Lecture 12

Algorand

Proof-of-Stake
“Virtual Mining”

Proof of Stake

- Bitcoin uses proof of work to address sybil attacks and implement consensus
  - Philosophy: Chance of “winning” in a block mining round proportional to your (hash) computing power
- Proof of Stake: Addresses sybil attacks by requiring that participants must have some “stake” (i.e., money) in the system
  - Philosophy: Chance of winning in a round proportional to your current stake

(Potential) Advantages

- In Proof of Stake based cryptocurrency, users (who have money in the system) are the miners
- Environment friendly
- No ASIC advantage
- 51% (or higher) majority assumption potentially more realistic

51% attack prevention argument

The Bitcoin economy is smaller than the world

Wealth *outside* Bitcoin has to move *inside*

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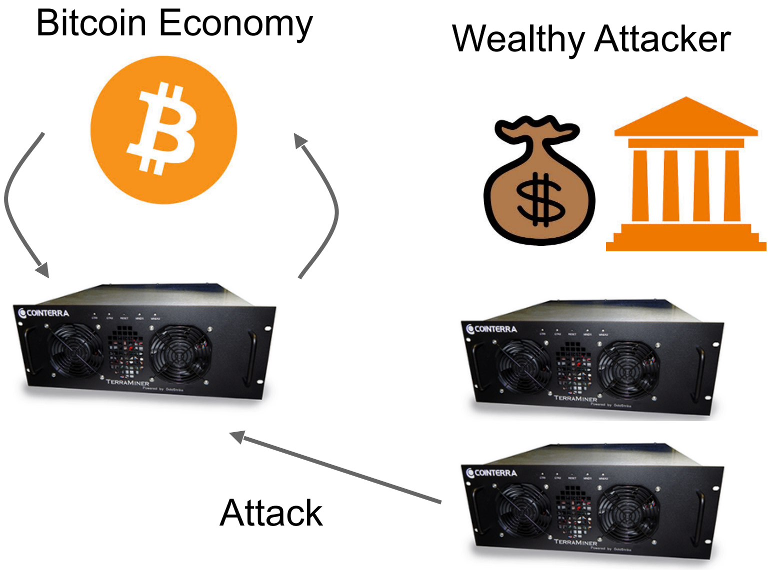

51% attack prevention argument

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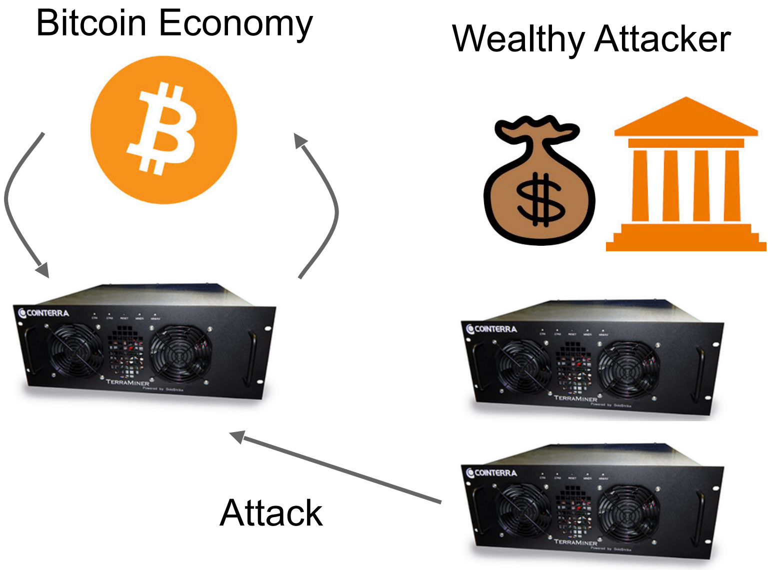

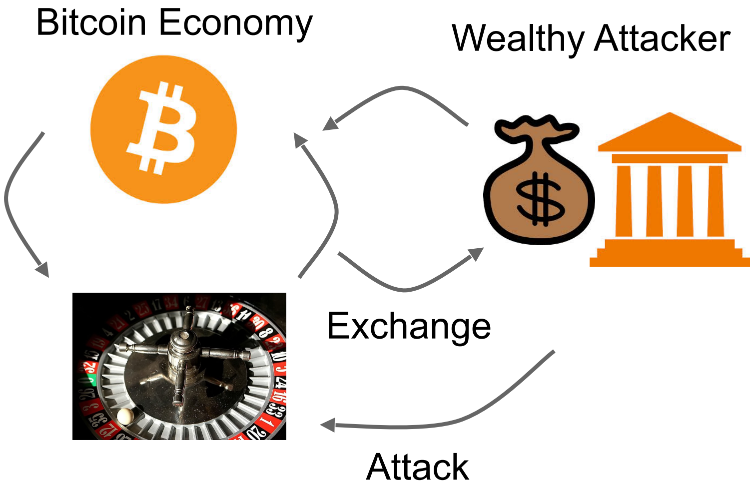

Examples of PoS based Cryptocurrencies

- Peercoin
- Blackcoin
- Nxt
- Neucoin
- …

PoS systems with security analysis

- Algorand [Gilad-Hemo-Micali-Vlachos-Zeldovich’17]
- Ourboros [Kiayias-Russel-David-Oliynykov’17]
- Snow white [Daian-Pass-Shi’17]
- …

Algorand: Main Highlights

- Proof of Stake based Cryptocurrency
- High throughput: ~1 min to confirm transactions vs an hour in Bitcoin
- Public ledger with low probability of forks
- Assumes 2/3-honest stake majority
- Uses a gossip communication protocol

Algorand: Main Highlights

- Adaptive adversary: May corrupt dynamically, as long as 2/3 majority assumption holds
- Achieves **Consistency** assuming “weak synchrony”
  - Network can be asynchronous for long bounded time period b, but then must have strong synchrony for short period s < b
- Achieves **Liveness** assuming “strong synchrony”
  - Most honest users (e.g., 95%) can send messages that will reach within a known time bound

Main Design Ingredients

- Users weighted by stake (to prevent sybil attacks)
- Builds on byzantine agreement (BA) protocol of Micali [ITCS’17] for consensus
- BA protocol executed between a small committee of users for scalability
- Committee chosen at random, using cryptographic techniques

Algorand Consensus: Main Highlights

- BA protocol in expectation terminates in only 4 steps (in “honest” case) or 13 steps (in “dishonest” case)
- Player replaceability: Players across different steps of BA protocol may not be the same
  - Possible because protocol does not require “private state”
- For each step, players chosen at random, non-interactively, in a “publicly verifiable” manner

Fast and Furious Byzantine Agreement

Micali [ITCS'17]

Byzantine Agreement

A protocol **P** is an (n,t) byzantine agreement protocol with soundness ***s*** if:

- for every value set V and adversary A who corrupts t out of n players,
- in an execution of **P** with A in which each player starts with value v_i in set V, each honest player halts with prob 1, outputting a value out_i, so as to satisfy, with prob ***s***, the following properties:

- **Agreement**: out_i = out_j for all honest players i and j
- **Consistency**: if for some v, $v_i = v$ for all honest players i, then $out_j = v$ for all honest players j

Binary BA vs Arbitrary value BA

- Binary BA: Input value set V is {0,1}
- [Turpin-Coan'84]: general reduction to convert binary BA into arbitrary value BA
  - assuming 2/3 honest majority
  - requires only two additional rounds of communication
- <u>This talk</u>: Focus on Binary BA

Why is Byzantine Agreement Hard?

- Protocol executed over point-to-point channels
- Adversarial parties may send different messages (including no message) to different honest parties
- BA over broadcast channels is trivial

Micali's Protocol: Main Intuition

Consider idealized Protocol P(r), where b_i is the initial input of party i:

- Each player i sends b_i to all other players
- A new random and independently selected bit c(r) appears in sky
- Player i updates bit b_i as follows:
  - If $\#_{i,r}(0) >= 2t+1$, set $b_i = 0$
  - If $\#_{i,r}(1) >= 2t+1$, set $b_i = 1$
  - Else, set $b_i = c(r)$

$\#_{i,r}$(b): Number of players from which i received b in "iteration" r

Quick Analysis

Assuming at least 2t+1 players are honest, P(r) achieves soundness 1/2

- If honest players start in agreement, then they remain in agreement
- If honest players do not start in agreement, then they end in agreement (on some bit) with probability 1/2

Implementing coin in sky using Crypto

Three Ingredients:

- **Unique Digital Signatures**: For every public key pk and message m, only one valid signature for m w.r.t. pk
  - Can be constructed from standard cryptographic assumptions
- **Hash function**: Modeled as a random oracle
- **Common random string R:** fixed at the start of the protocol execution, known to each party, and not controlled by adversary

Implementing coin in sky using crypto

ConcreteCoin(r): Each player i does the following,

- Send $v_i = SIG_i(R, r)$
- Compute m s.t. $H(v_m) <= H(v_i)$ for all i
- Set $c_i(r) = lsb(h)$, where $h = H(v_m)$

Think: What is the probability that $c_i(r) = c_j(r)$ for all honest i,j ?

Think: Why is $c_i(r)$ random?

Using ConcreteCoin(r)

Replacing coin in sky with ConcreteCoin(r) in P(r):

- If honest players start in agreement, then they remain in agreement
- If honest players do not start in agreement, then they end in agreement (on some bit) with probability 1/3
- Overall, the resulting protocol has soundness 1/3

Remaining Problem

Can we simply repeat the protocol indefinitely until agreement is reached?

- The honest players do not know that agreement is reached
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- Thus, the protocol may never terminate

Idea: Simply repeat say k = 200 times to ensure that agreement is reached, except with very small probability

Drawback: We waste many rounds since most times, agreement will be reached earlier

Micali’s Idea:

Protocol BBA* : It consists of sequential repetitions of P’(r), where each P’(r) consists of three correlated executions of P(r)

1. Execution of P(r) where c(r) = 0
2. Execution of P(r) where c(r) = 1
3. Execution of P(r) where c(r) is implemented via ConcreteCoin(r)

Note 1: In the first two executions, a party will terminate if it thinks agreement is reached

Note 2: While the number of repetitions of P’(r) are not fixed in advanced, the expected number of repetitions will be 3 (will follow from protocol analysis)

Notation:

1. A party i may at any point send special value b* (and HALT) meaning that in all future steps, other parties should think of i’s message as b
2. Counter γ which indicates how many times the 3-step loop has been executed. Initially set to 0
3. R denotes the common random string

PROTOCOL $BBA^\star$

(COMMUNICATION) STEP 1. [Coin-Fixed-To-0 Step] *Each player i propagates b_i.*

1.1 If $\#_i^1(0) \geq 2t+1$, then i sets $b_i = 0$, sends $0*$, outputs $out_i = 0$, and HALTS.

1.2 If $\#_i^1(1) \geq 2t+1$, then, then i sets $b_i = 1$.

1.3 Else, i sets $b_i = 0$.

(COMMUNICATION) STEP 2. [Coin-Fixed-To-1 Step] *Each player i propagates b_i.*

2.1 If $\#_i^2(1) \geq 2t+1$, then i sets $b_i = 1$, sends $1*$, outputs $out_i = 1$, and HALTS.

2.2 If $\#_i^2(0) \geq 2t+1$, then i set $b_i = 0$.

2.3 Else, i sets $b_i = 1$.

(COMMUNICATION) STEP 3. [Coin-Genuinely-Flipped Step] *Each player i propagates b_i and $SIG_i(R, \gamma)$.*

3.1 If $\#_i^3(0) \geq 2t+1$, then i sets $b_i = 0$.

3.2 Else, if $\#_i^3(1) \geq 2t+1$, then i sets $b_i = 1$.

3.3 Else, letting $S_i = \{j \in N : m_i^3(j) = SIG_j(R, \gamma)\}$,
i sets $b_i = c_i^{(\gamma)} \triangleq \texttt{lsb}(\min_{j \in S_i} H(SIG_i(R, \gamma)))$; increases γ_i by 1; and returns to Step 1.

Analysis

Claim 1: If at start of an execution of step 3, no player has halted and agreement has not been reached, then with prob 1/3, players will be in agreement at the end of the step

Analysis

Claim A: If at start of an execution of step 3, no player has halted and agreement has not been reached, then with prob 1/3, players will be in agreement at the end of the step

Proof: Consider 5 cases:

1. All honest i update b_i according to 3.1
2. All honest i update b_i according to 3.2
3. All honest i update b_i according to 3.3
4. Some honest i update b_i according to 3.1, others according to 3.3
5. Some honest i update b_i according to 3.2, others according to 3.3

Proof: Consider 5 cases:

1. All honest i update b_i according to 3.1
   - Agreement holds on 0
2. All honest i update b_i according to 3.2
   - Agreement holds on 1
3. All honest i update b_i according to 3.3
   - Agreement holds on c
4. Some honest i update b_i according to 3.1, others according to 3.3
   - Agreement on 0 reached with prob ½ (assuming c_i's are same)
5. Some honest i update b_i according to 3.2, others according to 3.3
   - Agreement on 1 reached with prob ½ (assuming c_i's are same)

Overall, when m is honest, agreement is reached with probability at least 1/2. m is honest with prob 2/3 (which means c_i's are same), so overall agreement prob is 1/3

Analysis (contd.)

Claim B: If at some step, agreement holds on bit b, then it continues to hold on bit b

Analysis (contd.)

Claim B: If at some step, agreement holds on bit b, then it continues to hold on bit b

Proof: If agreement held at the start of step, then all honest parties send bit b, which means $\#_i(b) >= 2t+1$

Analysis (contd.)

Claim B: If at some step, agreement holds on bit b, then it continues to hold on bit b

Proof: If agreement held at the start of step, then all honest parties send bit b, which means $\#_i(b) >= 2t+1$

Claim C: If at some step, a player i halts, then agreement will hold at the end of the step

Analysis (contd.)

Claim B: If at some step, agreement holds on bit b, then it continues to hold on bit b

Proof: If agreement held at the start of step, then all honest parties send bit b, which means $\#_i(b) >= 2t+1$

Claim C: If at some step, a player i halts, then agreement will hold at the end of the step

Proof: WLOG, suppose i halts in Coin-Fixed-To-0 step. Since $\#_i(0) >= 2t+1$, at least t+1 honest players sent 0. Thus, $\#_j(0) >= t+1$ for every other honest j. If $\#_j(0) >= 2t+1$, then j sets $b_j=0$ in step 1.1, else it sets $b_j=0$ in step 1.3. (<u>Main point: step 1.2 cannot be executed</u>)

Analysis (contd.)

Property 1: Consistency (if initial bit b for all honest players, then out_i=b)

Proof: Easily follows from Substep 1.1 or 2.2 (depending upon whether starting input was 0 or 1)

Property 2: Agreement (out_i = out_j for all honest i,j)

Proof: Follows from Claims A, B and C

Algorand using Byzantine Agreement

Main idea

- Users weighted by stake (to prevent sybil attacks)
- For every block generation round, a small committee of users is chosen at random, using crypto, based on user weights
- One of the committee members who has the highest “priority” proposes a block
- The committee then runs a BA protocol to reach consensus on the proposed block

Verifiable Random Functions

- On any input x, VRFsk(x) outputs (hash,proof)
- hash is uniquely determined given sk and x but indistinguishable from random to anyone who does not know sk
- Given pk and proof, anyone can check that hash corresponds to x

Notation

- W: total amount of currency units
- t: threshold, denoting expected number of users selected
- p: t/W
- w_i: stake/money of user i
- B(k;w,p): Prob of getting k successes in w trials, where prob of success in each trial is p (Binomial distribution)

$$\sum_{k=0}^{w} B(k;w,p) = 1$$

- Division of interval [0,1) into multiple consecutive internals

$$I_j = \Big[\sum_{k=0}^{j} B(k;w,p), \sum_{k=0}^{j+1} B(k;w,p) \Big)$$

Cryptographic Sortition

Sortition(sk,seed,p,w):

- $VRF_{sk}(seed) \rightarrow (hash, proof)$
- $j \rightarrow 0$
- While $\frac{hash}{2^{hashlen}} \notin \left[\sum_{k=0}^{j} B(k;w,p), \sum_{k=0}^{j+1} B(k;w,p) \right)$

  j++
- Return (hash,proof,j)

Cryptographic Sortition

- Any user can check on its own whether it was selected by using its sk, and then send a proof to others for the same
- User's whose hash is highest is the "block proposer"
- Users then run BA to reach consensus on proposed block

Consensus

- For each step of the consensus protocol, a different set of users is chosen (using cryptographic sortition algorithm)
- All users can passively participate in the protocol (by listening to the gossip network), and whenever selected for a step, they send a message based on what they heard so far on the network
- BA protocol has player replaceability; therefore using different users in each step is possible

Security Challenges

- For BA to have any security, a high majority of players must be honest
- Why can’t adversary simply corrupt all the committee members?
- Main Idea: Committee members for any step are disclosed only when they send their respective messages. If adversary corrupts now, its too late. The messages are already sent.

Security Challenges (contd.)

- How to select the threshold t?

- Use a threshold such that:
  - #good > threshold: for agreement
  - ½ #good + #bad <= threshold: to avoid forks

Other Points:

- The seed (used in sortition) has to be chosen carefully. Initially, it is set to be a common random string; later, for each round r, seed is determined from seed for round r-1 by using $VRFs_k$ of the block proposer in round r-1

- What are the chances of forks? – Forks can happen with some probability (if network has weak synchrony), but a recovery process can be used to eliminate fork assuming there is a strong synchrony period, using same BA procedure

Research Challenges

- Can we reduce the 2/3-honest majority assumption to 1/2 ?
- Consistency in Algorand requires strong synchrony periods interspersed between weak synchrony periods. Can this be relaxed?
- Liveness in Algorand requires strong synchrony. Can this be relaxed?