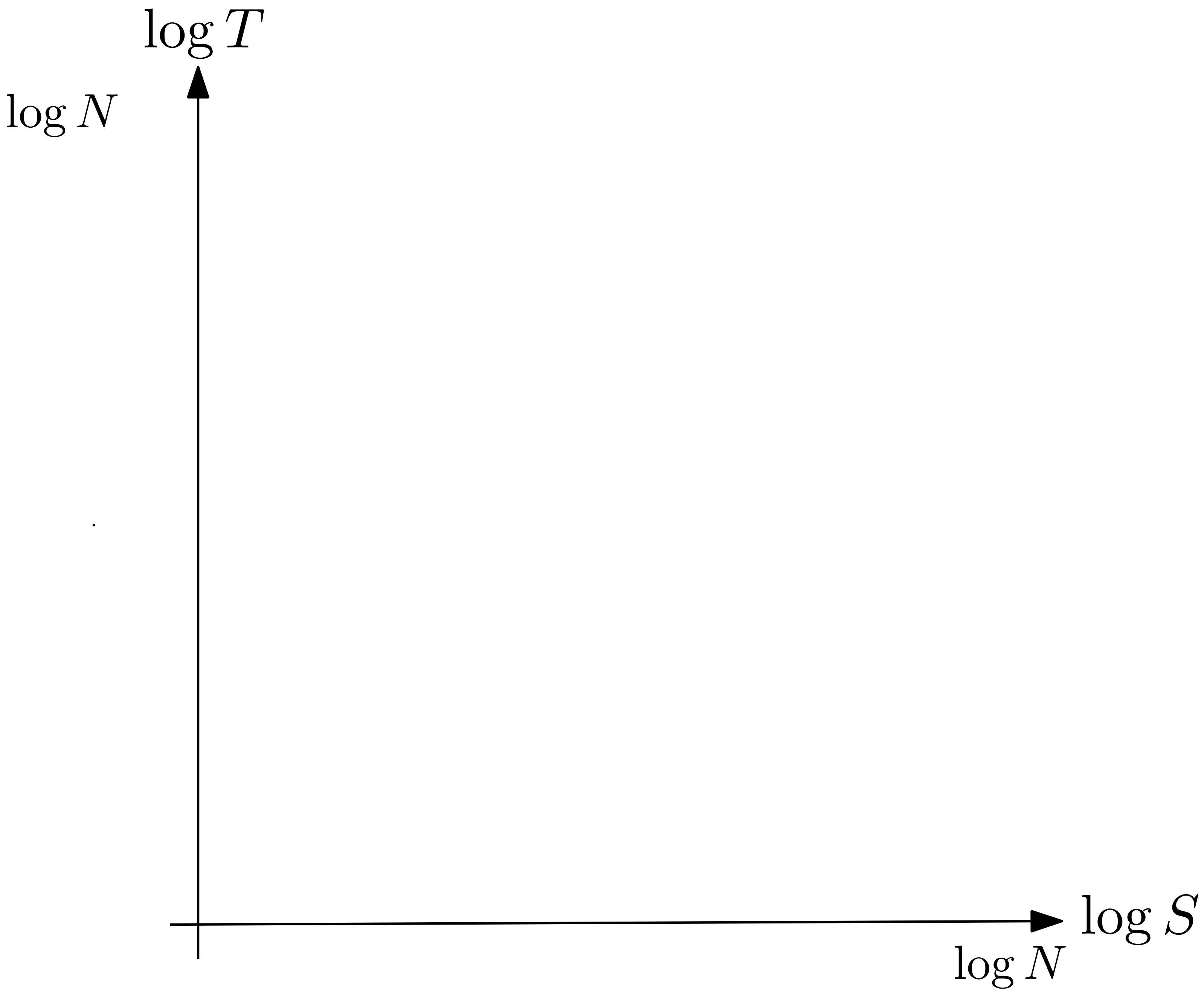
Caveat: holds for $T \leq N^{2/3}$

Function Inversion for PoS



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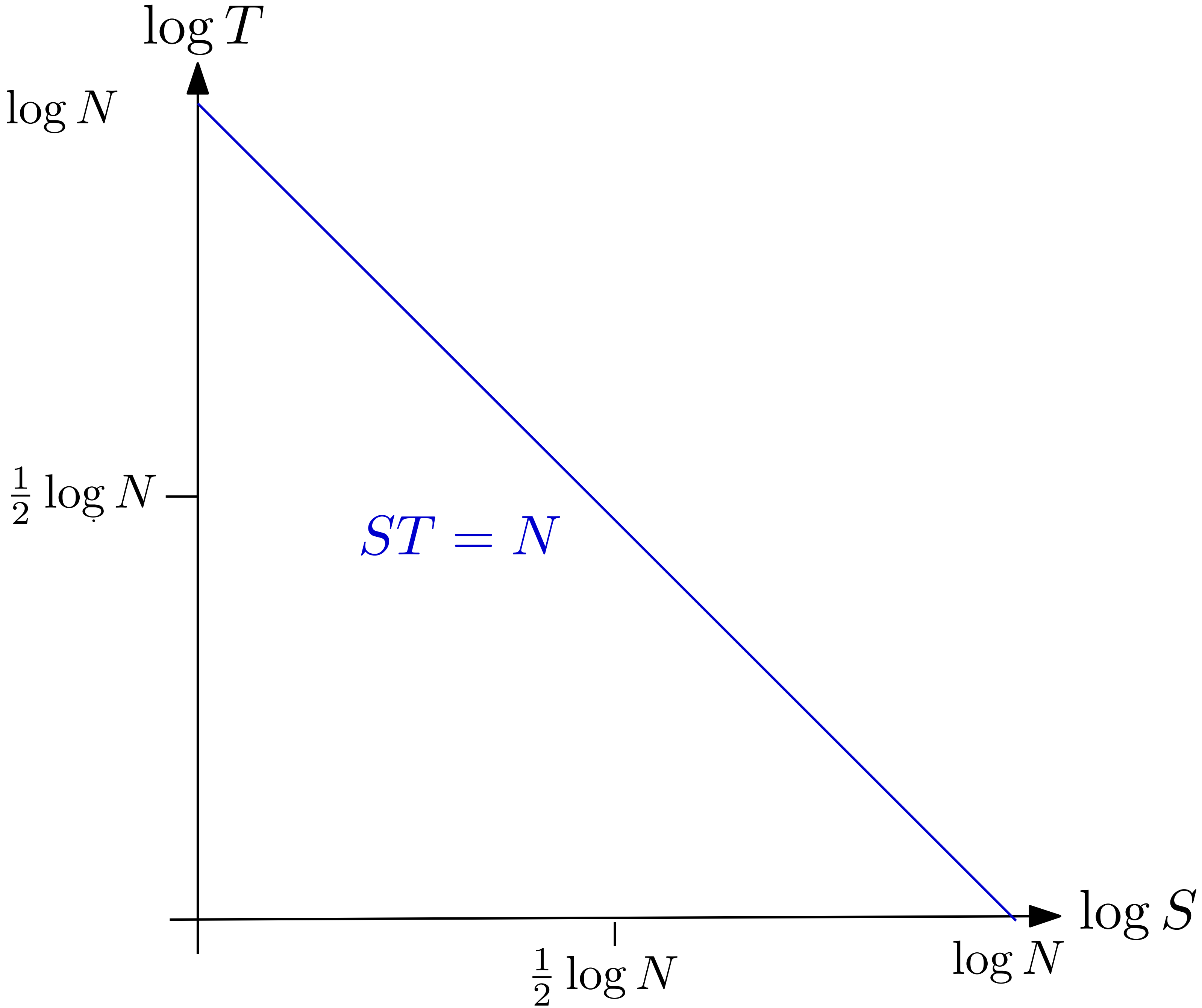

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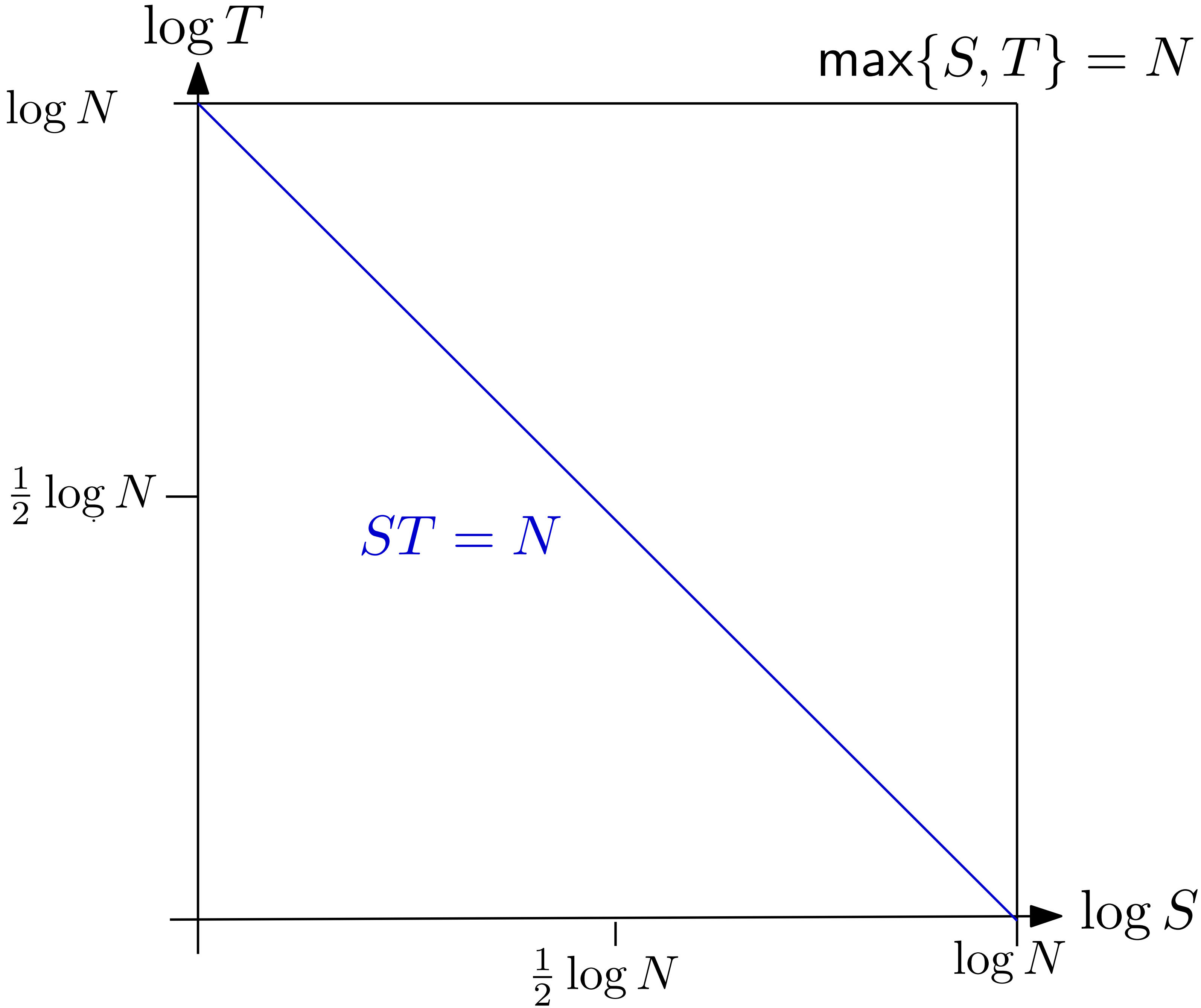

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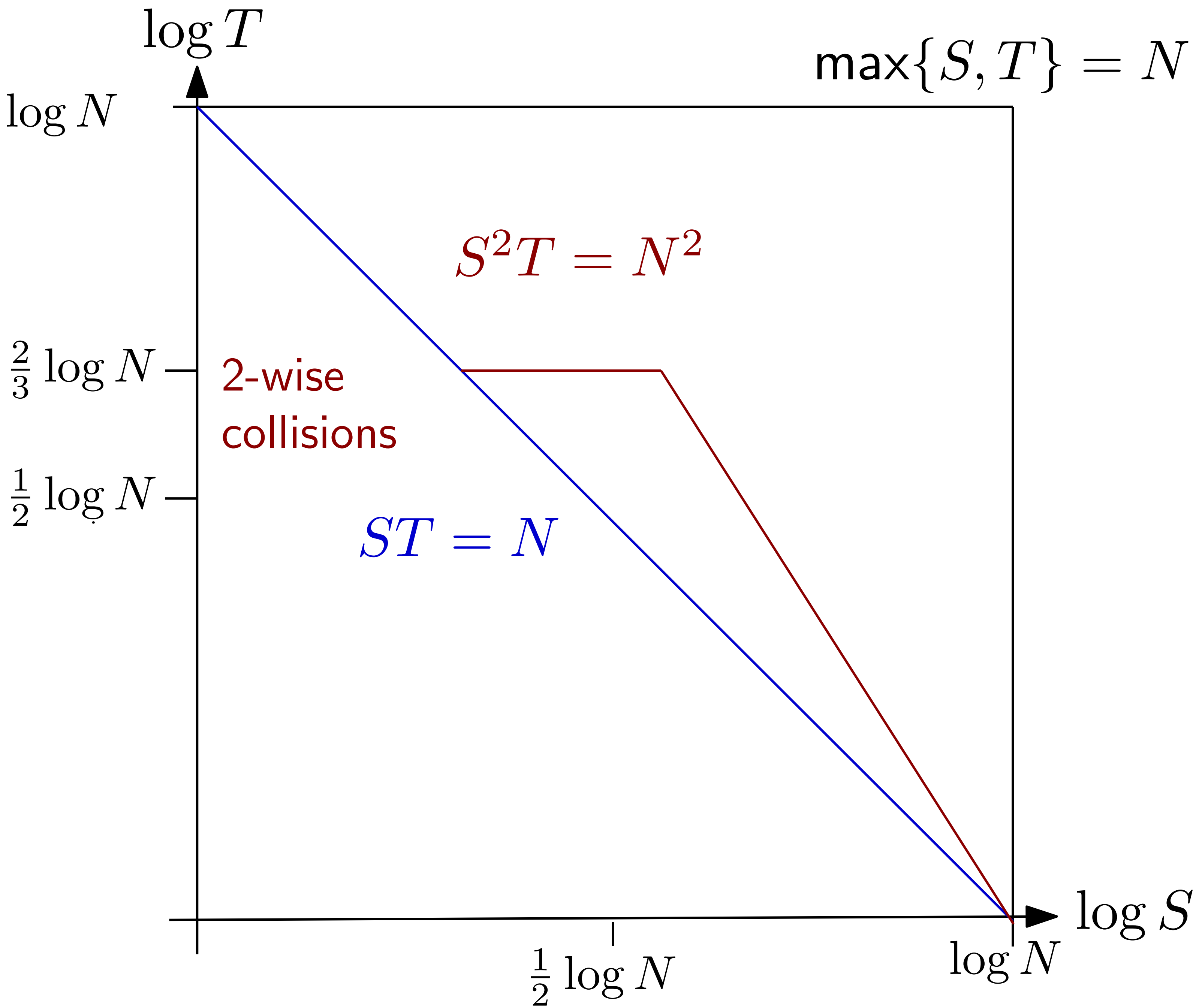

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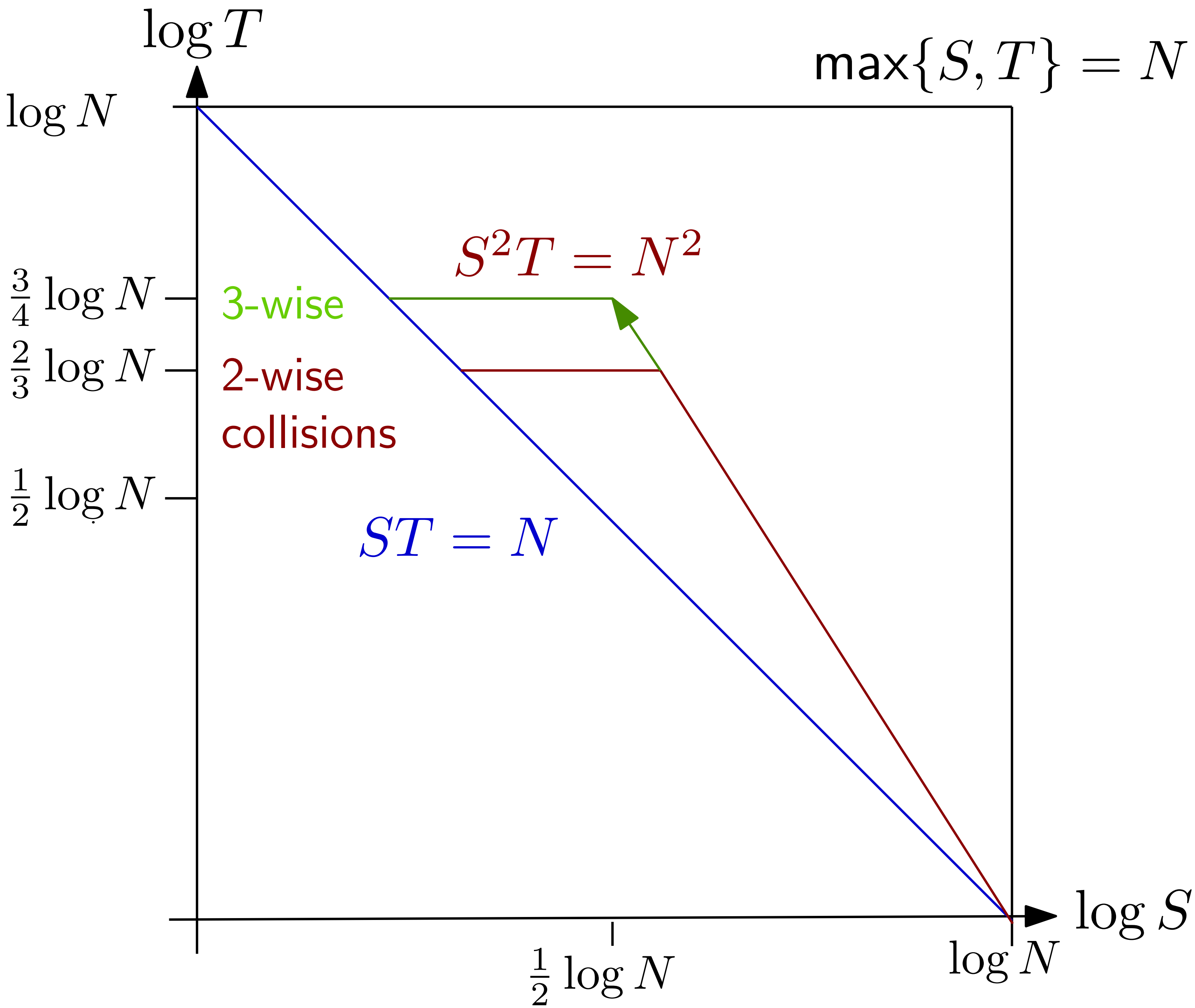

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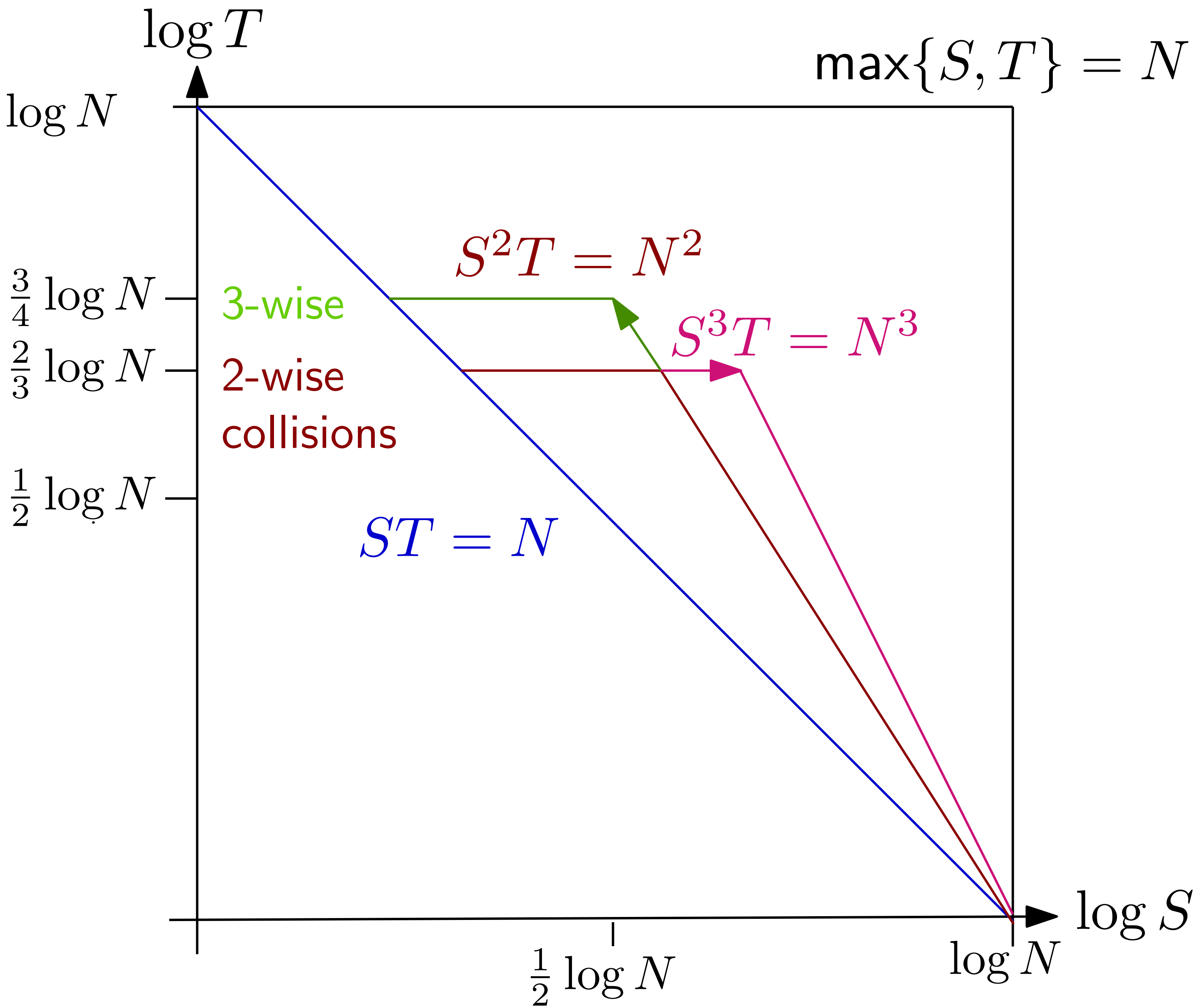

Proof Sketch

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- Compression argument:
  - Use $\mathcal{A}$ (given S bits advice and T queries per challenge) to "compress" f, g by X bits.
  - As random f, g are incompressible $\Rightarrow$ advice S was at least X.

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- $\mathcal{A}(y) \to g_f^{-1}(y)$ makes $\leq T$ queries total, of which T_g to g_f.
- Case 1, $T_g \leq \sqrt{T}$: compress g using $S \cdot T_g \geq N$ [Yao90] $\Rightarrow S^2 \cdot T \geq N^2$.
- Case 2 $T_g > \sqrt{T}$:
  - can use every g_f query (x, x') to compress an f value (as $f(x) = f(x') + 1$) if $f(x)$ is "fresh".
  - During $\approx N/T$ invocations of $\mathcal{A}$ most f "fresh", as every invocation "spoils" $\leq T$ f values.
  - Can compress total of $\frac{N}{T} \cdot T_g \geq \frac{N}{\sqrt{T}}$ values

    $\Rightarrow S \geq N/\sqrt{T} \Rightarrow S^2 \cdot T \geq N^2$.

Second Ingredient

Proofs of Sequential Work

aka Verifiable Delay Algorithm

Time-lock puzzles and timed-release Crypto

Ronald L. Rivest*, Adi Shamir**, and David A. Wagner***

Revised March 10, 1996

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Puzzle: given $(N = p \cdot q, x \in Z_N^*, T \in \mathbb{N})$ find $x^{2^T} \mod N$

requires T sequential squarings if p, q unknown:

$$x \to x^2 \to x^{2^2} \to \ldots x^{2^T} \mod N$$

Given p, q, compute $\phi(N) := (p-1)(q-1)$

$$m := 2^T \mod \phi(N) \mod N$$

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"Dedicated verifier": if one can efficiently verify

$y \stackrel{?}{=} 2^{2^T} \mod N$ one can also efficiently compute it.

Proofs of **Sequential** Work

- Function τ : challenge $\times$ time parameter $\rightarrow$ proof
- $\tau(c, t)$ can be computed making t sequential queries to some hash function H
- There's an efficient verification algorithm that outputs 1 on input $c, t, \tau(c, t)$.
- For random c, no algorithm making (slightly less than) t parallel queries to H can produce τ that passes verification

compute $\tau(c, t)$
making t
sequential
queries to H

$\mathcal{P}$

random challenge c
time parameter t

$\tau(c, t)$

$\mathcal{V}$

(efficiently)
verify $\tau(c, t)$

Publicly Verifiable Proofs of Sequential Work

Mohammad Mahmoody* Tal Moran† Salil Vadhan‡

February 18, 2013

- Prover needs not just T sequential time, but also T <u>space</u> to compute proof.
- Proof not unique: given a valid proof ϕ, can generate different accepting proofs $\phi' \neq \phi \Rightarrow$ GRINDING

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Simple Proofs of Sequential Work

Bram Cohen[1] and Krzysztof Pietrzak[2*]

[1] Chia Network, bram@chia.network
[2] IST Austria, pietrzak@ist.ac.at

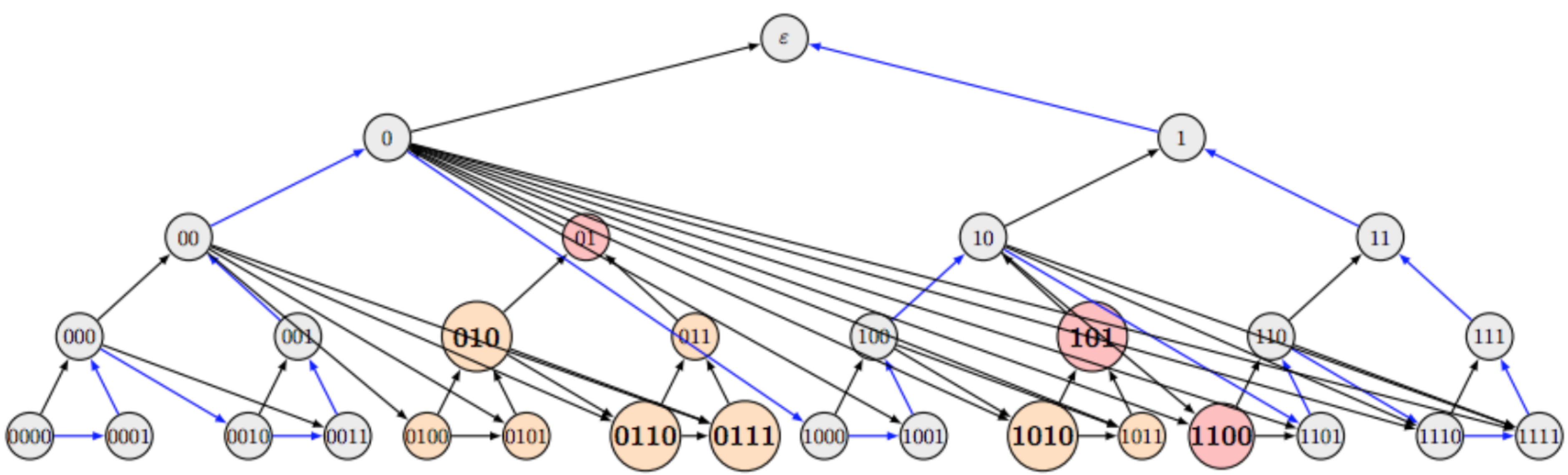
Super simple and efficient, prover just needs $\log(T)$ space, but still not unique!

Construction:

- $\mathcal{V}$ sends χ to $\mathcal{P}$, this defines hash (modelled as RO in proof) $H(.)$.
- $\mathcal{P}$ computes "labels" $\ell_{0000}, \ldots, \ell_{\varepsilon}$ of nodes where

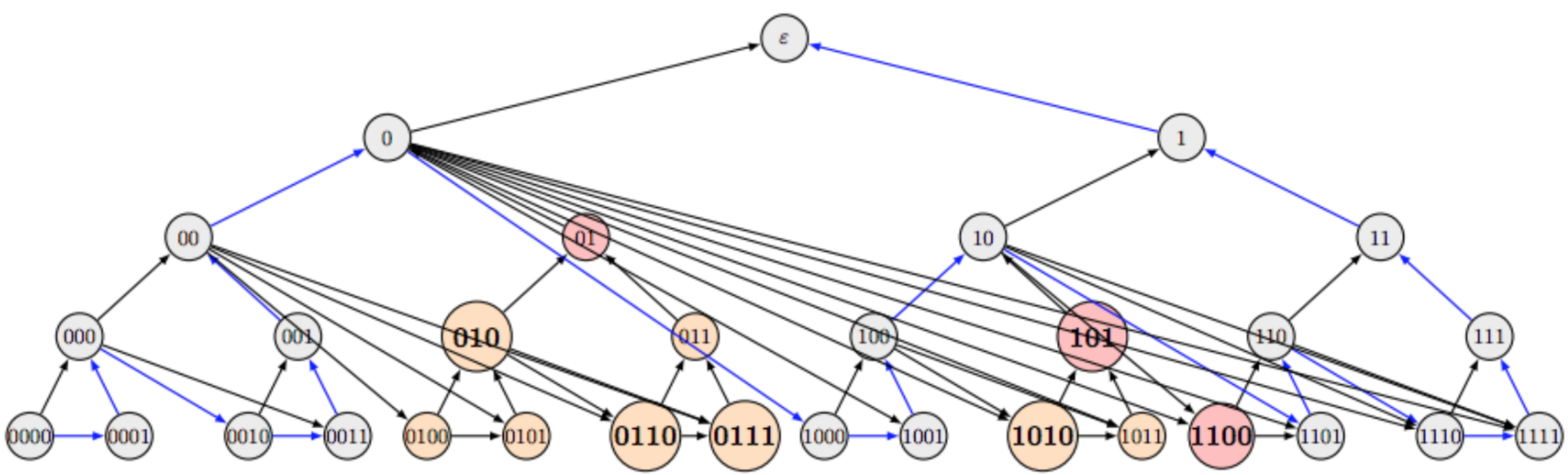
$$\ell_i = H(\ell_{p_1}, \ldots, \ell_{p_d}) \quad , \quad (p_1, \ldots, p_d) = parents(i)$$

  Sends label of root ℓ_{ε} to $\mathcal{V}$ (kinda Merkle-tree commitment to the ℓ_i's).
- $\mathcal{V}$ challenges $\mathcal{P}$ to open some random leaves with its parents, checks consistency.



Security

- After sending ℓ_ε $\mathcal{P}$ is "committed" to all labels.
- Assume $\mathcal{P}$ "cheated" on some labels $\ell_i \neq H(\ell_{p_1}, \ldots, \ell_{p_d})$. call those and all labels below them "bad"., let α be fraction of bad labels.
- $\mathcal{P}$ must have made $(1-\alpha)T$ sequential queries (in RO).
- $\mathcal{P}$ will fail if challenged to open a bad label $\Rightarrow$ will succeed on t random labels with prob. $\leq (1-\alpha)^t$.

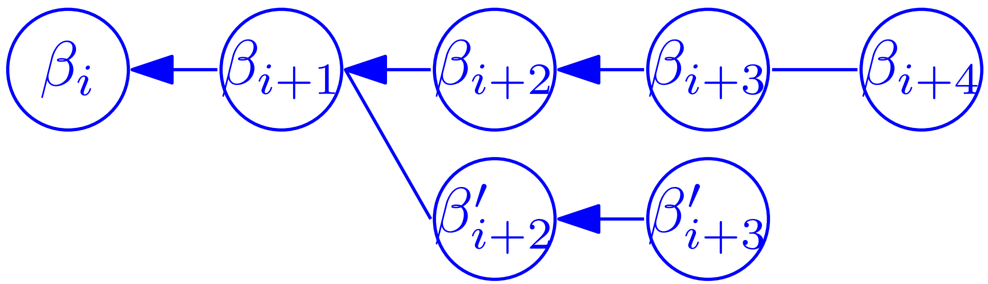


https://chia.net/

Bitcoin Mining Recap

Bticoin block $\beta_i = (\tilde{\beta}_i, \phi)$, $\tilde{\beta}_i = (i, H(\beta_{i-1}), pk, \tau_i)$ contains

- Transactions τ_i (consistent with chain so far).
- pk (block-reward and transactions fees go to pk)
- hash of previous block $H(\beta_{i-1})$
- PoW ϕ where $H(\phi, \tilde{\beta}_i) \leq treshhold$



Bitcoin mining:

Init: sample signature key-pair (pk, sk)

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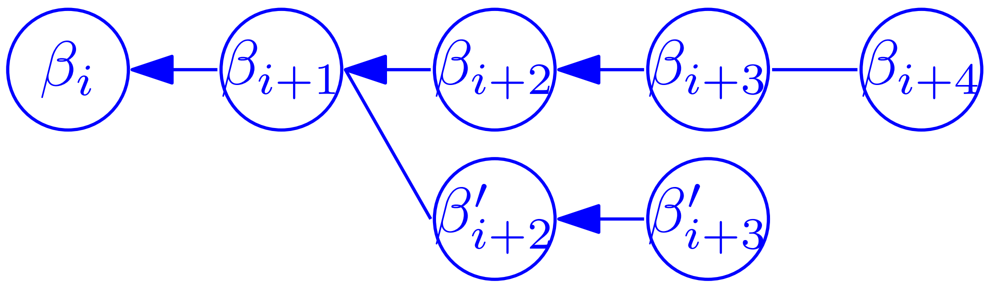

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Chia Building Blocks

Unique Digital Signatures

$$\forall m \ : \ \Pr[\mathsf{Sig.verify}(pk, m, \phi) = \mathsf{accept}] = 1$$

where $(pk, sk) \leftarrow \mathsf{Sig.keygen}$; $\phi \leftarrow \mathsf{Sig.sign}(sk, m)$

Unique : $(\mathsf{Sig.verify}(pk, m, \phi) = \mathsf{Sig.verify}(pk, m, \phi') = \mathsf{accept}) \Rightarrow (\phi = \phi')$

Unique & Signed Proofs of Space

$$S \leftarrow \mathsf{PoSpace.init}(pk, N)$$

$$\forall pk, N \ : \ \mathsf{PoSpace.verify}(c, \mathsf{PoSpace.prove}(S, pk, c)) = \mathsf{accept}$$

Weakly Unique : $\mathbb{E}_c \left[\{ \sigma \ : \ \mathsf{PoSpace.verify}(pk, c, \sigma) = \mathsf{accept} \} \right] = 1$

Signed : $\sigma = (\sigma', \mathsf{Sig.sign}(sk, \sigma'))$

Unique & Publicly Verifiable Proofs of Sequential Work

$$\forall t, c \ : \ \mathsf{PoSW.verify}(c, t, \mathsf{PoSW.prove}(c, t)) = \mathsf{accept}$$

$\mathsf{PoSW.prove}(c, t)$ should take almost sequential time t to compute

Unique : $(\mathsf{PoSW.verify}(c, t, \tau) = \mathsf{PoSW.verify}(c, t, \tau') = \mathsf{accept}) \Rightarrow \tau = \tau'$

Blockchain from Proofs of Space and Time

Space Miners (Farmers)

Initialization: $(pk, sk) \leftarrow$ Sig.KeyGen, $\Sigma \leftarrow$ PoSpace.Init(pk, N)

Mining: When new longest chain with head β_i observed:

compute $\phi \leftarrow$ PoSpace(Σ, c) for challenge $c := H(i, \beta_i, \tau_i, pk)$

gossip ϕ and define "quality" of ϕ as $q(\phi) := H(\phi)$.

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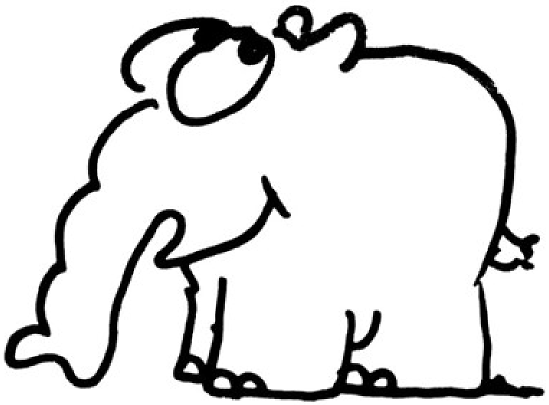
Time "Miners"

If PoSpace PoSpace ϕ observed, start computing

$\tau \leftarrow$ PoSW(challenge $= \phi$, time $= q(\phi) \cdot$ hardness parameter)

ONLY IF (given local view) this will be the first PoSW to finalize a block at this level.

Gossip τ once finished.



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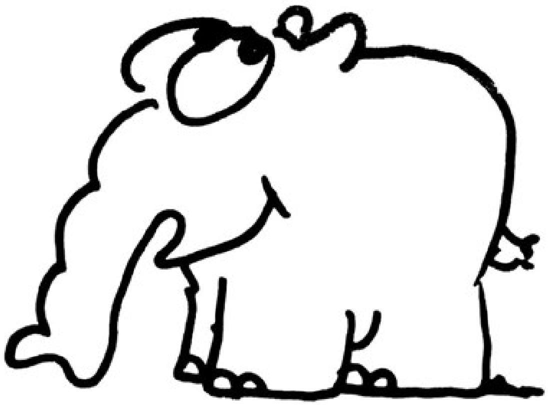

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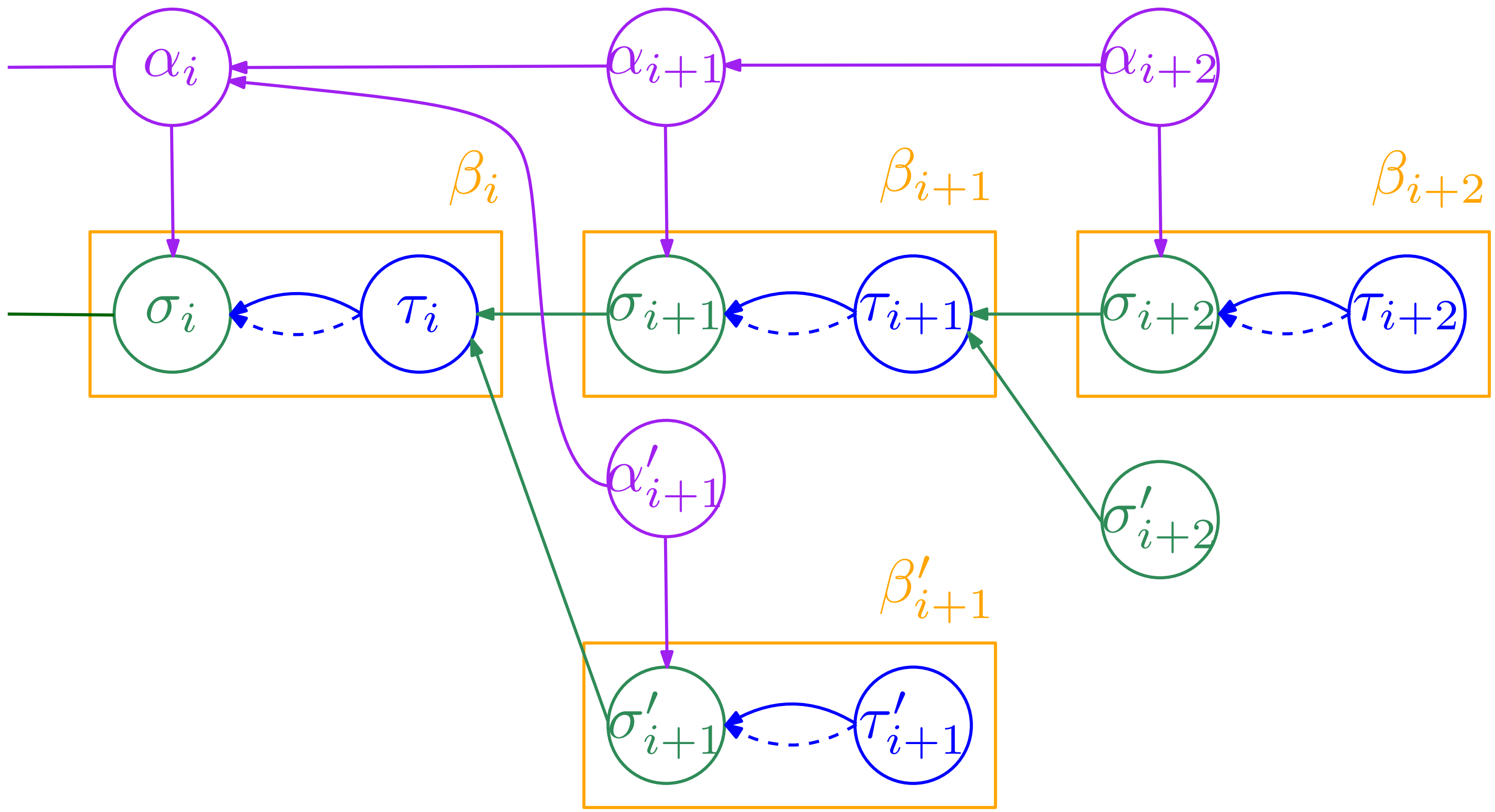
separate proofs from everything "graindable", Chia block format (next slide) kills the problem!

The Chia Block Format & (Non-)Grinding



A full block $\gamma_i = (\beta_i, \alpha_i)$ contains
$\beta_i = (i, (pk_i, \sigma_i), \tau_i)$ and $\alpha_i = (\phi_i, data_i)$

1. PoSpace.verify$(pk_i, H(\tau_{i-1}), \sigma_i, N) = 1$
2. PoSW.verify$(c, t, \tau_i) = 1$ where $c = H(\sigma_i)$, $t = 0.H(\sigma_i) \cdot T$
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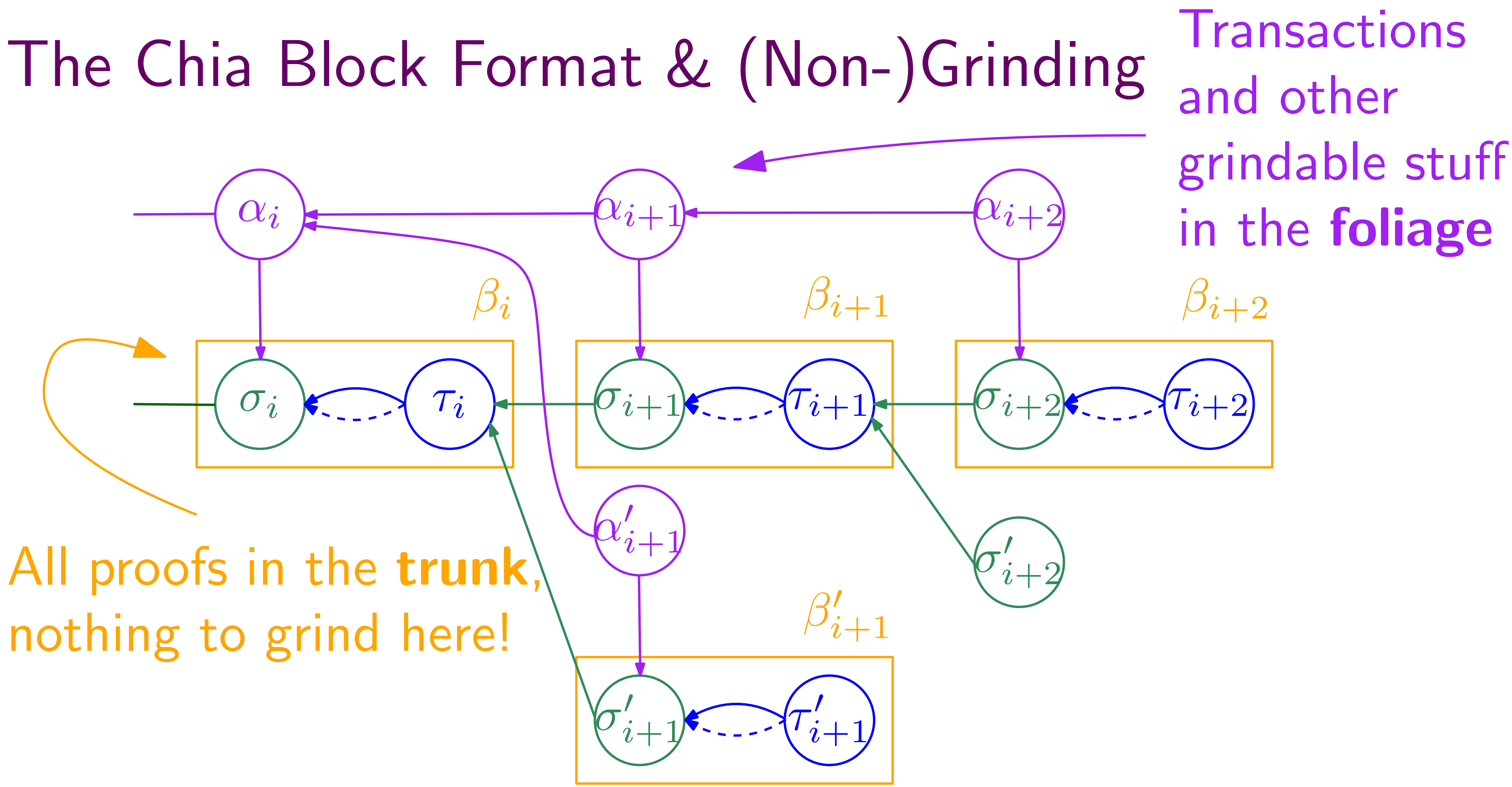

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Analysing Chain Growth

- h honest miners, each has one unit of space.
- adversarial miner with m units of space.
- every unit of space for every challenge gives a proof of quality uniform in $[0, 1]$.
- to finalize a proof of quality α takes time α (all PoSW equally fast).
- adversary can run infinite number of PoSW.
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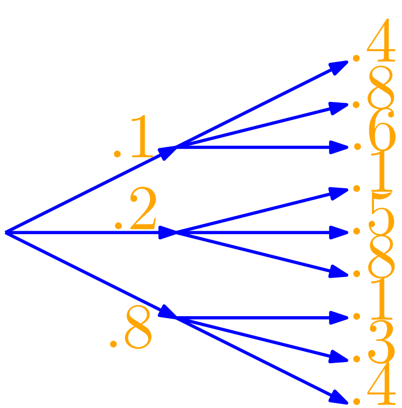
- Label every edge with random value from $[0,1]$.
- Random Variable $C^{\ell}_{\kappa,h}$ is length of shortest path we find when always following the κ best edges from root to a leave.



$C^{2}_{1,3} = .5$

$C^{2}_{\infty,3} = .3$

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- Label every edge with random value from $[0,1]$.
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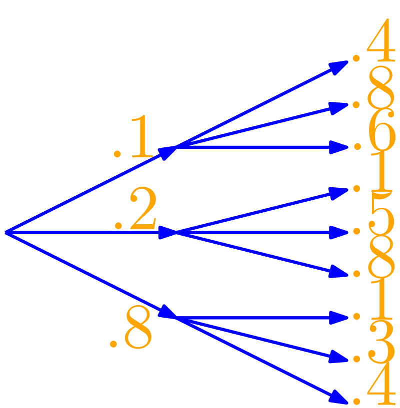


$C^2_{1,3} = .5$

$C^2_{\infty,3} = .3$

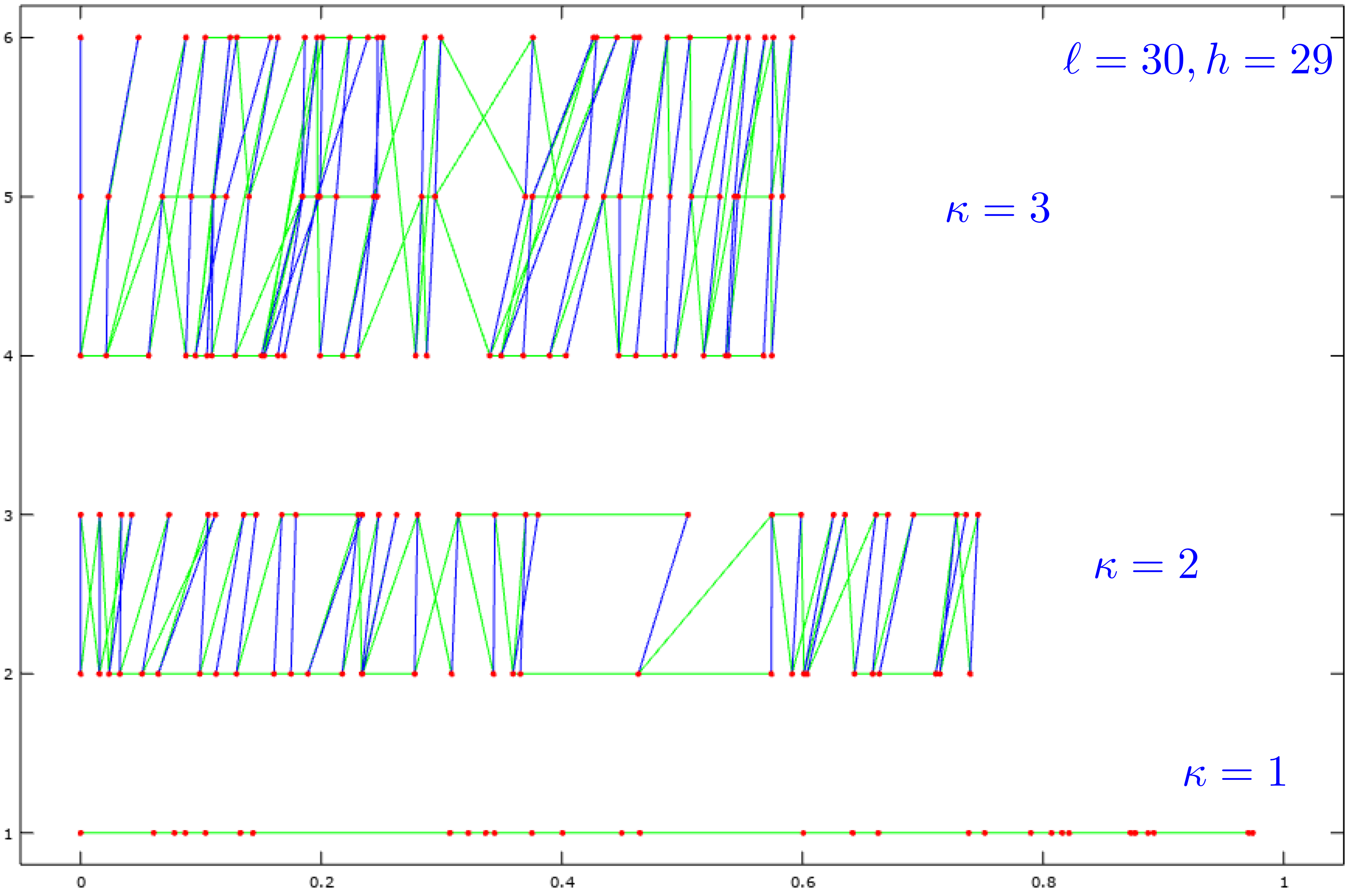
- $C^{\ell}_{\kappa,h}$ is expected time h honest miners need to grow chain of length ℓ.
- $C^{\ell}_{\infty,m}$ is expected time adversary controlling m space needs to grow chain of length ℓ.

Pseudocode For Sampling $C^{\ell}_{\kappa,h}$

Algorithm 1 sample $C^{\ell}_{\kappa,h}$

Input: κ, ℓ, h
$s[1,\ldots,\kappa] = 0$ ▷ initially we have κ paths of length 0
for $i = 1$ to ℓ **do** ▷ sample ℓ steps
    for $j = 1$ to κ **do** ▷ extend each of the κ states...
        for $k = 1$ to h **do** ▷ by h values...
            $p[j,k] = s[j] + \text{rand}([0,1])$ ▷ chosen uniform from $[0,1]$
        end for
    end for
    $z = \text{sort}(p[1,1],\ldots,p[\kappa,h])$ ▷ sort the $\kappa \cdot h$ values
    $s = z[1,\ldots,\kappa]$ ▷ new state are the κ shortest paths
end for
Return min(s)

Simulation of $C^{\ell}_{\kappa,h}$



What we know about $C^{\ell}_{\kappa,h}$

1. $C^{\ell}_{\kappa,h}$ is expected time h honest miners need to grow chain of length ℓ <u>without adversarial interference</u>
2. **No Slowdown Lemma**: an adversary with unbounded space and parallelism (but which cannot break the underlying signature scheme) cannot slow down the rate at which this chain grows.
3. We know exact expectation for $\kappa = 1$
$$E[C^{\ell}_{1,h}] = \frac{\ell}{h+1}$$
4. We can lower bound for $\kappa = \infty$
$$E[C^{\ell}_{\infty,h}] \geq \frac{\ell}{h+1} \cdot \frac{1}{e}$$

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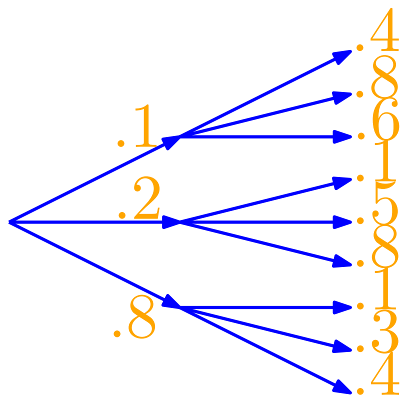
(Weak) Chain Quality Lemma: If $m < h/e$ (m space controlled by adversary, h honest space) then the fraction of honestly mined blocks is > 0.

Proof Sketch

- We can lower bound for $\kappa = \infty$

$$E[C^{\ell}_{\infty,h}] \geq \frac{\ell}{h+1} \cdot \frac{1}{e}$$



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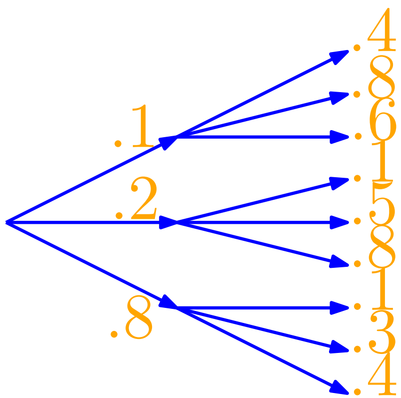

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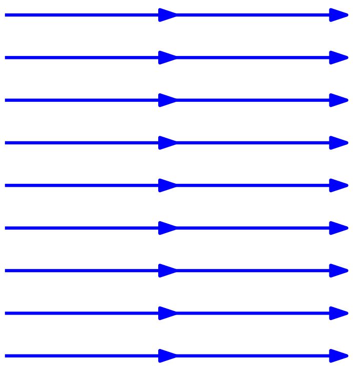
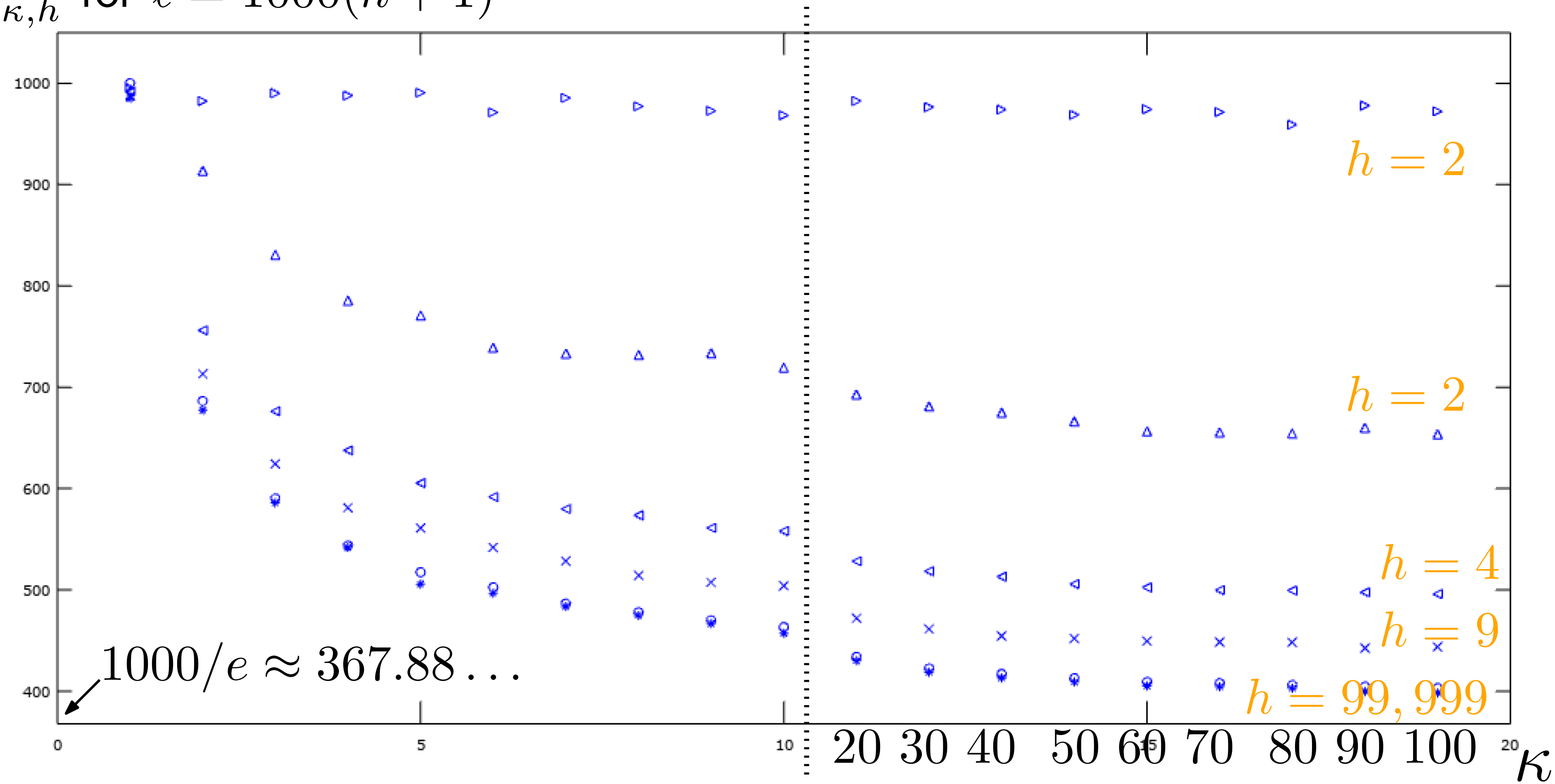


- Instead of analyzing shortest path in h-ary tree of depth ℓ, consider h^ℓ independent paths, prove that this tilts the bound in right direction.
- (Chernoff) Show that probability that any of those is shorter than x is $\ll \frac{1}{h^\ell}$.
- (Union Bound) Whp. all h^ℓ of them are shorter than x.

Choosing κ

$C^{\ell}_{\kappa,h}$ for $\ell = 1000(h+1)$



κ :	1	2	3	4	5	6	7	8	9
$C^{\ell}_{1,h}/C^{\ell}_{\kappa,h}$:	1	1.45	1.69	1.83	1.93	1.99	2.05	2.09	2
$e_\kappa \approx \frac{e}{C^{\ell}_{1,h}/C^{\ell}_{\kappa,h}}$:	2.71	1.86	1.60	1.47	1.40	1.36	1.32	1.29	1

Pseudocode

Space Miner Pseudocode (1/2)

Algorithm 2 SpaceMiner.init

Global Parameters: N

$\mathcal{C} \leftarrow$ Chain.init ▷ extract view from network

$(pk, sk) \leftarrow$ Sig.keygen ▷ generate a signature key pair

$S \leftarrow$ PoSpace.init(N, pk). ▷ run PoSpace initialisation with space N and identity pk to get a file S of size $|S| = N$.

Initalize a vector pos_count to all 0 ▷ see Remark **??**

Output:

$(pk, sk), S,$ pos_count ▷ State for SpaceMiner.mine

$\mathcal{C}$ ▷ State for Chain.update

Algorithm 3 SpaceMiner.loop

loop

  Wait for block(s) Γ to be received from the network

  $(\Gamma_f, \Gamma_n) \leftarrow$ Chain.update(Γ)

  $\forall \gamma \in \Gamma_f \ :$ SpaceMiner.mine(γ) ▷ Algorithm 4

end loop

Space Miner Pseudocode (2/2)

Algorithm 4 SpaceMiner.mine

Global Parameters: κ
Input: $\gamma_i = (\beta_i = (i, \sigma_i, \tau_i), \alpha_i)$. $\triangleright$ finalized, fresh & valid block for slot i
State: (pk, sk), S, pos_count
if pos_count$(i) = \kappa$ **then** $\triangleright$ already generated κ PoS for slot i
    return without output
end if
pos_count$(i) \leftarrow$ pos_count$(i) + 1$
$\sigma_{i+1} \leftarrow$ PoSpace.prove$(S, pk, \mathsf{H}(\tau_i))$ $\triangleright$ produce PoSpace
Generate data$_{i+1}$ $\triangleright$ application specific
$\phi_{i+1} \leftarrow$ Sig.sign$(sk, (\alpha_i, \sigma_{i+1}, \text{data}_{i+1})$ $\triangleright$ signature for signature chain

Chain.update$((i+1, \sigma_{i+1}), \alpha_{i+1} = (\phi_{i+1}, \text{data}_{i+1}))$ $\triangleright$ Cf. §**??**

Time Miner Pseudocode (1/3)

Algorithm 5 TimeMiner.init

$\mathcal{C} \leftarrow$ Chain.init ▷ extract view from network
Initalize a vectors finalized and running to all 0
Output:
finalized, running ▷ State for TimeMiner.mine/finalized/runPoSW
$\mathcal{C}$ ▷ State for Chain.update

Algorithm 6 TimeMiner.loop

loop
    Wait for block(s) Γ to be received from the network
    $(\Gamma_f, \Gamma_n) \leftarrow$ Chain.update(Γ)
    $\forall((i,\sigma),\alpha) \in \Gamma_n$: TimeMiner.mine(i,σ) ▷ Algorithm 7
    $\forall((i,\sigma,\tau),\alpha) \in \Gamma_f$: TimeMiner.finalized(i) ▷ Algorithm 9
end loop

Algorithm 7 TimeMiner.mine

Global Parameters: T, κ
Input: $\beta_i = (i, \sigma_i)$ ▷ non-finalized, fresh & valid block for slot i received
State: finalized, running
if finalize$[i] = \kappa$ **then** ▷ already finalized κ blocks for this slot
    return with no output
end if
$t := 0.\mathsf{H}(\sigma_i) \cdot T$ ▷ time required to finalize this block
if finalize$[i]$ + running$[i] < \kappa$ **then** ▷ $< \kappa$ proofs finalized or running
    start thread TimeMiner.runPoSW$(i, \mathsf{H}(\sigma_i), t)$ ▷ to finish at time now $+ t$
    running$[i]$ = running$[i] + 1$
end if
if finalize$[i]$ + running$[i] = \kappa$ **then** ▷ exactly κ proofs finalized or running
    if the slowest PoSW for slot i will finish at time $> t +$ now **then**
        abort the thread of this PoSW
        start thread TimeMiner.runPoSW$(i, \mathsf{H}(\sigma_i), t)$
    end if
end if

Time Miner Pseudocode (3/3)

Algorithm 8 TimeMiner.runPoSW

State: finalized, running
Input: $i, (c, t)$
$\tau_i \leftarrow$ PoSW(c, t) ▷ start PoSW, if not aborted will output proof τ_i in time t
finalized$[i] =$ finalized$[i] + 1$
running$[i] =$ running$[i] - 1$
Chain.update(τ_i)

Algorithm 9 TimeMiner.finalized

State: finalized, running
Input: i ▷ fresh, valid & finalized block for slot i was received
if running$[i] > 0$ and running$[i]$+finalized$[i] = \kappa$ **then**
    abort the thread TimeMiner.runPoSW for slot i scheduled to finish last
    running$[i] =$ running$[i] - 1$
end if
finalized$[i] = \min\{$finalized$[i] + 1, \kappa\}$

Some Open Problems

- (PoSW) Construct unique proofs of sequential work without heavy crypto machinery (SNARKs).
- (PoS) Is there a proof of space with non-interactive initialization and (at least asymptotically) optimal bounds?
- (Analysis) Better chain quality, persistience etc. analysis? Can we say something about rational (not just honest) miners?