Chosen-Ciphertext Security (II)

CS 601.442/642 Modern Cryptography

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Recall: Chosen-Ciphertext Attacks (CCA)

- Adversary can make decryption queries over ciphertext of its choice
- **CCA-1**: Decryption queries only before challenge ciphertext query
- **CCA-2**: Decryption queries before and after challenge ciphertext query
- No decryption query c should be equal to challenge ciphertext c^*

Last time: Construction of CCA-1 secure PKE

Today: Construction of CCA-2 secure PKE

Recall: CCA-2 Security

$\mathbf{Expt}_{\mathcal{A}}^{\mathsf{CCA2}}(b, z)$:

- $\mathsf{st} = z$
- $(pk, sk) \leftarrow \mathsf{Gen}(1^n)$
- Decryption query phase 1 (repeated poly times):
  - $c \leftarrow \mathcal{A}(pk, \mathsf{st})$
  - $m \leftarrow \mathsf{Dec}(sk, c)$
  - $\mathsf{st} = (\mathsf{st}, m)$
- $(m_0, m_1) \leftarrow \mathcal{A}(pk, \mathsf{st})$
- $c^* \leftarrow \mathsf{Enc}(pk, m_b)$
- Decryption query phase 2 (repeated poly times):
  - $c \leftarrow \mathcal{A}(pk, c^*, \mathsf{st})$
  - If $c = c^*$, output reject
  - $m \leftarrow \mathsf{Dec}(sk, c)$
  - $\mathsf{st} = (\mathsf{st}, m)$
- Output $b' \leftarrow \mathcal{A}(pk, c^*, \mathsf{st})$

CCA-2 Security (contd.)

Definition (IND-CCA-2 Security)

A public-key encryption scheme $(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ is IND-CCA-2 secure if for all n.u. PPT adversaries $\mathcal{A}$, there exists a negligible function $\mu(\cdot)$ s.t. for all auxiliary inputs $z \in \{0,1\}^*$:

$$\left|\Pr\left[\mathbf{Expt}_{\mathcal{A}}^{\mathsf{CCA2}}(1, z) = 1\right] - \Pr\left[\mathbf{Expt}_{\mathcal{A}}^{\mathsf{CCA2}}(0, z) = 1\right]\right| \leqslant \mu(n)$$

How to Construct CCA-2 secure Encryption?

- Why doesn't a CCA-1 secure scheme also achieve CCA-2 security?
- **Main problem:** An adversary may be able to modify the challenge ciphertext to obtain a new ciphertext of a *related* plaintext and then request its decryption in the second decryption query phase of IND-CCA-2. E.g., the adversary may be able to "maul" an encryption of x into an encryption of $x \oplus 1$ without knowing x. This is called *malleability attack*

  Think: Is the IND-CPA PKE scheme based on trapdoor permutations that we studied in the class *malleable*?
- **Solution Strategy:** Ensure that adversary's decryption query is "independent" of (and not just different from) the challenge ciphertext. That is, make the encryption *non-malleable*

CCA-2 Secure Public-Key Encryption

The first construction of CCA-2 secure encryption scheme was given by Dolev-Dwork-Naor.

Ingredients:

- An IND-CPA secure encryption scheme $(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$
- An adaptive NIZK proof $(\mathsf{K}, \mathsf{P}, \mathsf{V})$
- A strongly unforgeable one-time signature (OTS) scheme $(\mathsf{Setup}, \mathsf{Sign}, \mathsf{Verify})$. Assume, wlog, that verification keys in OTS scheme are of length n.

Construction

Construction of $(\mathsf{Gen}', \mathsf{Enc}', \mathsf{Dec}')$**:**

$\mathsf{Gen}'(1^n)$: Execute the following steps

- Compute CRS for NIZK: $\sigma \leftarrow \mathsf{K}(1^n)$
- Compute $2n$ key pairs of IND-CPA encryption scheme: $\left(pk_i^j, sk_i^j\right) \leftarrow \mathsf{Gen}(1^n)$, where $j \in \{0,1\}$, $i \in [n]$.
- Output $pk' = \left(\left\{pk_i^0, pk_i^1\right\}, \sigma\right)$, $sk' = \left(sk_1^0, sk_1^1\right)$.

Construction (contd.)

$\mathsf{Enc}'(pk', m)$: Execute the following steps

- Compute key pair for OTS scheme: $(SK, VK) \leftarrow \mathsf{Setup}(1^n)$.
- Let $VK = VK_1, \ldots, VK_n$. For every $i \in [n]$, encrypt m using $pk_i^{VK_i}$ and randomness r_i: $c_i \leftarrow \mathsf{Enc}\left(pk_i^{VK_i}, m; r_i\right)$
- Compute proof that each c_i encrypts the same message: $\pi \leftarrow \mathsf{P}(\sigma, x, w)$ where $x = \left(\left\{pk_i^{VK_i}\right\}, \{c_i\}\right)$, $w = (m, \{r_i\})$ and $R(x, w) = 1$ iff every c_i encrypts the same message m.
- Sign everything: $\Phi \leftarrow \mathsf{Sign}(SK, M)$ where $M = (\{c_i\}, \pi)$
- Output $c' = (VK, \{c_i\}, \pi, \Phi)$

Construction (contd.)

$\mathsf{Dec}'(sk', c')$: Execute the following steps

- Parse $c' = (VK, \{c_i\}, \pi, \Phi)$
- Let $M = (\{c_i\}, \pi)$
- Verify the signature: Output $\perp$ if $\mathsf{Verify}(VK, M, \Phi) = 0$
- Verify the NIZK proof: Output $\perp$ if $\mathsf{V}(\sigma, x, \pi) = 0$ where $x = \left(\left\{pk_i^{VK_i}\right\}, \{c_i\}\right)$
- Else, decrypt the first ciphertext component: $m' \leftarrow \mathsf{Dec}\left(sk_1^{VK_1}, c_1\right)$
- Output m'

Security (Intuition)

Consider decryption queries after adversary receives challenge ciphertext C^*:

- Let $C \neq C^*$ be a decryption query
- If verification key VK in C and verification key VK^* in challenge ciphertext C^* are same, then we can break the strong unforgeability of OTS
- If different, then VK and VK^* differ in at least one position $\ell \in [n]$:
    - Answer decryption query using the secret key $sk_\ell^{VK_i}$.
    - Don't need to know the secret keys $sk_i^{VK_i^*}$ for $i \in [n]$
    - Reduce to IND-CPA security of underlying encryption scheme

Security (Hybrids)

- H_0: (Honest) Encryption of m_0
- H_1: Compute CRS σ in public key and proof π in challenge ciphertext using NIZK simulator
- H_2: Choose VK^* in the beginning during Gen'
- H_3: For any decryption query $C = (VK, \{c_i\}, \pi, \Phi)$:
  - If $VK = VK^*$ and $\mathsf{Verify}(VK, (\{c_i\}, \pi, \Phi), \Phi) = 1$, then abort
  - Else, let $\ell \in [n]$ be such that VK^* and VK in c differ at position ℓ. Set $sk' = \left\{ sk_i^{\overline{VK}_i^*} \right\}$, $i \in [n]$, where $\overline{VK}_i^* = 1 - VK_i^*$. Decrypt c by decrypting c_ℓ (instead of c_1) using $sk_\ell^{\overline{VK}_\ell^*}$.
- H_4: Change every c_i^* in C^* to encryption of m_1
- H_5: Compute CRS σ in public key and proof π in challenge ciphertext honestly. This experiment is same as (honest) encryption of m_1.

Indistinguishability of Hybrids

- $H_0 \approx H_1$: ZK property of NIZK
- $H_1 \approx H_2$: Generating VK^* early or later does not change the distribution
- $H_2 \approx H_3$: We argue indistinguishability as follows:
  - First, we argue that probability of aborting is negligible. Recall that $c \neq c^*$ by the definition of CCA-2. Then, if $VK = VK^*$, it must be that $(\{c_i\}, \pi, \Phi) \neq (\{c_i^*\}, \pi^*, \Phi^*)$. Now, if $\mathsf{Verify}(VK, (\{c_i\}, \pi), \Phi) = 1$, then we can break strong unforgeability of the OTS scheme.
  - Now, conditioned on not aborting, let ℓ be the position s.t. $VK_\ell \neq VK_\ell^*$. Note that the only difference in H_2 and H_3 in this case might be the answers to the decryption queries of adversary. In particular, in H_2, we decrypt c_1 in c using $sk_1^{VK_1}$. In contrast, in H_3, we decrypt c_ℓ in c using $sk_\ell^{\overline{VK_\ell^*}}$. Now, from soundness of NIZK, it follows that except with negligible probability, all the c_i's in c encrypt the same message. Therefore decrypting c_ℓ instead of c_1 does not change the answer.

Indistinguishability of Hybrids (contd.)

- $H_3 \approx H_4$: IND-CPA security of underlying PKE
- $H_4 \approx H_5$: ZK property of NIZK

Combining the above, we get $H_0 \approx H_5$.